

Voluntary Disclosure with Evolving News

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Abstract

We study a dynamic voluntary disclosure setting where the manager's information and the firm's value evolve over time. The manager is not limited in her disclosure opportunities but disclosure is costly. The results show that the manager discloses even if this leads to a price *decrease* in the current period. The manager absorbs this price drop in order to increase her option value of withholding disclosure in the future. That is, by disclosing today she can improve her continuation value. The results provide a number of novel empirical predictions, which, among others, include that firms who are more timely with their disclosures are more likely to be met with a negative market reaction.

1 Introduction

A firm's informational environment is generally characterized by continuous inflows of new information. For example, advances made through research and development could lead to patents and eventual product launches. Similarly, the firm's direction or strategy may change based on current or projected industry conditions. Firm managers must continuously decide whether to release such new information to investors or the public, even if there is no legal obligation to do so. Accordingly, the process of price discovery for the firm typically involves voluntary information disclosures by firm executives regarding the firm's present situation.

Casual observation and findings in the empirical literature further motivate us to study voluntary disclosure in the presence of evolving news. A few studies have documented that firms' disclosure decisions vary with their performance (e.g., Kothari et al. (2009), Sletten (2012)). Moreover, while the extant theoretical literature has shown that firms release information to improve their market valuations, voluntary corporate disclosures which lead to price decreases or a *negative* market reaction are pervasive in practice. Indeed, numerous studies have documented that firms often voluntarily release information which is met with a negative market reaction (e.g., Skinner (1994), Soffer et al. (2000), Matsumoto (2002), Baik and Jiang (2006), Anilowski et al. (2007), and Kross et al. (2011), among others). The goal

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of this paper is to investigate the theoretical underpinnings of firm disclosure behavior in the presence of evolving information, and to find an endogenous explanation for this anomalous yet enduring empirical regularity.

Our setting is one where the manager privately observes the firm's fundamental value in each of two periods. The manager may choose to disclose, at a cost (such as a proprietary or certification cost), her private information of the firm's value in each period. The model has two key components. The first component, which is the central nuance of this paper, is that the firm's value between periods evolves according to a simple, correlated process. This allows disclosure in the present period to influence market beliefs in the future. Moreover, the manager must take into consideration potential changes in the firm value when deciding her disclosure policy in the present period. The second feature of our model is that, at the beginning of the second period, the firm distributes its cash flows as dividends. This ultimately serves as a signal of the underlying firm value given non-disclosure in the previous period.

Our main result shows that first-period disclosure by the manager whose value is at the disclosure threshold *always results in a price decrease* relative to non-disclosure (Theorem 1). Stated differently, the threshold-type manager receives a higher price by keeping quiet in the first period than from disclosing. This occurs because early disclosure increases the option value of withholding information in the future. In this sense, early disclosure *generates* a real option for the manager. Specifically, by disclosing in the first period, this raises the second-period disclosure threshold and helps to protect the manager if firm value declines in the future. However, if it turns out to be the case that firm value has improved in the second period, the manager can simply disclose this value to the market. This leads the manager to disclose *excessively* in equilibrium.

We note that the economic forces driving the main result are in contrast to extant dynamic voluntary disclosure models. Previous models of dynamic disclosure generally involve a manager who can generate a real option from *concealing* information in the present period (e.g., Acharya et al. (2011), Guttman et al. (2014)). These models are dynamic but entail a constant firm value. In contrast, in our setting we find that the manager can improve her option value of disclosure in the future by *revealing* information in the current period. Hence, we find that allowing firm value to change over time leads to significantly different disclosure incentives and behavior. We note that this improved option value from early disclosure prevails even when the manager has a countervailing incentive to withhold information, such as in the form of exogenous positive news which may overstate the firm's value (as in Acharya

et al. (2011)).

The manager faces two conflicting real options when making her disclosure decision in the first period. On one hand, withholding disclosure allows for the possibility that realized cash flows may overstate the firm profitability, thus resulting in a more favorable price. On the other hand, the firm value may decline in the future. As we show, early disclosure gives the manager more flexibility to conceal future bad news. The evolving nature of the firm leads the option value generated from disclosure to dominate the real option from keeping quiet. Consequently, the manager is inclined to disclose even if this hurts the first-period price. The result follows from three key equilibrium properties.

The first two properties concern the unique equilibrium disclosure threshold in the second period, given that the manager did not disclose in the previous period. First, we find that there is limited upside of the impact from strong dividends (and thus public news) on the second-period disclosure threshold. While positive news always improves the second-period disclosure threshold, it is still the case that the manager would have disclosed in the first period if her private information was sufficiently high. The upside of strong positive news is thus mitigated by the manager's non-disclosure in the first period. Likewise, as the second equilibrium property, we find that the second-period threshold increases in the first-period threshold at a rate less than the autocorrelation (and hence less than one). While a higher first-period threshold implies that the second-period firm value must also be high, increases in the first-period threshold do not fully "carry over" to the second period. The reason is that, upon non-disclosure in the first period, the market updates its beliefs regarding the evolved second-period value using the conditional expectation for *the set of all non-disclosing types*. The market thus determines the *average* evolved firm value, which leads the second-period threshold to increase in the first-period threshold at a slower rate.

Third, we find that, for the threshold-type manager, the second-period disclosure threshold is *always lower* if the manager had concealed information in the first period than if she had disclosed information. This implies that the threshold-type manager's non-disclosure price in the second period is strictly higher if she had disclosed her private information in the first period. This occurs since the manager is pooled with the other first-period non-disclosing firms, and since it is unlikely that the signal from realized dividends will push the market expectation of the evolved value to be at least the first-period threshold level. Conversely, by disclosing, the second period threshold increases in the disclosed value at a rate *equal* to the autocorrelation (in contrast to the second property above). Hence, by disclosing in the present period, the manager can positively influence the market's belief in

the following period by raising that period’s disclosure threshold. In other words, disclosure in the present period increases the option value of withholding disclosure in the *following* period. These three equilibrium properties lead the manager to reveal her information in the first-period, even if she endures a strictly lower market valuation by doing so (relative to concealing information).

The results of the model provide a rich set of novel empirical predictions. Our results concerning the price decrease upon disclosure occurs for some firms who disclose in the first period, or who are more *timely* with their voluntary disclosures (as defined by Skinner (1997)). The results imply that firms who are more timely with their disclosures are more likely to be met with a market reaction which is negative. Hence, the results identify a salient feature—the timeliness of disclosure—as an important determinant of the market reaction. This is perhaps surprising, as we would not expect that a manager who is more transparent, in the sense of disclosing information in a more timely manner, to be “punished” by the market.

Additionally, the results show how positive skewness can arise following joint releases of disclosure and public (news) announcements, which is in dissonance to previous voluntary disclosure models. The results of Acharya et al. (2011) imply negative skewness when public news announcements are followed by disclosure. In contrast, the results of our model imply that returns can exhibit positive skewness when disclosures are made after public news announcements. This occurs since the manager begins disclosure in the second period (after non-disclosure in the first period) when the fundamental value exceeds the market belief based on the public signal (dividends). When the public signal is high, this implies that the underlying fundamental is also high. However, due to the evolving nature, the fundamental may improve in a greater magnitude than the public signal, thus crossing the threshold and compelling the manager to disclose. This kind of positive skewness is not possible in Acharya et al. (2011) as the manager always preempts good news announcements in their setting.

Furthermore, the model provides predictions concerning disclosure timeliness as related to firm properties. The results of the model imply that firms are more timely (or have less delay) with their voluntary disclosures when: *(i)* there is greater information asymmetry between the firm and the market; *(ii)* the firm’s cash flows have relatively high autocorrelation; *(iii)* there is *less* uncertainty regarding the firm’s future value; and *(iv)* the firm has relatively high disclosure costs. These predictions, as well as others, are discussed more thoroughly in Section 4.

1.1 Related Literature

Grossman (1981) and Milgrom (1981) first studied static voluntary disclosure and showed that, in the absence of disclosure costs, the agent always reveals her private information in equilibrium.¹ Jovanovic (1982) and Verrecchia (1983) extend this result by examining a static disclosure setting where information release is costly. We build from these studies and incorporate disclosure costs as the basic friction which prevents unraveling.

Our model is related to the literature on dynamic voluntary disclosure. Einhorn and Ziv (2008) and Marinovic and Varas (2016) also consider settings in which the firm value evolves over time. Einhorn and Ziv (2008) examine a repeated game in which disclosures made in the present affect the market’s perception that a future-period manager has received material information. Importantly, Einhorn and Ziv (2008) assume that the manager’s private information (cash flows) is always made common knowledge at the end of each period, which removes strategic considerations regarding future market beliefs of firm value. Moreover, Einhorn and Ziv (2008) assume that the manager is purely myopic (or short-lived) in the sense that she only seeks to maximize the firm’s price in the current period, whereas we assume the manager prefers to maximize both short and long-term prices (though we analyze the purely myopic case to establish a benchmark result).

Marinovic and Varas (2016) investigate a continuous-time, binary disclosure model where the firm’s value fluctuates according to a Markov process. They assume that the firm faces a risk of litigation when bad news is withheld, and thus *not* disclosing is costly. The model here differs from Marinovic and Varas (2016) primarily in that litigation risk is a fundamental feature of their setting. In contrast, we investigate dynamic disclosure without imposing an exogenous cost of withholding disclosure.

Our setting is also related to a stream of literature in dynamic disclosure where the manager may choose the timing of her disclosure, but the underlying value of the firm does not change. Acharya et al. (2011) investigate a model where an exogenous correlated signal is publicly revealed at a known time. Their results show clustering of announcements in bad times, where the manager discloses immediately if the public signal is sufficiently low. Relatedly, Guttman et al. (2014) consider a two-period model where the manager may receive two independent signals of the firm value in each period. They show that the market value

¹This is commonly referred to as the “unraveling result.” Grossman (1981) and Milgrom (1981) show that, if disclosure is costless, then another friction, such as lack of common knowledge that the agent received information, must be present in order to prevent unraveling. This latter friction was first explored by Dye (1985) and Jung and Kwon (1988). Voluntary disclosure models typically include either disclosure costs or uncertainty regarding the agent’s information endowment to prevent unraveling.

of the firm is higher if one signal is disclosed in the second period rather than if one signal is disclosed in the first period. The main difference in our setting and Acharya et al. (2011) and Guttman et al. (2014) is that we assume that firm value changes over time. Moreover, a driving force in both Acharya et al. (2011) and Guttman et al. (2014) is that the manager can improve her option value by concealing information, whereas we find the opposite force.

Shin (2003, 2006) considers disclosure in a binomial setting where projects may either succeed or fail. The equilibrium constructed is one where the manager follows a “sanitation strategy” where only project successes are disclosed in the interim period. In a similar vein, Goto et al. (2008) extend Shin’s (2003) framework to include risk-averse investors. The present setting varies from Shin (2003, 2006) and Goto et al. (2008) in that we are more focused on intertemporal considerations of voluntary disclosure.

Another stream in the disclosure literature considers voluntary disclosure in settings where the manager has additional private information concerning her type. This allows disclosure to entail an additional signaling value. Teoh and Hwang (1991) consider a binary disclosure setting where firms, in addition to value, have private type information that cannot be revealed. They find that high-type firms may disclose bad news, whereas low-type firms do not. Beyer and Dye (2012) examine a setting where the manager may either be forthcoming or strategic, and find that the strategic manager may disclose bad news in order to build a reputation for being forthcoming. Our setting differs from these models as the value structure is interdependent between periods and the manager does not have additional private information. This paper is also related to models where disclosure is not verifiable. In particular, Stocken (2000) considers a repeated game of unverifiable disclosure and shows that the equilibrium entails truthful disclosure by the sender. This implies that the sender discloses bad news in order to build credibility investors (the receiver). In contrast, our model features verifiable disclosure and the private signal realizations of the sender are correlated over time.

The paper proceeds as follows. Section 2 outlines the model, while Section 3 presents the main results. Section 4 considers comparative statics and empirical implications. In Section 5 we examine an extended setting where the disclosure cost may have a long-term impact on firm profitability, as well as an extended setting where discounting is present. The final section concludes. All proofs are relegated to the Appendix.

