

# Market Timing, Investment, and Risk Management

\*Preliminary and Incomplete; Do not circulate.

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## Abstract

Financing can be cheaper in certain periods than others. For example, in crisis periods, firms face tougher financing terms than in normal times. We develop an analytically tractable dynamic framework for firms facing stochastic financing opportunities. Financially constrained firms choose intertemporal equity issuance, internal cash accumulation, corporate investment, risk management and payout policies in an environment subjective to time-varying financing shocks. Firms value financial slack and hence build cash reserves to mitigate financial constraints. However, the short duration of cheap financing opportunities induce firms to rationally time the equity market. With fixed costs of external financing, lumpy equity issuance when financing is cheap is optimal even when the firm has immediate needs for investment. The firm trades off the benefits of cheap financing with the costs of raising funds and carrying cash. For firms with sufficiently high cash, the precautionary motive for cash dominates the market timing motive. For firms with sufficiently low cash, the market timing motive dominates. Anticipating external financing in relatively cheap terms, the marginal value of cash is getting lower when the firm is closer to equity issuance. The marginal value of cash increases with cash for the low financial slack region and hence investment decreases with cash as a result of increasing marginal value of cash. Finally, the interaction between the finite duration of relatively cheap financing opportunities and the fixed external financing cost can encourage a firm to engage in risk taking in derivatives markets by being a speculator rather than a hedger.

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# 1 Introduction

What are the key factors that determine corporate equity issues? And, how do external equity financing costs affect corporate investment and cash holdings? There has been an ongoing and lively debate between the proponents of the *pecking order view* (Myers and Majluf, 1984) and the proponents of the *market timing view* (Loughran and Ritter, 1995, 1997, and Baker and Wurgler, 2002) on these questions. According to the pecking order view (in very broad terms) firms issue equity only as a last resort when they have exhausted all other forms of financing. According to the market timing view firms issue equity when the cost of equity capital is low. Underlying the differences between these two views are two fundamentally different conceptions of investor rationality.

Under the pecking order view, investors are assumed to be *fully rational* and to fully understand firms' motives for issuing equity. Thus, when equity issuers have better information than investors about the value of their business, investors tend to interpret equity issues as negative signals about the value of the business: They infer that issuers are more likely to issue equity when the stock is overvalued, and therefore lower their valuation of the firm in response to a new equity issue. The most direct evidence in support of this view are the studies by Asquith and Mullins (1986), Masulis and Korwar (1986), and Mikkelson and Partch (1986) among others, who have found that the average stock price announcement effect of a new common stock issue is a decline in stock price of the order of  $-3\%$  to  $-4.5\%$ .

Under the market timing view, investors are generally assumed to have *behavioral biases* such as overconfidence, which may lead them to sometimes overvalue stocks. Moreover, behaviorally biased investors are assumed to not fully undo a temporary overvaluation when they see firms issue more stock. As a result, firms may be able to take advantage of investors' behavioral biases and benefit from timing their equity issues in periods when investors are particularly favorably disposed towards their firm.<sup>1</sup>

Partly in reaction to the technology bubble and the dot-com IPO wave of the late 1990s, several studies have shown that firms do indeed try to time equity markets. Building on early findings of

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<sup>1</sup>It might also be possible to build an efficient market timing theory with fully rational investors who have time-changing risk-aversion, but such a theory has not yet been developed. One of the difficulties for such a theory is to translate changing risk-preferences into changes in the cost of equity capital *relative* to debt capital.

market timing by Loughran and Ritter (1995, 1997) in particular, Baker and Wurgler (2002) and Huang and Ritter (2009) have shown that market timing has a significant and persistent effect on firm leverage. Similarly, Graham and Harvey's (2001) survey of CFO corporate finance practices finds that most CFOs attempt to time equity markets. Fama and French (2005) also argue that the pecking order view is rejected by the data, to the extent that many firms are seen to tap equity markets even though they have not exhausted other financing opportunities. Yet the study by DeAngelo, DeAngelo and Stulz (2009) appears to contradict the market timing hypothesis, and suggests that firms' equity issuance policies are better explained by their concern for rebuilding cash reserves. They find that most firms with high market-to-book ratios do not issue stock and most firms who do issue stock look as if they are cash constrained.