2 Model of Dynamic Disclosure

Our baseline setting is a discrete, two-period model. This parsimonious setting captures the main insight and clearly illustrates the economic forces driving the results. The firm generates a cash flow s_t in each period ($t = 0, 1$). We assume that a risk-neutral firm manager privately observes the mean of cash flows, y_0 , in time 0, and that (s_0, y_0) is a bivariate normal variable with zero mean and correlation $\rho > 0$.² Specifically, we assume that $\sigma_s = \sigma_y/\rho$, where σ_s and σ_y are volatility parameters of s_0 and y_0 , respectively. We note that the results of the model are not qualitatively affected if $\sigma_s \neq \sigma_y/\rho$. We assume this for ease of exposition so that the mean of s_0 can simply be represented by y_0 . Thus, conditional on y_0 , the cash flow s_0 is given by

$$s_0 = y_0 + w_0,$$

where w_0 is normally distributed with mean zero and variance $(1 - \rho^2)\sigma_s^2$.³ This may be interpreted such that y_0 is the profitability of the underlying fundamental and w_0 is an industry or macroeconomic shock to cash flows.

Upon learning y_0 , the manager may disclose the information to the market, in which case it becomes public information. We assume that disclosure is verifiable in the sense that the manager cannot manipulate the disclosed value. Disclosure is also assumed to be costly for the firm, where $c > 0$ is incurred upon disclosure. The disclosure cost can be interpreted, for instance, as a certification cost, whereby the manager must hire an auditor to certify that the information disclosed is factual. Alternatively, the disclosure may be relevant to proprietary information that could be adopted by competitor firms. Indeed, a wide-scale survey of executives at large public firms finds evidence consistent with this view: “Nearly three-fifths of survey respondents agree or strongly agree that giving away company secrets is an important barrier to more voluntary disclosure” (Graham et al. (2005, p. 62)).⁴

After the manager makes her disclosure decision at time 0, the market, composed of risk-neutral investors, determines the date 0 price of the firm. Then, s_0 is realized and the cash flow net of the disclosure cost (if the manager had disclosed) is distributed to shareholders.

We allow the mean of cash flows to evolve in the sense that new developments may have

²The zero-mean assumption on (s_0, y_0) is without loss of generality.

³Including noise in the cash flow prevents the market from filtering out the mean cash flow perfectly upon observing dividends in the event that the manager does not disclose.

⁴Empirical evidence of proprietary costs has been documented by Berger and Hann (2007), Bens et al. (2011), and Ellis et al. (2012). Other costs of disclosure—arranging press releases, conference calls, and meetings with analysts—are nontrivial and impose time costs on the manager and monetary costs on the firm.

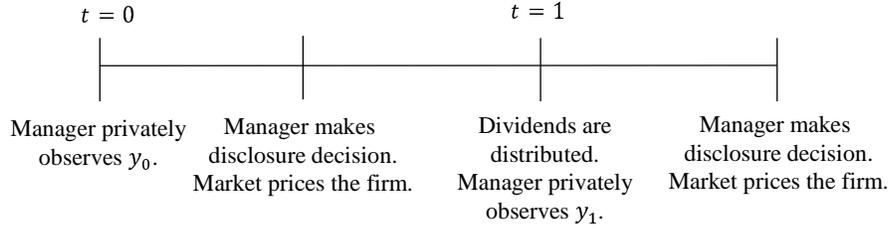


Figure 1: Timeline.

occurred between time 0 and time 1 such that the underlying firm profitability improves or declines. This is captured by the time 1 mean cash flow, given by:

$$y_1 = \kappa y_0 + \eta,$$

where $\kappa \in (0, 1]$ denotes autocorrelation of the mean cash flow, and η is a normal variable with mean zero and variance σ_η^2 . We assume that η and (s_0, y_0) are independent. Regardless of the time 0 disclosure decision, the manager privately observes y_1 . The distribution of η is common knowledge. We assume that the second-period cash flow s_1 is simply given by $s_1 = y_1$.⁵ At time 1, after observing y_1 the manager may disclose y_1 to the market. The market then determines the time 1 price of the firm after observing the manager's disclosure decisions at time 0 and at time 1, and the cash flow in the first period. A timeline of model is presented in Figure 1.

The cum dividend price in each period satisfies:

$$\begin{aligned}
 p_0 &= E[s_0 - cd_0 + s_1 - cd_1 | \Omega_0] \\
 p_1 &= E[s_1 - cd_1 | \Omega_1],
 \end{aligned}$$

where d_t is an indicator equal to one if the manager discloses in time t and zero otherwise. Ω_t denotes the market's information set at time t ; Ω_0 includes d_0 and the manager's disclosure strategy, and Ω_1 includes s_0, d_0, d_1 , and the manager's disclosure strategy.

The manager is risk neutral and thus her objective is to maximize the sum of the current market price and the expected market price:

$$\max_{d_0, d_1} p_0 + E[p_1 | y_0].$$

⁵Allowing (s_1, y_1) to be bivariate normal would not qualitatively affect the results.

The manager is concerned with the market price at all times as it is often the case that an executive’s compensation includes bonuses which are determined in part by share price.⁶ For simplicity, we assume that there is no discounting by the manager or the market. We discuss the quantitative effects of discounting in Section 5.1.

3 Equilibrium

In this section, we characterize the equilibrium of our baseline setting. Before we begin the analysis of the dynamic model, we first analyze the myopic benchmark, which will be helpful in the ensuing analysis.

3.1 Myopic benchmark

In this special case, we assume that the manager is myopic and simply aims to maximize the price of the current period. This is a variant of the static costly disclosure model studied by Jovanovic (1982) and Verrechia (1983). The main difference is that the non-myopic market must still take into account the expected cash flow of the second period when pricing the firm in the first period. This setting provides a point of comparison with the fully dynamic main model and also allows us to more precisely convey how evolving news affects the non-myopic manager’s disclosure strategy.

As the game ends after the second period, the manager’s disclosure strategy in the second period is identical in both the myopic and non-myopic setting. Therefore, in this benchmark case we focus on the manager’s disclosure strategy in the first period.

Since the price, and thus the manager’s payoff, from disclosure is increasing in her private information y_0 , any equilibrium strategy must be a *disclosure threshold strategy*. We let x^* denote the equilibrium myopic disclosure threshold in the first period, defined whereby the manager discloses if and only if $y_0 \geq x^*$. For ease of the analysis, we introduce the function $\delta(x)$, which is the negative expectation of a standard normal variable conditional on being truncated above at x :

$$\delta(x) = -E[\xi | \xi < x] = \phi(x)\Phi(x)^{-1}, \tag{1}$$

where ξ is a standard normal variable, and $\phi(\cdot)$ and $\Phi(\cdot)$ are the density function and distribution function of the standard normal distribution, respectively.

⁶A similar assumption regarding the manager’s utility function is made in previous dynamic voluntary disclosure models, such as Acharya et al. (2011) and Guttman et al. (2014).

If the threshold-type manager (i.e., $y_0 = x^*$) discloses at time 0, then the time 0 price $p_0^d(x^*)$ is given by

$$p_0^d(x^*) = E[s_0 - c + s_1 - cd_1 | \Omega_0^d] = (1 + \kappa)x^* - c(1 + \alpha_d), \quad (2)$$

where Ω_0^d is the information available to the market when the manager discloses, and $\alpha_d = E[d_1 | \Omega_0^d]$ is the probability of disclosure at time 1 given disclosure at time 0. In the next section, we show that this probability is independent of the first-period threshold.

If the threshold-type manager does not disclose at time 0, the time 0 price is given by

$$\begin{aligned} p_0^n(x^*) &= E[s_0 + s_1 - cd_1 | \Omega_0^n] \\ &= (1 + \kappa)E[y_0 | y_0 < x^*] - c\alpha_n(x^*) = -(1 + \kappa)\sigma_y \delta \left(\frac{x^*}{\sigma_y} \right) - c\alpha_n(x^*), \end{aligned} \quad (3)$$

where Ω_0^n is the information available to the market when the manager does not disclose, and $\alpha_n(x^*) = E[d_1 | \Omega_0^n]$ is the probability of disclosure at time 1 given non-disclosure at time 0. In the next section, we show that this probability depends on the time 0 threshold.

Since the myopic manager is indifferent between disclosure and non-disclosure at x^* , we see that x^* is given by the following condition:

$$c = (1 + \kappa)\sigma_y v \left(\frac{x^*}{\sigma_y} \right) + c(\alpha_n(x^*) - \alpha_d), \quad (4)$$

where $v(x) = x + \delta(x)$. The left-hand side is the expected total disclosure cost when the manager discloses at time 0. The right-hand side is the size of undervaluation plus the expected disclosure cost at time 1. The myopic disclosure threshold x^* provides a useful benchmark which is frequently used for comparison and in the analysis of the dynamic non-myopic case. The following proposition establishes existence and uniqueness of this threshold:

Proposition 1 *There exists a unique myopic disclosure threshold x^* such that the manager discloses if and only if $y_0 \geq x^*$.*

In the Appendix, we also show that $v(x)$ is non-negative and increasing in x , which implies that $\delta(x)$ is decreasing in x . This property will be helpful in the following analysis.

3.2 Second-Period Disclosure

We now turn to our main setting where the manager considers both period's prices in the first period. In solving the equilibrium strategy for the dynamic setting, we begin with the manager's decision at time 1 after she has learned y_1 . There are two possible paths the manager could have taken prior to time 1: disclosure or non-disclosure in time 0. Below, we analyze each case separately.

Suppose that the time 0 disclosure decision can be characterized by some threshold x_0 , such that the manager discloses her private information only if $y_0 \geq x_0$. For now, we keep the time 0 disclosure threshold exogenous and fixed as we analyze the second-period disclosure decision (we endogenize the time 0 decision in the following section). At date 1, the manager will choose to disclose her private information if and only if the expected cash flow at date 1 exceeds the market price absent disclosure plus the disclosure cost.

Time 1 disclosure decision when $d_0 = 1$

First, we consider the case where the manager had disclosed her private information at time 0, i.e., $d_0 = 1$. The manager will also disclose at time 1 if her payoff from disclosure exceeds that from remaining quiet:

$$y_1 - c > E[y_1 | \Omega_1^d],$$

where $\Omega_1^d = \{y_0, y_1 < x_d(y_0)\}$ is the information available to the market when the manager had disclosed and she is not disclosing currently, and $x_d(y_0)$ denotes the disclosure threshold at date 1 given that the disclosed value at date 0 is y_0 . In the case where the manager had previously disclosed the mean cash flow at time 0, the realization of cash flow s_0 does not deliver additional information to the market that is relevant to y_1 . The equilibrium threshold satisfies:

$$x_d(y_0) = c + E[y_1 | y_0, y_1 < x_d(y_0)] = \kappa y_0 + \eta^*,$$

where η^* solves

$$c = \eta^* - E[\eta | \eta < \eta^*] = \sigma_\eta v\left(\frac{\eta^*}{\sigma_\eta}\right), \quad (5)$$

and $v(\cdot)$ is defined as in the previous section.

Proposition 2 *There exists a unique equilibrium disclosure threshold satisfying equation (5).*

The existence and uniqueness of η^* can be shown similarly as in Proposition 1. Based on

this threshold, we have that the ex ante likelihood of disclosure at time 1 given that there was disclosure in time 0 is given by $\alpha_d = \Phi(-\eta^*/\sigma_\eta)$.

The threshold $x_d(y_0)$ has an intuitive interpretation; when the realized η is sufficiently high, this pushes the new firm value to be above $x_d(y_0)$ and induces disclosure by the manager. Moreover, the disclosure of y_0 in the first period can raise the option value of disclosure in the second period, as the disclosure threshold $x_d(y_0)$ is increasing in y_0 . Hence, when the manager discloses a high y_0 in the first period, she has positively influenced the market's belief of y_1 through her disclosure, which carries through as a comparatively higher valuation in the absence of disclosure in the second period. In this sense, early disclosure of positive news in the first period can *increase* the option value of disclosure in the second period. We note that this is a key distinction between the present framework and the unchanging environment of Acharya et al. (2011), as early disclosure in the latter setting eliminates the option value.

Notably, the disclosure threshold at time 1 increases at a rate κ when the disclosed value at time 0 increases by one. As the firm's fundamental value follows a mean-reverting process with autocorrelation κ , we see that disclosure at time 0 has full impact on $x_d(y_0)$, and thus the option value upon initial disclosure. As we will see in the following section, this property becomes a salient factor that influences the time 0 disclosure decision.

Time 1 disclosure decision when $d_0 = 0$

We now consider the case where the manager did not disclose at date 0, i.e., $d_0 = 0$. In this case, the manager will disclose at date 1 if and only if

$$y_1 - c > E[y_1 | \Omega_1^n], \quad (6)$$

where $\Omega_1^n = \{s_0, y_0 < x_0, y_1 < x_n(x_0, s_0)\}$ is the information available to the market when the manager has not disclosed in both periods, and $x_n(x_0, s_0)$ denotes the disclosure threshold at date 1 given non-disclosure, realized cash flows s_0 , and disclosure threshold x_0 at date 0.

Since the manager did not disclose in time 0, the market does not observe y_0 . However, the distribution of dividends (which is equal to cash flows s_0) by the firm provides investors with information regarding y_0 . As we will see, this signal gives the manager a potential benefit from withholding disclosure in the first period. For example, a positive industry or macroeconomic shock w_0 to cash flows may lead investors to overstate the value of y_0 after observing dividends s_0 . Consequently, this may result in a more generous price in the

second period absent disclosure through inflated market beliefs of y_1 . Hence, this effectively provides the manager with a real option of withholding disclosure in the first period.

Upon observing the first-period cash flows, the market believes that y_0 is normally distributed with mean $f s_0 = \frac{\rho \sigma_y}{\sigma_s} s_0$ and variance $\sigma_z^2 = (1 - \rho^2) \sigma_y^2$, i.e., $y_0 = f s_0 + z$, where z is a normal variable with mean zero and variance σ_z^2 . The information that the manager had not previously disclosed additionally implies that the random variable z is truncated above at $g = x_0 - f s_0$. Thus, the market price at date 1 absent disclosure is given by

$$E[y_1 | \Omega_1^n] = \kappa f s_0 + E[\kappa z + \eta | z < g, \kappa z + \eta < x_n(x_0, s_0) - \kappa f s_0].$$

From equation (6), we find that the equilibrium threshold satisfies:

$$x_n(x_0, s_0) = \kappa f s_0 + \epsilon^*(g),$$

where $\epsilon^*(g)$ solves

$$c = \epsilon^*(g) - E[\kappa z + \eta | z < g, \kappa z + \eta < \epsilon^*(g)]. \quad (7)$$

Thus, $\epsilon^*(g)$ is the mean-adjusted disclosure threshold for the manager. The following result establishes existence and uniqueness of $\epsilon^*(g)$:

Proposition 3 *There exists a unique fixed point satisfying (7).*

We see that $x_n(x_0, s_0)$ depends on the realization of cash flows s_0 , as well as the manager's time 0 disclosure threshold, captured by the term $\epsilon^*(g)$. We first discuss the impact of the market's observation of cash flows s_0 given non-disclosure at $t = 0$. When s_0 is high, this has both a direct effect on $x_n(x_0, s_0)$ and an indirect effect through $\epsilon^*(g)$. First, a high s_0 raises the disclosure threshold $x_n(x_0, s_0)$, since s_0 and y_0 are positively correlated and y_t is autocorrelated. This implies that the manager's non-disclosure price in the second period is more favorable when higher cash flows are observed. This direct effect is reflected in the first term of $x_n(x_0, s_0)$. We see that the direct impact is stronger when s_0 is more informative (high f).

Second, we find that the direct effect of high cash flows is somewhat mitigated by the fact that the manager did not disclose in the first period, as captured through the indirect effect of s_0 on $\epsilon^*(g)$. Consider the case where a large s_0 is observed such that the difference between the posterior belief of y_0 and the first-period threshold ($f s_0 - x_0$) is large. Without considering information from $t = 0$, the market updates the expectation of y_0 to $f s_0$. However, investors must take into consideration the fact that the manager did not disclose in the first period,

and consequently must account for the value of cash flows relative to the threshold level of disclosure at time 0, x_0 . This implies that, even if period-one cash flows are very high, it is still the case that the manager's information at time 0 was not sufficiently positive to induce disclosure. This indirect effect is captured by the gap $g = x_0 - fs_0$, which affects $\epsilon^*(g)$. The following lemma allows us to more precisely see the effects of s_0 and x_0 on $x_n(x_0, s_0)$.

Lemma 1 $\epsilon^*(g)$ is increasing in g at a rate less than κ , i.e.,

$$0 < \frac{d\epsilon^*(g)}{dg} < \kappa.$$

Lemma 1 states that $\epsilon^*(g)$ is increasing in g , which implies that $\epsilon^*(g)$ is decreasing in s_0 . As mentioned previously, high cash flows can positively influence the market's belief, but the upside of a high s_0 is limited as a sufficiently high-type firm would have disclosed at time 0. Hence, $\epsilon^*(g)$ is decreasing in s_0 , which serves to mitigate the effect of s_0 on the threshold $x_n(x_0, s_0)$. However, the net effect of an increase in s_0 always results in an increase in $x_n(x_0, s_0)$. This can be seen from the property $-\kappa f < \frac{\partial \epsilon^*(g)}{\partial s_0} = -f \frac{d\epsilon^*(g)}{dg} < 0$, which implies that, when s_0 increases by one, $x_n(x_0, s_0)$ increases by less than κf . Hence, a high first-period cash flow is always beneficial, but this benefit is somewhat mitigated by the manager's non-disclosure in the first period.

Next, we discuss the impact of the first-period disclosure threshold x_0 on $x_n(x_0, s_0)$. Lemma 1 states that $\epsilon^*(g)$ is increasing in g , which implies that the threshold $x_n(x_0, s_0)$ is increasing in x_0 . This property is straightforward, as less disclosure at time 0 (higher x_0) means that, for the same value of s_0 , this is likely to be an indication of high y_0 and thus of high y_1 . This implies that the non-disclosure price at time 1 will be relatively higher as x_0 increases.

However, what is striking is that $\frac{d\epsilon^*(g)}{dg} < \kappa$, which indicates that an increase in x_0 by one results in an increase of $x_n(x_0, s_0)$ by *less* than the autocorrelation κ . This implies that the disclosure threshold in the first period does not fully "carry over" to the second period. First, note that an increase in the threshold x_0 overall improves the market's beliefs in the second period, but also increases the set of first-period non-disclosing firms. This latter effect puts an additional disadvantage from increasing the first-period threshold x_0 . More specifically, the conditional expectation of y_0 given the observed dividends and non-disclosure, $E[y_0 | s_0, y_0 < x_0]$, does not increase in line with increases in the threshold x_0 . Thus, the increase in the first-period threshold by a small amount, Δ , results in the market updating their beliefs of y_1 based on the fact that $y_0 \leq x_0 + \Delta$, and from the observed dividends s_0 . In this sense, the

market's belief of y_1 considers the evolution from $E[y_0|s_0, y_0 \leq x_0 + \Delta]$, which is an increase from $E[y_0|s_0, y_0 < x_0]$ by not more than Δ . Hence, the market is determining the *average* evolved firm value based on its information set, which implies that an increase in x_0 by Δ results in an increase of the non-disclosure price in the second period by less than $\kappa\Delta$.

We see that there is some limitation to the benefits of non-disclosure in the first period, as the threshold level does not fully carry over to the second period and increases in s_0 are mitigated by non-disclosure. While $x_n(x_0, s_0)$ increases in x_0 at a rate strictly less than κ , the threshold $x_d(y_0)$ increases in the disclosed value y_0 at a rate equal to κ . This difference (together with the fact that $0 < \partial x_n(x_0, s_0)/\partial s_0 < \kappa f$) is a significant driving force of the main result that we will see in the following section.

We now present an important equilibrium property which describes the difference in the threshold-type manager's behavior at time 1 depending on the disclosure history.

Lemma 2 *The threshold-type manager ($y_0 = x_0$) will begin to disclose at a lower value of y_1 in the second period if she had not disclosed at time zero than if she had disclosed, i.e., $x_n(x_0, s_0) < x_d(x_0) \equiv \kappa x_0 + \eta^*$. Moreover, we have that (i) $\epsilon^*(g) - \kappa g \rightarrow \eta^*$ and $\frac{d\epsilon^*(g)}{dg} \rightarrow \kappa$, as $g \rightarrow -\infty$, and (ii) $\epsilon^*(g) \rightarrow \bar{\epsilon}$ and $\frac{d\epsilon^*(g)}{dg} \rightarrow 0$, as $g \rightarrow \infty$, where $\bar{\epsilon}$ is defined in the Appendix.*

Lemma 2 indicates that, upon non-disclosure in $t = 0$, the threshold-type manager *always* begins disclosure at a lower realization of y_1 than if she had disclosed in $t = 0$. This implies that the threshold-type manager's second-period non-disclosure price is always *lower* if she had kept quiet in the first period rather than if she had disclosed y_0 . In other words, by disclosing in period one, the threshold-type manager can raise her non-disclosure price, and thus her option of keeping quiet, in the second period. This is perhaps surprising, as the result holds independent of the cash flows s_0 .

This equilibrium property occurs due to the evolving nature of the firm value. To more clearly see the intuition, consider the case where observed cash flows are sufficiently high such that the market assigns the highest possible value following non-disclosure in $t = 0$. As initial non-disclosure implies that $y_0 \leq x_0$, the market's belief of y_0 upon observing sufficiently high cash flows becomes x_0 . That is, under the best situation that the threshold-type manager can imagine, the market will assign a value that is identical to the threshold-type's y_0 . Thus, given any x_0 , a sufficiently high s_0 implies that whether or not the threshold-type manager discloses at time 0 does not deliver any additional information to the market. Consequently, in this extreme case, the second-period thresholds will be identical, $x_n(x_0, \infty) = x_d(x_0)$.

Now, as the value of the observed cash flows decreases, the market's belief of y_0 upon observing s_0 deviates further from the threshold-type's y_0 , as the market places greater

weight on the possibility that $y_0 < x_0$ upon observing an intermediate value of s_0 . This implies that the threshold-type manager ($y_0 = x_0$) becomes relatively more under-valued by the market as s_0 decreases. At the same time, the non-disclosure price in the second period decreases (given non-disclosure in $t = 0$). The manager with the threshold-type x_0 is thus relatively more inclined to disclose in the second period as she is unlikely to realize the benefits from an over-stated first-period cash flow s_0 .

In this sense, the market's belief of y_1 considers the evolution from $E[y_0|s_0; y_0 \leq x_0]$, or a value that is likely to be *less than* x_0 . Hence, the market is determining the *average* evolved firm value based on its information set, which implies that the market is, in expectation, assigning an evolved value that is less than the threshold-type's y_1 . Put differently, non-disclosure by the manager in the present period affects the market's belief of the future value. This is "costly" in the sense that a high-type manager may be leaving money on the table in *future* periods by not disclosing today. The manager can thus positively influence the market's future beliefs, and thus the non-disclosure price in the subsequent period, by disclosing today. In this light, the manager can *increase* her option value of non-disclosure tomorrow by not concealing information in the present period.

We next examine properties of the likelihood of disclosure in $t = 1$. Recall that $\alpha_n(x_0)$ denotes the manager's ex ante probability of disclosing in period two given that she did not disclose in period one, and α_d is the corresponding probability given that she disclosed in period one.

Lemma 3 *The ex ante likelihood of disclosure at time 1 given that there was non-disclosure in time 0 has the following properties: (i) $\alpha_n(x_0) \rightarrow \alpha_d$ and $\alpha'_n(x_0) < 0$ as $x_0 \rightarrow -\infty$, and (ii) $\alpha_n(x_0) \rightarrow \bar{\alpha}_n$ and $\alpha'_n(x_0) > 0$ as $x_0 \rightarrow \infty$, where $\bar{\alpha}_n$ is defined in the Appendix.*

Property (i) of Lemma 3 is intuitive; $x_0 \rightarrow -\infty$ implies that the manager always discloses in $t = 0$. Hence, the market's belief of the likelihood of disclosure at time 1 approaches α_d . Property (ii) similarly examines the disclosure likelihood as $x_0 \rightarrow \infty$, i.e., when the manager never discloses in $t = 0$. Intuitively, two separate effects occur as x_0 increases. First, the increased set of first-period non-disclosing types results in a higher threshold in the second period (Lemma 1). Thus, the ex ante likelihood of disclosure in the second period is decreasing as x_0 increases. Second, an increase in x_0 results in a larger set of non-disclosing period-one values that will ultimately disclose in the second period. This occurs since, as x_0 increases, a larger set of non-disclosing types are, *on average*, being under-valued in the second period. Recall from Lemma 1 that $x_n(x_0, s_0)$ does not increase in line with increases

in x_0 . This implies that some managers who previously had not disclosed in the first period are more inclined to disclose in the second period. As x_0 increases, we are increasing this set of managers and thus $\alpha_n(x_0)$ increases. Lemma 3 implies that the former effect dominates the latter one initially, but then the latter effect becomes dominant.