In this paper we propose the first dynamic model of equity market timing, which may reconcile to a large extent the seemingly contradictory evidence of these studies. A dynamic model of market timing must be able to account for the facts that most firms only issue equity intermittently, and when they do go to the equity market, that they issue large amounts, as Fama and French (2005) have underscored. Moreover, the theory must account for the greater propensity to issue equity when the cost of equity capital is low. Accordingly, our dynamic theory of market timing builds on our model of corporate precautionary cash holdings in Bolton, Chen and Wang (2009) (BCW) by introducing stochastic financing opportunities. In this model we can analyze how equity market timing interacts with a firm's cash reserves, investment, and dynamic hedging policies.

The four main building blocks of the model are: 1) a long-run constant-returns-to-scale production function with *i.i.d.* earnings shocks, convex investment adjustment costs, and a constant capital depreciation rate (as in Hayashi (1982)); 2) stochastic external equity financing costs; 3) constant cash carry costs; and 4) dynamic hedging opportunities through futures contracts. As the firm's external equity financing costs are sometimes low, we are able to study how the firm optimally times equity markets, and how it adjusts its investment, payout, and hedging policies to changing financing opportunities.

Our model can also be applied to study how firms' financing and investment decisions are affected by the impending risk of a financial crisis, which freezes up financial markets. Similarly, it can be applied to the situation of a firm in the midst of a financial crisis looking ahead to a return to normally functioning capital markets.

We show that, although the firm's equity issuance is driven by the need for cash, it is still optimal for firms to time equity markets. If the state of the world in which equity is cheap is (nearly) permanent then the firm only issues equity when it runs out of cash. If, however, the firm knows that the transition probability out of this favorable state is significantly different from zero then it will optimally time the market by issuing new equity before it runs out of cash. These results are consistent with the findings in both DeAngelo, DeAngelo and Stulz (2009) and in Fama and French (2005) and Huang and Ritter (2009). Moreover, we show that as the transition probability out of the favorable state rises the firm will issue equity in that state sooner. Similarly, the firm delays cash payouts to shareholders more, the more likely is the probability that equity market conditions will worsen. Finally, the firm also scales back its investment as the probability of exiting the good state rises. Overall, the firm's cash inventory rises in anticipation of a significant worsening of equity financing opportunities. These results also confirm the conjecture of Bates, Kahle and Stulz (2009), who have found that strikingly the average cash-to-asset ratio of US firms has nearly doubled in the past quarter century, and who attribute this rise to firms' perceived increase in risk. These results also help explain the investment and financing policies of many US non-financial firms in the years prior to the financial crisis of 2007-2008, to the extent that these firms had forebodings of a likely financial crisis .

We model a financial crisis by assuming that equity markets freeze up and asset sales are nearly impossible in a crisis state. We show that a firm entering the crisis state with a lot of cash will barely change its investment policy and will simply delay somewhat payouts to shareholders. The lower its cash reserves, however, the more the firm cuts back on investment and the more it engages in asset sales – to the extent that these are still feasible – in an effort to preserve cash and survive. These results are broadly in line with the findings of Campello, Graham and Harvey (2009) who surveyed a large sample of CFOs in the midst of the crisis and found that the more financially constrained their firms were the more they cut back on investment and the more they engaged in asset sales. If there is a redeeming aspect to crises it could be that looking ahead the future is expected to get brighter. The firm in the crisis state anticipates that with some probability it will exit the state and be able to return to more favorable equity market conditions. We show that, as one might expect, when the probability of exiting the crisis rises the firm invests more and is less conservative in its payout policy. One surprising result, when the firm is unable to