The analysis in the second-period disclosure decision shows that the manager must weigh two different real options. The first stems from the fact that the profitability changes over time—by disclosing today, the manager can increase the disclosure threshold, and thus her option value, in the second period. This option enhances the incentive for disclosure in the first period. The second real option arises from the noisy cash flow s_0 . The manager can keep quiet in the first period in order to take advantage of a potentially high cash flow. Conversely, this option strengthens the incentive for non-disclosure in the first period. These countervailing forces are salient in the analysis of the first-period disclosure decision.

3.3 First Period Disclosure

We now analyze the manager's time 0 disclosure decision. If the threshold-type manager ($y_0 = x_0$) discloses at time 0 ($d_0 = 1$), the price $p_0^d(x_0)$ in that period is given by equation (2). At date 1, depending on the new mean cash flow, the payoff to the manager is equal to either $y_1 - c$ if $y_1 > x_d(y_0)$, or $x_d(y_0) - c$ if $y_1 \leq x_d(y_0)$. Thus, the expected utility of the threshold-type manager upon initial disclosure is given by:

$$p_0^d(x_0) + E[y_1 - c + (x_d(y_0) - y_1)^+ | y_0 = x_0] = p_0^d(x_0) + \kappa x_0 - c + u_d. \quad (8)$$

The first term in the left-hand side of equation (8) is the manager's first-period payoff from disclosure, which is simply the time 0 market price. The second term is the manager's expected second-period payoff, which includes the option value of disclosure, given by:

$$u_d = E[(\eta^* - \eta)^+]. \quad (9)$$

Observe that equation (9) is similar to that of an American put option, where the manager can exercise the option to disclose when the realization of η exceeds η^* . Or, equivalently, the manager exercises the option to hide information when the realization of η is lower than the threshold.

Conversely, if the threshold-type manager does not disclose at time 0, the market price in that period, $p_0^n(x_0)$, is given by equation (3). At time 1, the market price is either $y_1 - c$ from disclosure or $x_n(x_0, s_0) - c$ from non-disclosure. Thus, the expected utility of the manager

upon non-disclosure in the first period is given by:

$$p_0^n(x_0) + E[y_1 - c + (x_n(x_0, s_0) - y_1)^+ | y_0 = x_0] = p_0^n(x_0) + \kappa x_0 - c + u_n(x_0),$$

where the option value upon non-disclosure in the first period, denoted by $u_n(x_0)$, is given as:

$$u_n(x_0) = E[(x_n(x_0, s_0) - \kappa x_0 - \eta)^+]. \quad (10)$$

Similar to equation (9), the above equation also resembles an American put option, where the manager exercises the disclosure option when the realization of η exceeds the threshold $x_n(x_0, s_0) - \kappa x_0$. The difference between the put option we have developed in equation (10) and the classic put option model is that the equivalent of the strike price in our put option is itself a random variable. Thus, we can clearly see that the manager does not disclose initially in hopes of taking advantage of either a high realization of cash flow s_0 , which increases the strike price, or a low realization of η , which decreases the mean cash flow. The equilibrium first-period disclosure threshold thus satisfies:

$$p_0^d = p_0^n(x_0) + u_n(x_0) - u_d. \quad (11)$$

We have two possible cases:

- Case 1: $p_0^n(x_0) < p_0^d(x_0)$. In this case, the market price upon disclosure at the first-period disclosure threshold is higher than the non-disclosure market price. In order for this to be the case, the value of the put option upon non-disclosure in time 0 must be higher than the value of the put option upon disclosure, i.e., $u_n(x_0) > u_d$. Hence, the option value of delay in the first period is sufficiently high such that the manager withholds disclosure comparatively more often in the first period. As a result, the price *increases* upon disclosure, as the manager bears additional undervaluation due to the put option from non-disclosure in time 0. This is similar to the excessive delay result presented in Proposition 4 of Acharya et al. (2011).
- Case 2: $p_0^n(x_0) > p_0^d(x_0)$. Here, the market price upon disclosure is below the non-disclosure market price in the first period. This occurs when the value of the put option upon non-disclosure is *lower* than the value of the put option upon initial disclosure, i.e., when $u_n(x_0) < u_d$. Hence, by disclosing at time 0, the manager can *increase* the option value in the second period. This follows from the analysis in Section 3.2; by disclosing in time 0, the manager can raise the threshold $x_d(y_0)$. Interestingly, in

this case, the market price at time 0 *decreases* upon disclosure by the manager. This implies that the manager is disclosing excessively in time 0, and does so even in cases in which the market price drops after disclosure. In other words, to improve the option value in the second period, the manager delays less and even sacrifices a higher market price in the first period. This is in contrast to the result in Acharya et al. (2011), as the manager's ex ante disclosure can only improve the market price in their setting.

We examine the equilibrium condition (11) and find that Case 2 always occurs in the unique equilibrium.

Theorem 1 *There exists a unique fixed point satisfying equation (11). Moreover, Case 2 always occurs. Also, the first-period dynamic disclosure threshold is lower than the myopic disclosure threshold: $x_0 < x^*$.*

Theorem 1 states that, when the firm value evolves over time, the threshold-type manager discloses even though this results in a lower first-period price. In other words, by keeping quiet at time 0, the manager's price would have been higher. The benefit of disclosing in the first period is the possibility that the fundamental value drops in the future. In that case, the manager can hide the reduced value and accept the non-disclosure price in the second period. On the other hand, if it turns out to be the case that the fundamental value has improved, the manager can simply disclose this value to the market. Correspondingly, by disclosing at time 0, the manager obtains the put option u_d whose strike price is η^* .

Conversely, disclosure in the first period sacrifices the option value, $u_n(x_0)$, from the public signal provided by dividends s_0 . If the manager conceals information in $t = 0$, she allows herself the possibility that the public signal increases the second-period threshold to a value that exceeds the innovation in firm profitability. Then, the manager can continue to hide information and enjoy the benefit from non-disclosure. However, as shown by Lemmas 1 and 2, there is an endogenous limitation to the upside of non-disclosure in the first period. This arises from the fact that the market updates its beliefs of the evolved fundamental value based on the average of the set of non-disclosing firms, which limits the benefit of a high x_0 . Moreover, the second-period threshold does not increase in line with increases in s_0 , thus limiting the benefit of a high s_0 . This leads the second-period threshold following non-disclosure in $t = 0$ to be strictly less than the threshold following disclosure. Put differently, the threshold-type manager gains the put option $u_n(x_0)$ with the strike price $x_n(x_0, s_0) - \kappa x_0$, but this strike price is always lower than η^* . Since the value of the put option price is increasing in its strike price, the option value upon disclosure is always greater

than the option value upon non-disclosure. Hence, we find that disclosure by the threshold-type always results in a *decrease* in the time 0 market price.

The main economic force driving the result is that the manager can generate an option value from *revealing* information in the first period, thus inducing excessive disclosure. This is in contrast to extant dynamic disclosure models, which generally feature an option value that is generated from *concealing* information, and results in excessive delay of disclosure. Note that the evolution of the firm value is essential for this result; under the unchanging environment, the option value upon disclosure is always zero. Hence, we have identified the key mechanism—time-varying firm value—which *endogenously* generates excessive disclosure, or, in other words, disclosure which results in a price drop (relative to non-disclosure).

Theorem 1 also helps to explain a pervasive finding in the empirical literature, whereby firms voluntarily release information even though this is met with a negative market reaction (Skinner (1994, 1997), Soffer et al. (2000), Matsumoto (2002), Baik and Jiang (2006), Anilowski et al. (2007), Kross et al. (2011)). Below, we further discuss several empirical implications that arise from this setting.

4 Equilibrium Properties and Empirical Predictions

Our model provides a theoretical link between the equilibrium disclosure threshold and the downward price jump from disclosure. In this section, we illustrate how these endogenous variables respond when an exogenous variable shifts. First, we establish the following result regarding the volatility of cash flow s_0 .

Proposition 4 *The first-period disclosure threshold x_0 is independent of the volatility of actual cash flows, σ_s .*

We find that the first-period disclosure threshold does not vary with changes in the volatility of the first-period cash flow. This is perhaps counter-intuitive, as we would expect the option value from non-disclosure, $u_n(x_0)$, to be more valuable for the manager when σ_s is higher. However, an increase in σ_s also has the opposing effect whereby investors place comparatively less weight on the realization of cash flows when it conveys relatively less information about the firm’s mean cash flow. We find that these two effects off-set each other and lead x_0 to be unaffected by changes in σ_s .

More precisely, at $t = 0$, from the perspective of the threshold-type manager ($y_0 = x_0$), the gap between the previous firm value and the investors’ posterior belief, $g = x_0 - fs_0 =$

$(1 - f)x_0 - fw_0$, is normally distributed with mean and variance:

$$\mathbb{E}[g|y_0 = x_0] = (1 - f)x_0, \quad (12)$$

$$\text{Var}(g|y_0 = x_0) = \rho^2(1 - \rho^2)\sigma_y^2. \quad (13)$$

Note that the variance of the gap, g , is independent of the volatility of actual cash flows, since the variance of w_0 is $(1 - \rho^2)\sigma_s^2$ and $f = \rho\sigma_y/\sigma_s$. That is, when actual cash flow is more volatile, investors place less weight on the announcement of s_0 and the manager anticipates this. This implies that the option value upon non-disclosure, $u_n(x_0)$, and thus the first-period disclosure threshold, is independent of the volatility of the first-period cash flow. We next examine the limiting behavior of the first-period threshold.

Proposition 5 *We have following limiting behavior of the first-period disclosure threshold: as $|\rho| \rightarrow 1$, then $x_0 \rightarrow x^*$; and as $\kappa \rightarrow 0$, then $x_0 \rightarrow x^*$.*

We see that the first-period disclosure threshold is equal to the myopic one as $|\rho| \rightarrow 1$. This occurs because the manager's option upon non-disclosure becomes less relevant for the first-period disclosure decision, as investors have more precise information regarding y_0 as $|\rho|$ increases. Consequently, the market eventually recovers the non-disclosed mean firm value when s_0 and y_0 are perfectly correlated. This diminishes the manager's incentive to disclose excessively in the first period relative to the myopic case, and hence we have that $x_0 = x^*$. Similarly, when the mean cash flows are independent of each other, i.e., $\kappa = 0$, the first-period disclosure decision is irrelevant for the second-period decision. This effectively results in the manager becoming myopic.

4.1 Empirical Predictions

We now analyze changes in the first-period disclosure threshold x_0 through changes in the persistence of cash flows (κ), disclosure cost (c), time 0 uncertainty (σ_y), and volatility in the change in firm value (σ_η). Since the first-period threshold embeds two real options, where each option also indirectly embeds the second-period threshold, we employ numerical analyses to gain insight into the directions of the effects. Below, we more specifically discuss the underlying intuition and numerical results for the change in each parameter.

These comparative results also provide novel empirical implications. To provide further context for the analysis, we consider a slightly augmented version of our baseline setting

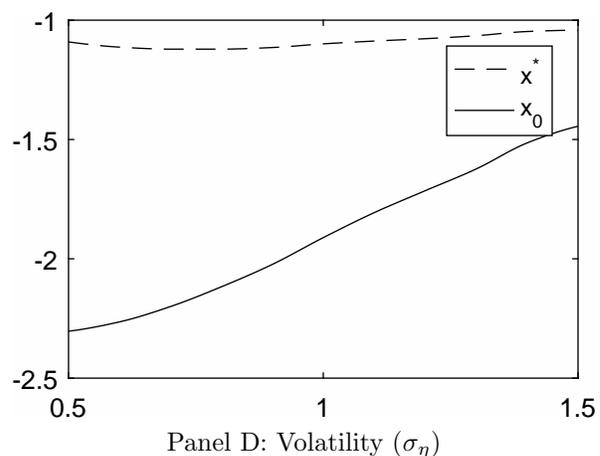
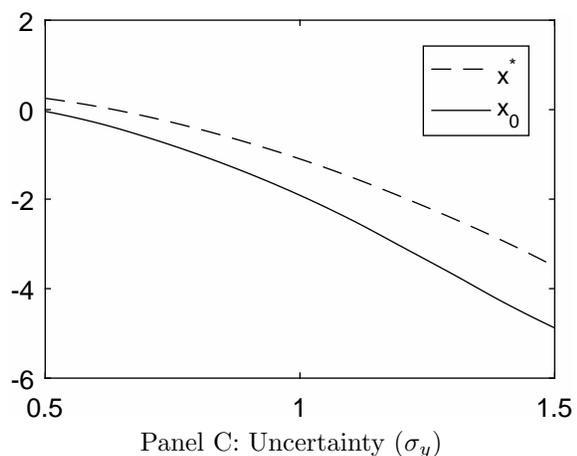
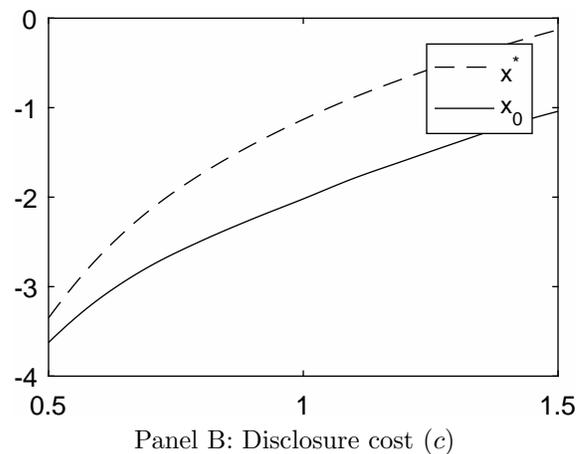
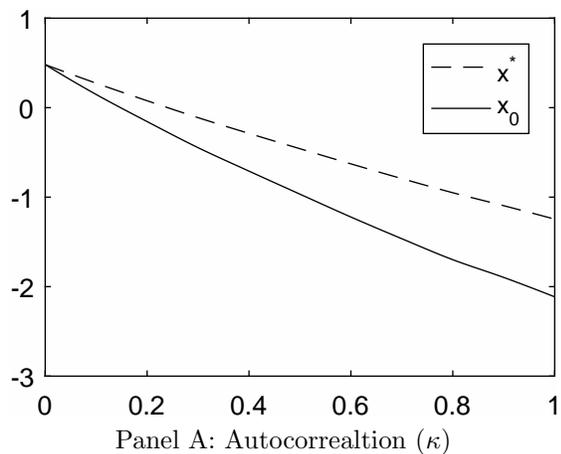


Figure 2: Effect of changes in parameters on disclosure threshold. The baseline parameters are: $\sigma_y = 1$, $\sigma_\eta = 1$, $\sigma_s = 2$, $c = 1$, $\rho = 0.5$, and $\kappa = 0.9$.

where there is a mandatory earnings announcement in a third period whereby the second-period cash flows (or third-period cash flows) are announced to the market. The manager does not have any discretion in this additional third period. None of our results in Section 3 are affected by this new assumption, however, we consider this framework as it allows us to more clearly describe the following notion of disclosure “timeliness,” or alternatively, delay in disclosure. We adopt this notion from empirical studies which investigate disclosure timeliness (e.g., Skinner (1997), Billings (2008)). Timely disclosure generally refers to the practice by firms to voluntarily release information well in advance of a mandatory earnings announcement. Moreover, the earlier a voluntary disclosure is made prior to the mandatory disclosure date, the more timely the disclosure is considered. With respect to our model, more timely disclosure (or less delay in disclosure) corresponds to more disclosure in the first period (i.e., a lower first-period disclosure threshold).

Our results provide a number of implications regarding disclosure timeliness. As we discuss in more detail below, we find that there is less delay in disclosure when (i) the firm’s cash flows are relatively more persistent; (ii) the firm has a relatively low disclosure cost; (iii) there is relatively greater information asymmetry between the firm and the market; and (iv) there is relatively *less* uncertainty regarding future changes in firm value.

Autocorrelation

We see in Panel A of Figure 2 that the first-period threshold is decreasing in κ . As κ rises, this implies that the firm’s cash flows exhibit greater persistence. Consequently, the first-period mean cash flow y_0 becomes more salient for the market’s beliefs in the second period as κ increases. Recall that, in the absence of disclosure in the first period, Lemma 2 shows that the market updates its beliefs of y_1 taking into consideration the first-period disclosure threshold x_0 and the dividends s_0 . Hence, as κ increases, the market places greater weight on x_0 and s_0 when determining their beliefs of y_1 given non-disclosure in the first period.

To see this more clearly, we note that there are two effects on the option from concealing information, $u_n(x_0)$, as κ increases. The first is on the relative second-period “undervaluation” endured by the threshold type. As the market uses the average of the first-period non-disclosing firms, the threshold-type’s option value from withholding disclosure decreases in κ . This occurs because the market places greater weight on a value that is ex-ante less than x_0 when updating their beliefs as κ increases. This effect makes the option from withholding disclosure less valuable. The second effect is on the possibility that observed dividends s_0 may overstate underlying profitability y_0 . As κ increases, the market also places relatively

greater weight on first-period dividends s_0 given non-disclosure in period 1, as cash flows are comparatively more persistent between periods. This effect further incentivizes the manager to withhold disclosure as the option $u_n(x_0)$ becomes more valuable.

We thus have two countervailing effects as κ increases. Consistent with Lemma 1 and Theorem 1, we find that the first effect dominates, and that $u_n(x_0)$ is accordingly *decreasing* in κ (see Panel A of Figure 4). The first-period threshold x_0 also decreases in κ as the manager has less incentive to withhold disclosure in $t = 0$. Thus, the results of the model imply that we should expect more timely voluntary disclosure or less delay in disclosure when there is *greater* autocorrelation or more persistence in cash flows.

Cost of disclosure

In Panel B, we show the effect of changes in the cost of disclosure. When c is low, it is less costly for the manager to take advantage of the option value from disclosure, u_d . This leads the manager to disclose relatively more often in the first period in order to generate the option value u_d . Interestingly, as c increases, u_d becomes *more* valuable for the manager. This occurs because disclosure in the first period has a greater impact on the market's beliefs in the second period, since it is less likely that the manager discloses in the second period due to a higher c . Correspondingly, we find that the first-period price upon disclosure and the non-disclosure price are both increasing in c (Figure 3).

The empirical literature investigating the proprietary nature of voluntary disclosure has generally found less disclosure in more competitive industries or segments (e.g., Botosan and Stanford (2005), Verrecchia and Weber (2006), Berger and Hann (2007)). While these findings are consistent with our prediction, the results of the model provide insights into the temporal dimension of costly voluntary disclosure. Specifically, we should expect more delay in disclosure in industries which are characterized by relatively high proprietary costs, such as firms in more competitive industries, or relatively high certification costs, such as firms or industries which are more tightly regulated. This implies further that disclosures which are made in more competitive industries pertain to information which is more mature or advanced in nature, as the firm is relatively more inclined to disclose in the second period rather than the first period when c is high.

Firm volatility

We examine the effect of changes in the volatility of the initial firm value, σ_y , in Panel C. We see in Figure 2 that the first-period dynamic threshold is decreasing in σ_y . First note that

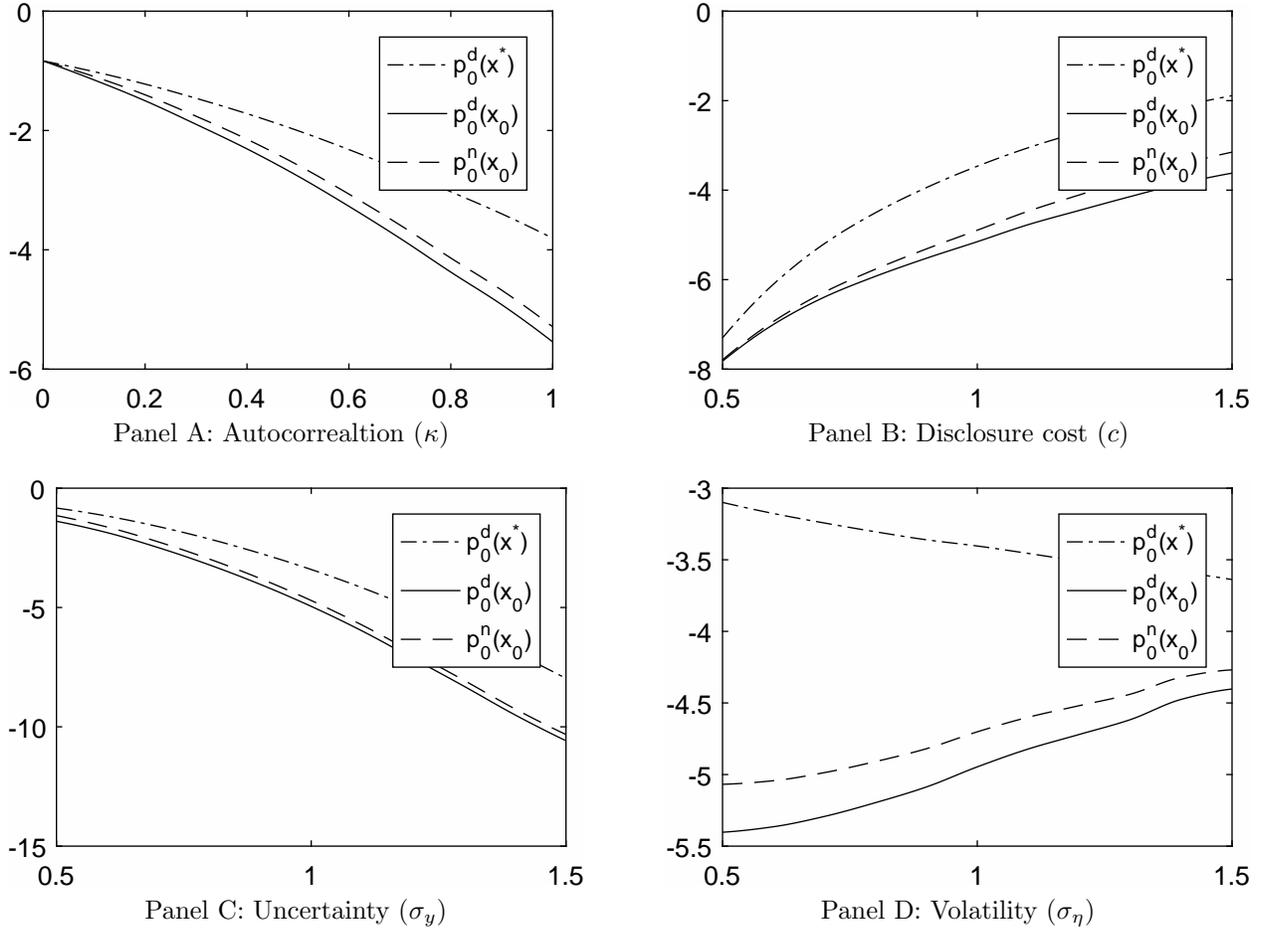
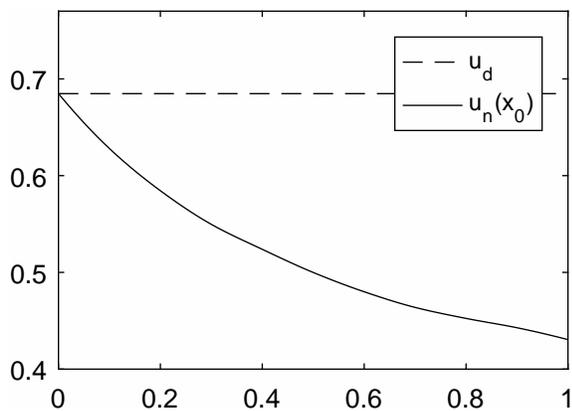
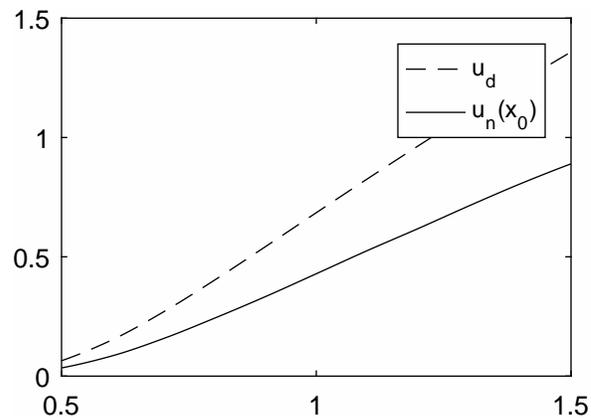


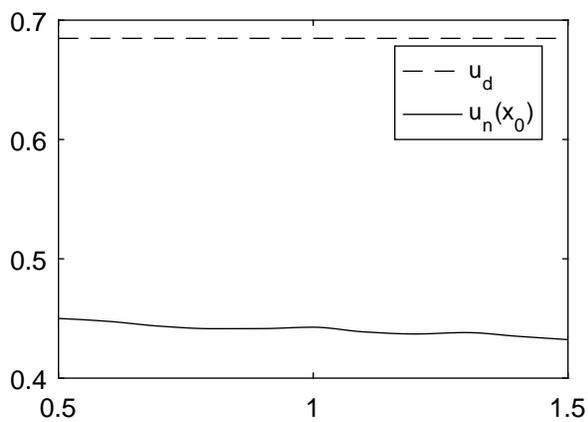
Figure 3: Effect of changes in parameters on threshold-type price. The baseline parameters are: $\sigma_y = 1$, $\sigma_\eta = 1$, $\sigma_s = 2$, $c = 1$, $\rho = 0.5$, and $\kappa = 0.9$.



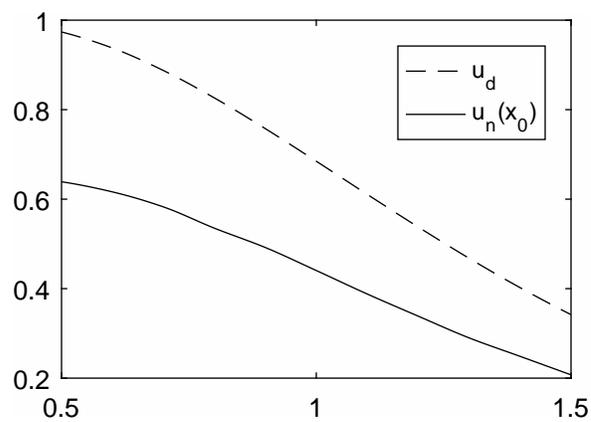
Panel A: Autocorrelation (κ)



Panel B: Disclosure cost (c)



Panel C: Uncertainty (σ_y)



Panel D: Volatility (σ_η)

Figure 4: Effect of changes in parameters on option value. The baseline parameters are: $\sigma_y = 1$, $\sigma_\eta = 1$, $\sigma_s = 2$, $c = 1$, $\rho = 0.5$, and $\kappa = 0.9$.

changes in σ_y do not affect the option value generated from disclosure, u_d (Figure 4). This is natural as σ_y becomes irrelevant if the market learns y_0 perfectly from disclosure. The change in first-period threshold x_0 is thus driven by the manager's incentives given non-disclosure in $t = 0$. As σ_y increases, the market's prior information regarding first-period cash flows y_0 becomes less informative. Consequently, the observed dividends s_0 and the disclosure threshold x_0 become relatively more informative for the market given non-disclosure, and hence have a relatively greater impact on market beliefs as σ_y increases.

As in the case of the autocorrelation κ , this has two countervailing effects on the manager's incentives—the manager can benefit from potentially high dividends s_0 , but is also hurt by the truncation imposed by x_0 . As foreshadowed by Lemmas 1 and 2, we see that the latter effect is dominant, and the manager discloses more often in the first period as σ_y increases.

We can interpret σ_y as the market's ex ante level of uncertainty regarding y_0 , or as the information asymmetry between the manager and the market at time 0. We predict that firms whose information environments generally involve greater information asymmetry or greater uncertainty will also have more timely voluntary disclosures. Some evidence of this has been found by Anantharaman and Zhang (2011) and Balakrishnan et al. (2014), who document that firms increase their frequency of earnings guidance, a type of voluntary disclosure, in response to decreases in analyst following of the firm (i.e., an increase in information asymmetry).

Interestingly, we find that an increase in the volatility of the change in firm value, σ_η , *increases* the first-period dynamic threshold x_0 in Panel D. To see this, first consider the effect of an increase in σ_η on the option value upon disclosure, u_d (Figure 4). In this case, disclosure in $t = 0$ is relatively less informative of the evolved underlying profitability in $t = 1$ as σ_η increases. This implies that the impact of first-period disclosure is less salient for second-period beliefs, and hence the put option generated from early disclosure is relatively less valuable. This effect incentivizes the manager to disclose less often in period one.

However, we see that the put option generated from non-disclosure, $u_n(x_0)$, is also decreasing in σ_η . This occurs since the observed dividends s_0 are relatively less informative about y_1 given non-disclosure as σ_η increases, thus leading the put option from non-disclosure to be less valuable. This effect rather incentivizes the manager to disclose more often in the first period. While both option values are decreasing in σ_η , and have disparate effects on x_0 , we see in Figure 4 that the difference between the two put options, $u_n(x_0) - u_d$ is decreasing in σ_η . This implies that the put option from keeping quiet becomes more appealing (or less unappealing) relative to u_d as σ_η increases. The net effect is that the manager discloses less

often in $t = 0$, as u_d decreases at a faster rate than $u_n(x_0)$.

The parameter σ_η depicts uncertainty regarding the future profitability of the firm. This corresponds to firms in industries which are characterized as having relatively greater uncertainty in their long-run or future value, such as firms with high R&D expenses, greater executive turnover, or firms in rapidly evolving industries. We thus predict that firms with greater uncertainty in future value are *less* timely with their voluntary disclosures.

4.2 Relation to Empirical Literature

There is a sizable empirical literature on voluntary disclosure. The present model helps to shed light on some of the documented empirical regularities. The large-scale survey of executives by Graham et al. (2005) finds evidence in support of voluntary disclosure as embedding a real option. We formalize this notion and find that the unique equilibrium first-period disclosure strategy incorporates both a real option from delay and from disclosure. The model also helps to explain the patterns found in Kothari et al. (2009) and Sletten (2012). Kothari et al. (2009) find evidence supporting the hypothesis that managers withhold information over time up to a threshold before issuing a disclosure. In a similar vein, Sletten (2012) documents that firms disclose information more often following negative shocks to share prices. The results of the model show that there is a larger price decline in the second period for a manager who had not disclosed in the first period (Lemma 2). Hence, first-period non-disclosing firms are more prone to disclose in the second period, especially following an unfavorable public realization of s_0 —consistent with the findings of Sletten (2012).

Moreover, our analysis in Section 3.3 shows that the price decreases upon disclosure relative to non-disclosure in the first period. This occurs since the put option from disclosure, u_d exceeds the put option from non-disclosure, $u_n(x_0)$. Indeed, the larger u_d is relative to $u_n(x_0)$, the more often the manager discloses in the first-period and the greater the price drop upon non-disclosure. However, in the second period we do not observe a price decrease upon disclosure relative to non-disclosure. This implies that disclosures which are more timely, or firms that are more timely in their voluntary release of information, are more likely to be met with a market reaction which is negative. We thus identify a salient feature—disclosure timeliness—which may help to assess the type of market reaction following disclosure. Hence, the results of the model predict that firms which are more timely with their voluntary disclosures are more frequently met with a market reaction which is *negative*, or that these disclosures are often “bad news” in nature. This also helps to reconcile numerous results in the empirical literature which have found that bad news disclosures are more frequent than

disclosures of good news (see, e.g., Skinner (1994, 1997), Soffer et al. (2000), Matsumoto (2002), Baik and Jiang (2006), Anilowski et al. (2007), Kross et al. (2011)).

The results of the model also have implications for the skewness of observed returns. As documented by Beedles (1979), as well as several other studies, individual stock returns tend to have a positive skewness. In environments where information is learned by the market over time, the model helps to explain how positive skewness can arise when there is disclosure after public news releases. In the case where the manager had not disclosed in the first period, the manager discloses in period 2 if her evolved firm value is greater than the disclosure threshold (Proposition 3). There are two broad cases. First, if the observed dividends are sufficiently negative (i.e., a bad news announcement), market sentiment deteriorates and this triggers disclosure by the manager. This results in a *negative* skewness following a disclosure. A similar implication is made in Acharya et al. (2011). However, in the model of Acharya et al. (2011), the release of private information after public news cannot generate *positive* skewness. In contrast, in the present model, due to the changing nature of the firm value, this situation can arise. For example, suppose that the observed dividends vastly overstates y_0 . In this case, the news is positive and market sentiment improves. However, the fundamental firm value may also be improving, and to such a magnitude that it overcomes the second-period disclosure threshold $x_n(x_0, s_0)$, resulting in disclosure by the manager. This implies that, even though the public signal is releasing good news, there is also disclosure of good news by the firm, which thus leads to positive skewness in the stock return after disclosure following public news announcements. This feature cannot arise in Acharya et al. (2011) as good news announcements are always preempted by the manager in the equilibrium of their setting.

5 Extensions

In this section, we consider two extensions to the baseline setting. First, we allow discounting of the second-period cash flows by the manager and the market. We then examine the setting where disclosure has a long-term impact on the firm by hurting the firm's profitability. For ease of exposition, we assume that the autocorrelation is equal to one, $\kappa = 1$, in the following two extensions.

5.1 Discounting

We introduce discounting in both the first-period market price and the manager's utility function. Suppose that the market discounts time 1 cash flows with a discount factor $\beta \in [0, 1]$, and the manager discounts the time 1 price with a discount factor $\lambda \in [0, 1]$. The time 1 disclosure decision is not affected by discounting. However, the time 0 prices are now given by

$$\begin{aligned} p_0^d(x_0) &= (1 + \beta)x_0 - c(1 + \beta\alpha_d), \\ p_0^n(x_0) &= -(1 + \beta)\sigma_y\delta\left(\frac{x_0}{\sigma_y}\right) - c\beta\alpha_n(x_0). \end{aligned}$$

Similarly, the manager is now maximizing $p_0 + \lambda E[p_1|y_0]$. Then, the time 0 disclosure threshold is determined by

$$p_0^d(x_0) = p_0^n(x_0) + \lambda(u_n(x_0) - u_d).$$

We characterize the equilibrium disclosure behavior of the manager in the limit cases in the following proposition.

Proposition 6 *As $\beta \rightarrow 0$ and $\lambda \rightarrow 0$, we have that $x_0 \rightarrow x^{**}$, where x^{**} is the static disclosure threshold and solves the following equation:*

$$c = \sigma_y v\left(\frac{x^{**}}{\sigma_y}\right). \quad (14)$$

As $\beta \rightarrow 1$ and $\lambda \rightarrow 0$, we have that $x_0 \rightarrow x^$. As $\beta \rightarrow 0$ and $\lambda \rightarrow 1$, we have that $x_0 < x^{**}$. As $\beta \rightarrow 1$ and $\lambda \rightarrow 1$, the model becomes the baseline one. The first-period threshold x_0 is decreasing in β if $\alpha_n(x_0) > \alpha_d$, and is decreasing in λ .*

As $\beta \rightarrow 0$ and $\lambda \rightarrow 0$, both the market and the manager become myopic. The market cares only about the first-period cash flow when pricing the firm and the manager also maximizes the first-period price. Thus, the first-period threshold approaches the static threshold x^{**} . As $\beta \rightarrow 1$ and $\lambda \rightarrow 0$, only the manager becomes myopic, in which we attain the myopic benchmark presented in Section 3.1. As $\beta \rightarrow 0$ and $\lambda \rightarrow 1$, the market becomes myopic, and the manager continues to have two real options. Since the option value upon disclosure is higher, the equilibrium continues to exhibit excessive disclosure relative to the static case. Finally, as $\beta \rightarrow 1$ and $\lambda \rightarrow 1$, the model becomes the baseline one.

As long as $\alpha_n(x_0) > \alpha_d$, as the market becomes less myopic, the more excessively the manager discloses in the first period. As β increases, the difference between the disclosure and non-disclosure price increases given that the likelihood of disclosure in the second period is higher upon non-disclosure: $\alpha_n(x_0) > \alpha_d$. Thus, the manager is compelled to disclose excessively to increase the option value generated from disclosure, u_d . Conversely, as the manager becomes less myopic, the less she delays disclosure. This occurs because the difference between the two option values is weighed more heavily in the manager's first-period utility, and hence she is willing to begin disclosure at a relatively lower realization of y_0 .

5.2 Long-term Impact of Disclosure

In the baseline model, we assume that disclosure entails a cost to the firm of $c > 0$. This is meant to capture the familiar notion that firms may endure certification or verification costs (such as fees paid to an auditing firm) when releasing information, or from potential loss in revenue due to competitors adopting innovations. While our assumption of the disclosure cost in the baseline model is consistent with the extant literature, in this section we consider an alternative specification where disclosure in the first period impacts the underlying profitability of the firm. We capture this by assuming that the cost from disclosure in the first period is also present in the second period, regardless of whether or not the manager discloses in the second period. Specifically, we assume that the time 1 mean cash flow evolves according to the following process:

$$y_1 = y_0 - \ell cd_0 + \eta,$$

and first-period dividends are given by $s_0 - (1 - \ell)cd_0$. We continue to assume that $\kappa = 1$ for ease of exposition. In this case, time 1 cash flows evolve based on $y_0 - \ell c$, rather than on y_0 as in the baseline setting. Moreover, dividends at time 0 are decreased by $(1 - \ell)c$ due to disclosure in the first period. The parameter $\ell \in [0, 1]$ controls the duration of the disclosure cost. Prices at time 0 are now given by

$$\begin{aligned} p_0^d(x_0) &= 2x_0 - c(1 + \alpha_d), \\ p_0^n(x_0) &= -2\sigma_y\delta\left(\frac{x_0}{\sigma_y}\right) - c\alpha_n(x_0). \end{aligned}$$

The second-period disclosure threshold following disclosure in the first period is given as

$$x_d(y_0) = y_0 - \ell c + \eta^*.$$

We can similarly characterize the first-period disclosure decision:

$$p_0^d(x_0) = p_0^n(x_0) + u_n(x_0) - u_d + \ell c.$$

We see that the first-period threshold exhibits the following properties under this alternative setting:

Proposition 7 *The first-period disclosure threshold x_0 is increasing in ℓ . We have $x_0 < x^*$ as $\ell \rightarrow 0$, and $x_0 > x^*$ as $\ell \rightarrow 1$.*

When $\ell = 0$, we have the baseline model and find excessive disclosure relative to the myopic benchmark. Conversely, when $\ell = 1$, first-period disclosure severely deteriorates the firm's profitability. Moreover, this loss is not covered by the option value from disclosure, which is relatively higher than the option value gained from non-disclosure. This additional penalty to disclosure diminishes the manager's option value from early disclosure, and consequently leads to excessive delay in equilibrium. This occurs because the disclosure cost is essentially double-counted in the manager's utility, as a single disclosure hurts the price in both periods. While this occurs in the extreme case of $\ell = 1$, Proposition 7 implies that we can find an $\bar{\ell} \in (0, 1)$ such that, for $\ell < \bar{\ell}$, the manager discloses excessively (i.e., $x_0 < x^*$) and the price decreases upon disclosure relative to non-disclosure.

6 Conclusion

Voluntary information release is an ubiquitous activity by firms and is central to the process of price discovery. Firms are often timely in their release of information, and also frequently disclose information voluntarily that is met with a negative market reaction. We capture these empirical regularities in a parsimonious setting where the firm's underlying profitability evolves over time. We find that the manager may voluntarily disclose information even if this results in a lower price than if she had concealed the information. The manager endures this price drop for the purpose of generating a real option which allows her to conceal information more often in the future. The implication is that disclosure in the present period positively influences the market's beliefs concerning the evolution of the firm's value, and thus increases the price upon non-disclosure in a future period. We find that this result holds even in the face of a public signal which may overstate the firm's value, thus providing the manager with an option value from delaying disclosure.

The results of this study provide a rich set of avenues for future research. The results provide implications for firm or industry characteristics for which we are more likely to observe timely disclosure, or disclosure that is met with a negative market reaction. This includes firms for which there is less uncertainty regarding future earnings, greater information symmetry, or higher autocorrelation in cash flows. The results also help to explain how positive skewness can emerge when public news announcements and voluntary disclosures are made in tandem.

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Appendix

A Proofs

A.1 Proof of Propositions 1 and 2

To prove Proposition 1, we first prove the following Lemma:

Lemma A1 Define $v(x) = x + \delta(x)$. Then, $v(x)$ is non negative, and increasing in x . Furthermore, $\delta(x)$ is weakly decreasing in x . Finally, $\lim_{x \rightarrow -\infty} \delta(x) = -x$, $\lim_{x \rightarrow \infty} \delta(x) = 0$, $\lim_{x \rightarrow -\infty} \delta(x)v(x) = 1$, and $\lim_{x \rightarrow \infty} \delta(x)v(x) = 0$.

Proof of Lemma A1. First, we want to show that $\delta(x) \geq -x$ so that $v(x) \geq 0$. When $x \geq 0$, clearly it holds. For $x < 0$, define $R(x) = \delta(x)^{-1}$. Then, we want to show that $R(x) \leq -\frac{1}{x}$ for $x < 0$. The first derivative is

$$R'(x) = 1 + xR(x) \tag{A.1}$$

and we also have

$$\lim_{x \rightarrow -\infty} xR(x) = -1 \tag{A.2}$$

Suppose that at any point $x_1 < 0$, $R(x_1) > -\frac{1}{x_1}$, i.e. $x_1R(x_1) < -1$ by contradiction. Then, by (A.1) $R'(x) < 0$ and $R(x)$ would continue to increase with decreasing x . This also implies that $xR(x)$ would continue to decrease, hence we should have $xR(x) < -1$ for any $x \leq x_1$, which contradicts (A.2). Therefore we show that $R(x) \leq -\frac{1}{x}$, i.e. $\delta(x) \geq -x$ for $x < 0$ too.

Next, we want to show that $v'(x) > 0$. The first derivative of $v(x)$ is given by

$$v'(x) = 1 - \delta(x)v(x)$$

Notice that this is variance of a standard normal variable ξ conditional on $\xi < x$. Since this must be positive, we have $v'(x) > 0$. This also implies that $\delta(x)v(x) < 1$ and $-1 < \delta'(x) = -\delta(x)v(x) \leq 0$ since $\delta(x) > 0$ and $v(x) \geq 0$.

Finally, since $\delta(x)$ is the negative mean of a standard normal variable with one sided truncation of upper tail at x , we have that $\delta(x) \rightarrow -x$ as $x \rightarrow -\infty$ and $\delta(x) \rightarrow 0$ as $x \rightarrow \infty$. This also implies that $\delta'(x) = -\delta(x)v(x) \rightarrow -1$ as $x \rightarrow -\infty$ and $\delta'(x) = -\delta(x)v(x) \rightarrow 0$ as $x \rightarrow \infty$. ■

Proposition 1 immediately follows from Lemma A1 and Lemma 3 and Proposition 2 follows from Lemma A1.

A.2 Proof of Proposition 3 and Lemma 1

We first establish the following Lemma:

Lemma A2 Define a function $k(x, y; s)$ for any $s > 0$

$$k(x, y; s) = \int_{-\infty}^y v(x - sz) \frac{\phi(z)}{\Phi(y)} dz$$

We have following properties of $k(x, y; s)$:

$$k_x(x, y; s) > 0, \quad \text{and} \quad k_x(x, y; s) + \frac{1}{s}k_y(x, y; s) > 0.$$

Proof of Lemma A2. The first derivative with respect to x is given by

$$k_x(x, y; s) = \int_{-\infty}^y (1 - \delta(x - sz)v(x - sz)) \frac{\phi(z)}{\Phi(y)} dz > 0$$

The second inequality is due to lemma A1. Take the first derivative with respect to y :

$$\begin{aligned} k_y(x, y; s) &= \delta(y) \left[v(x - sy) - \int_{-\infty}^y v(x - sz) \frac{\phi(z)}{\Phi(y)} dz \right] \\ &= \delta(y) \left[v(x - sy) - v(x - sy) \frac{\Phi(y)}{\Phi(y)} \Big|_{-\infty}^y - s \int_{-\infty}^y (1 - \delta(x - sz)v(x - sz)) \frac{\phi(z)}{\Phi(y)} dz \right] \\ &= -s\delta(y) \int_{-\infty}^y (1 - \delta(x - sz)v(x - sz)) \frac{\phi(z)}{\Phi(y)} dz \end{aligned}$$

The second equality holds since we have

$$\Phi(z) = \left(1 - \frac{1}{z^2} + \mathcal{O}\left(\frac{1}{z^2}\right) \right) \frac{\phi(z)}{|z|}$$

and thus

$$\lim_{z \rightarrow -\infty} v(x - sz)\Phi(z) = \lim_{z \rightarrow -\infty} \frac{x - sz}{|z|} \left(1 - \frac{1}{z^2} + \mathcal{O}\left(\frac{1}{z^2}\right) \right) \phi(z) = 0$$

Finally, we have

$$k_x(x, y; s) + \frac{1}{s}k_y(x, y; s) = \int_{-\infty}^y (1 - \delta(x - sz)v(x - sz))(\delta(z) - \delta(y)) \frac{\Phi(z)}{\Phi(y)} dz > 0$$

The second inequality is due to $\delta(z) > \delta(y)$ for $z < y$ and $\delta(\cdot)v(\cdot) < 1$. ■

Now, we can express (7) using $k(x, y; s)$:

$$\begin{aligned} c &= \epsilon^*(g) - E[E[\kappa z + \eta | z, \eta < \epsilon^*(g) - \kappa z] | z < g] \\ &= \epsilon^*(g) - E \left[\kappa z - \sigma_\eta \frac{\phi\left(\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta}\right)}{\Phi\left(\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta}\right)} \Big| z < g \right] \\ &= \epsilon^*(g) + \sigma_\eta \int_{-\infty}^g \left[-\frac{\kappa z}{\sigma_\eta} + \delta\left(\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta}\right) \right] \frac{\phi\left(\frac{z}{\sigma_z}\right)}{\sigma_z \Phi\left(\frac{g}{\sigma_z}\right)} dz \\ &= \sigma_\eta k\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}; \frac{\kappa \sigma_z}{\sigma_\eta}\right), \end{aligned} \tag{A.3}$$

where $g = x_0 - \rho \sigma_y s / \sigma_s$. By Lemma A2, given g the right hand side of (A.3) is increasing in $\epsilon^*(g)$ so that there exists a unique fixed point. Next, totally differentiate (A.3), then we have

$$0 < \frac{d\epsilon^*(g)}{dg} = -\frac{\frac{\sigma_\eta}{\sigma_z} k_y\left(\frac{\epsilon^*}{\sigma_\eta}, \frac{g}{\sigma_z}; \frac{\kappa \sigma_z}{\sigma_\eta}\right)}{k_x\left(\frac{\epsilon^*}{\sigma_\eta}, \frac{g}{\sigma_z}; \frac{\kappa \sigma_z}{\sigma_\eta}\right)} < \kappa$$

by Lemma A2.

A.3 Proof of Lemmas 2 and 3

To prove Lemma 2, notice that as $g \rightarrow -\infty$, $\epsilon^*(g)$ solves

$$\begin{aligned} c &= \lim_{g \rightarrow -\infty} \sigma_\eta \int_{-\infty}^g v \left(\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta} \right) \frac{\phi\left(\frac{z}{\sigma_z}\right)}{\sigma_z \Phi\left(\frac{g}{\sigma_z}\right)} dz \\ &= \lim_{g \rightarrow -\infty} \left[\sigma_\eta v \left(\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta} \right) \frac{\Phi\left(\frac{z}{\sigma_z}\right)}{\Phi\left(\frac{g}{\sigma_z}\right)} \Big|_{-\infty}^g - \kappa \int_{-\infty}^g \left\{ 1 - \delta \left(\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta} \right) v \left(\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta} \right) \right\} \frac{\Phi\left(\frac{z}{\sigma_z}\right)}{\Phi\left(\frac{g}{\sigma_z}\right)} dz \right] \\ &= \lim_{g \rightarrow -\infty} \sigma_\eta v \left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta} \right) \end{aligned}$$

Thus, we have $\epsilon^*(g) - \kappa g \rightarrow \eta^*$ as $g \rightarrow -\infty$. This also implies that $\lim_{g \rightarrow -\infty} \frac{d\epsilon^*(g)}{dg} = \kappa$.

Now, suppose that $g \rightarrow \infty$. Then, we have $\epsilon^*(g) \rightarrow \bar{\epsilon}$, where $\bar{\epsilon}$ solves

$$c = \lim_{g \rightarrow \infty} \sigma_\eta \int_{-\infty}^g v \left(\frac{\bar{\epsilon} - \kappa z}{\sigma_\eta} \right) \frac{\phi\left(\frac{z}{\sigma_z}\right)}{\sigma_z \Phi\left(\frac{g}{\sigma_z}\right)} dz = \sigma_\eta \int_{-\infty}^{\infty} v \left(\frac{\bar{\epsilon} - \kappa z}{\sigma_\eta} \right) \frac{\phi\left(\frac{z}{\sigma_z}\right)}{\sigma_z} dz \quad (\text{A.4})$$

This also implies that $\lim_{g \rightarrow \infty} \frac{d\epsilon^*(g)}{dg} = 0$. We can now show that

$$x_n(x_0, s_0) = \kappa f s_0 + \epsilon^*(g) < \kappa x_0 + \eta^* = x_d(x_0) \leftrightarrow \epsilon^*(g) - \kappa g < \eta^*,$$

for any g since we have that $\lim_{g \rightarrow -\infty} \epsilon^*(g) - \kappa g = \eta^*$ and that $\frac{d}{dg}(\epsilon^*(g) - \kappa g) < 0$.

Next, to prove Lemma 3 we define the following function for any $s > 0$:

$$F(x, y, s) = \int_{-\infty}^y \Phi(-x + sz) \frac{\phi(z)}{\Phi(y)} dz$$

and establish the following properties.

Lemma A3 $F_x(x, y, s) < 0$, $F_y(x, y, s) > 0$, $sF_x(x, y, s) + F_y(x, y, s) < 0$, $\lim_{y \rightarrow -\infty} F(x, y, s) = \Phi(-x + sy)$.

Proof of Lemma A3. Take the partial derivative with respect to x :

$$F_x = - \int_{-\infty}^y \phi(-x + sz) \frac{\phi(z)}{\Phi(y)} dz < 0$$

Take the partial derivative with respect to y :

$$\begin{aligned} F_y &= \delta(y) \left[\Phi(-x + sy) - \int_{-\infty}^y \Phi(-x + sz) \frac{\phi(z)}{\Phi(y)} dz \right] \\ &= s\delta(y) \int_{-\infty}^y \phi(-x + sz) \frac{\Phi(z)}{\Phi(y)} dz > 0 \end{aligned}$$

Thus, we have

$$sF_x(x, y, s) + F_y(x, y, s) = s \int_{-\infty}^y \phi(-x + sz) (\delta(y) - \delta(z)) \frac{\Phi(z)}{\Phi(y)} dz < 0$$

Finally, $F(x, y, s)$ can be expressed as

$$F(x, y, s) = \Phi(-x + sy) + s \int_{-\infty}^y \phi(-x + sz) \frac{\Phi(z)}{\Phi(y)} dz$$

which implies that $\lim_{y \rightarrow -\infty} F(x, y, s) = \lim_{y \rightarrow -\infty} \Phi(-x + sy)$. ■

Now, the ex ante likelihood of disclosure at time 1 given nondisclosure at time 0 can be expressed using the function $F(x, y, s)$:

$$\begin{aligned} \alpha_n(x_0) &= \Pr(y_1 > x_n(x_0, s_0) | y_0 < x_0) \\ &= E[E[1(y_1 > x_n(x_0, s_0)) | s_0, z < g]] \\ &= E\left[\int_{-\infty}^g \Phi\left(-\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta}\right) \frac{\phi\left(\frac{z}{\sigma_z}\right)}{\sigma_z \Phi\left(\frac{g}{\sigma_z}\right)} dz\right] \\ &= E\left[F\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}, \frac{\kappa \sigma_z}{\sigma_\eta}\right)\right], \end{aligned}$$

where $g = x_0 - fs_0$ and the last expectation is done with respect to s_0 . Taking x_0 to $-\infty$, i.e. $g \rightarrow -\infty$, then we have

$$\alpha_n(x_0) = \lim_{g \rightarrow -\infty} F\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}, \frac{\kappa \sigma_z}{\sigma_\eta}\right) = \lim_{g \rightarrow -\infty} \Phi\left(-\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta}\right) = \Phi\left(-\frac{\eta^*}{\sigma_\eta}\right) = \alpha_d.$$

The second equality is due to Lemma A3 and the third one is due to Lemma 2. Taking x_0 to ∞ , i.e. $g \rightarrow \infty$, then we have $\alpha_n(x_0) \rightarrow \bar{\alpha}_n$, where $\bar{\alpha}_n$ is given by

$$\bar{\alpha}_n = \lim_{g \rightarrow \infty} F\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}, \frac{\kappa \sigma_z}{\sigma_\eta}\right) = \int_{-\infty}^{\infty} \Phi\left(-\frac{\bar{\epsilon} - \kappa z}{\sigma_\eta}\right) \frac{\phi\left(\frac{z}{\sigma_z}\right)}{\sigma_z} dz.$$

Lastly, we can take the first derivative of $\alpha_n(x_0)$:

$$\alpha_n(x_0)' = \frac{1}{\sigma_z} E\left[\frac{\sigma_z}{\sigma_\eta} \frac{d\epsilon^*(g)}{dg} F_x + F_y\right].$$

By Lemma 2 and A3, we have $\alpha_n(x_0)' \rightarrow \frac{1}{\sigma_z} E\left[\frac{\kappa \sigma_z}{\sigma_\eta} F_x + F_y\right] < 0$ as $x_0 \rightarrow -\infty$ and $\alpha_n(x_0)' \rightarrow \frac{1}{\sigma_z} E[F_y] > 0$ as $x_0 \rightarrow \infty$.

A.4 Proof of Theorem 1

We begin by computing the option values. The option value upon initial disclosure is given by

$$\begin{aligned} u_d &= \int_{-\infty}^{\eta^*} (\eta^* - \eta) \frac{1}{\sqrt{2\pi\sigma_\eta^2}} e^{-\eta^2/2\sigma_\eta^2} d\eta \\ &= \eta^* \Phi\left(\frac{\eta^*}{\sigma_\eta}\right) + \sigma_\eta \delta\left(\frac{\eta^*}{\sigma_\eta}\right) \Phi\left(\frac{\eta^*}{\sigma_\eta}\right) \\ &= c\Phi\left(\frac{\eta^*}{\sigma_\eta}\right). \end{aligned}$$

The last equality holds by the definition of η^* . We similarly compute the manager's option value upon non-disclosure:

$$\begin{aligned} u_n(x_0) &= E \left[(\epsilon^*(g) - \kappa g - \eta)^+ \right] \\ &= E \left[E \left[(\epsilon^*(g) - \kappa g - \eta)^+ \mid w_0 \right] \right] \\ &= E \left[\sigma_\eta \Phi \left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta} \right) v \left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta} \right) \right], \end{aligned}$$

where the last expectation is done with respect to w_0 and $g = (1 - f)x_0 - fw_0$. The equilibrium condition (11) for the first-period disclosure threshold can be rewritten as

$$c = (1 + \kappa)\sigma_y v \left(\frac{x_0}{\sigma_y} \right) + u_d - u_n(x_0) + c(\alpha_n(x_0) - \alpha_d). \quad (\text{A.5})$$

We define the function

$$f(x) = (1 + \kappa)\sigma_y v \left(\frac{x}{\sigma_y} \right) + u_d - u_n(x) + c(\alpha_n(x) - \alpha_d).$$

Differentiating, we have

$$f'(x) = (1 + \kappa) \left[1 - \delta \left(\frac{x}{\sigma_y} \right) v \left(\frac{x}{\sigma_y} \right) \right] - (1 - f)E \left[\Phi \left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta} \right) \left(\frac{d\epsilon^*(g)}{dg} - \kappa \right) \right] + c\alpha_n(x)'.$$

Note that we use $(\Phi(x)v(x))' = \Phi(x)$. Take x to $-\infty$, then by Lemmas A1 and 3,

$$\begin{aligned} \lim_{x \rightarrow -\infty} f(x) &= 0, \\ \lim_{x \rightarrow -\infty} f'(x) &= \lim_{x \rightarrow -\infty} c\alpha_n(x)' < 0. \end{aligned}$$

Taking x to ∞ , we have

$$\begin{aligned} \lim_{x \rightarrow \infty} f(x) &= \lim_{x \rightarrow \infty} (1 + \kappa)\sigma_y v \left(\frac{x}{\sigma_y} \right) + u_d + c(\bar{\alpha}_n - \alpha_d) = \infty, \\ \lim_{x \rightarrow \infty} f'(x) &= 1 + \kappa + \lim_{x \rightarrow \infty} c\alpha_n(x)' > 0. \end{aligned}$$

Thus, there exists a unique x solving $c = f(x)$. Suppose that x_0 is such x . Then, we have

$$\begin{aligned} u_d - u_n(x_0) &= c\Phi \left(\frac{\eta^*}{\sigma_\eta} \right) - E \left[\sigma_\eta \Phi \left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta} \right) v \left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta} \right) \right] \\ &> c \left[\Phi \left(\frac{\eta^*}{\sigma_\eta} \right) - \Phi \left(\frac{\eta^*}{\sigma_\eta} \right) \right] \\ &> 0, \end{aligned}$$

since $\epsilon^*(g) - \kappa g < \eta^*$ by Lemma 2 and $\Phi(x)v(x)$ is an increasing function. This implies that the myopic threshold x^* should be higher than x_0 since $f(x)$ is increasing at $x = x_0$.

A.5 Proof of Propositions 4 and 5

The mean and variance of g conditional on the initial mean cash flow is independent of σ_s . Thus, $\epsilon^*(g)$ is also independent of σ_s and so does $u_n(x_0)$.

When $|\rho| \rightarrow 1$, upon observing s_0 investors can recover y_0 perfectly. Thus, two option values are identical, which implies $x_0 = x^*$. When $\kappa \rightarrow 0$, the information of nondisclosure is irrelevant for the second period decision. This implies $x_0 = x^*$.

A.6 Proof of Proposition 6

The equilibrium condition for the first-period disclosure threshold can be rewritten as

$$c = (1 + \beta)\sigma_y v \left(\frac{x_0}{\sigma_y} \right) + \lambda(u_d - u_n(x_0)) + c\beta(\alpha_n(x_0) - \alpha_d). \quad (\text{A.6})$$

Clearly, the equilibrium condition becomes (14) when $\beta = \lambda = 0$. When $\beta = 1$ and $\lambda = 0$, the model becomes the myopic benchmark and we have $x_0 = x^*$. When $\beta = 0$ and $\lambda = 1$, the equilibrium condition becomes

$$c = \sigma_y v \left(\frac{x_0}{\sigma_y} \right) + u_d - u_n(x_0).$$

Since $u_d > u_n(x_0)$, the first-period threshold x_0 should be lower than the static one x^{**} . Finally, when $\beta = \lambda = 1$, the model becomes the baseline one.

Set the function $f(x_0)$ equal to the right hand side of (A.6). Totally differentiate $f(x_0)$, then we have

$$\frac{\partial x_0}{\partial \beta} = - \frac{\sigma_y v \left(\frac{x_0}{\sigma_y} \right) + c(\alpha_n(x_0) - \alpha_d)}{f'(x_0)} < 0,$$

when $\alpha_n(x_0) > \alpha_d$ since $f'(x_0) > 0$ if x_0 satisfies (A.6). Again, totally differentiate $f(x_0)$, then we have

$$\frac{\partial x_0}{\partial \lambda} = - \frac{u_d - u_n(x_0)}{f'(x_0)} < 0,$$

since $u_d > u_n(x_0)$ and $f'(x_0) > 0$ if x_0 satisfies (A.6).

A.7 Proof of Proposition 7

The equilibrium condition for the first-period disclosure threshold can be rewritten as

$$c = 2\sigma_y v \left(\frac{x_0}{\sigma_y} \right) + u_d - u_n(x_0) + c(\alpha_n(x_0) - \alpha_d - \ell). \quad (\text{A.7})$$

Clearly, the equilibrium condition becomes (A.5) when $\ell = 0$. When $\ell = 1$, the equilibrium condition becomes

$$c = 2\sigma_y v \left(\frac{x_0}{\sigma_y} \right) + u_d - u_n(x_0) - c + c(\alpha_n(x_0) - \alpha_d).$$

Since $u_d < c$, we have $u_d - u_n(x_0) - c < 0$ and the first-period threshold x_0 is lower than the myopic one x^* which solves (4).

Set the function $f(x_0)$ equal to the right hand side of (A.7). Totally differentiate $f(x_0)$, then we have

$$\frac{\partial x_0}{\partial \ell} = \frac{c}{f'(x_0)} > 0,$$

since $f'(x_0) > 0$ if x_0 satisfies (A.7).