engage in asset sales in the crisis state, is that the firm's payout boundary in that state may be non-monotonic in the probability of exiting the crisis. For very high and very low probabilities the firm pays out sooner than for intermediate probabilities. The reason is that when the firm is stuck in the crisis state for a long time the value of its investment opportunities is so low that it is best to payout cash to shareholders. When the probability of exiting the crisis is very high then the prospect of raising cheap equity in the future also encourages the firm to pay out more dividends in the crisis state. It is only for intermediate probabilities, when the value of the firm's investment opportunities is relatively high, but the risk of staying in a prolonged crisis is also high, that the firm is more conservative in its payout policy. This latter result, while intriguing is likely to be more of a curiosity than a quantitatively significant observation.

In contrast, the other surprising result of our analysis is important. We show that a major non-convexity in the firm's value – or the firm's average  $Q$  – as a function of its cash-to-asset ratio emerges as a result of equity market timing. When the firm times the market by choosing to issue equity before it runs out of cash, its value function is necessarily non-convex to the right of the (endogenous) equity issuance boundary. To see this, note first that when the firm issues equity it incurs a fixed issuance cost and raises a lumpy amount of cash. Second, note that a condition of optimality in equity issuance is that the marginal value of a dollar at the issuance boundary must be the same as the marginal value of the dollar at the return point for the firm's cash-to-asset ratio. This latter observation implies that the firm's value function is locally convex around the equity issuance boundary. This non-convexity in turn translates into a non-monotonicity in the firm's shadow value of cash, which in turn results in a non-monotonicity of investment as a function of the firm's cash-to-asset ratio.

The basic economics of this result can be explained as follows. When the firm's cash-to-asset ratio decreases and approaches the issuance boundary the firm increases its investment and thereby accelerates the depletion of its cash stock, as it is eager to time the favorable equity markets. By burning through its cash stock the firm brings forward the point at which it taps equity markets and replenishes its cash stock. However, when the firm's cash-to-asset ratio is already high it benefits less from timing the market and it is more concerned with preserving rather than renewing its cash hoard. This is why the firm responds locally to a small reduction in its cash-to-asset ratio by cutting back on investment and thereby replenish its cash stock. Our results are yet another

warning of the complex dynamic interactions between firm savings and investment: when firms engage in market timing, not only is the cash-sensitivity of investment likely to be non-monotonic, but investment itself is non-monotonic.

This finding also has important consequences for the firm's dynamic hedging policy. Indeed, we show that the non-convexity in the firm's value function caused by market timing can induce the firm to load up on rather than hedge systematic risk. We are thus able to account for the possibility of risk-taking through derivatives positions, which far from being excessive, is efficient and maximizes the value of the firm. Note that these risk-taking benefits exist even when the firm has no leverage and are driven purely by market timing.

## 2 The Model

We build on the model of Bolton, Chen, and Wang (2009) by introducing stochastic investment and external financing opportunities. The recent financial crisis among other large shocks to financial markets strongly suggests that firms' financing opportunities in reality are time-varying. In crisis periods, in particular, the cost of equity financing can be quite steep. Firms must adapt to these stochastic financing opportunities and change their financing and investment policies to be able to time favorable market conditions and hedge against unfavorable market conditions.

We capture the firm's stochastic investment and financing opportunities using a regime-switching model. The firm can be in one of two states 1, 2. In each state, the firm faces potentially different financing and investment opportunities. With a constant probability  $\zeta_1\Delta$  (or  $\zeta_2\Delta$ ), the firm will move from state 1 to 2 (or from 2 to 1) over a short period  $\Delta$ . Finally, we use  $s_t$  to denote the state the firm is in at time  $t$ .

### 2.1 Production technology

The firm employs only physical capital as an input for production and, the price of physical capital is normalized to unity. We denote by  $K$  and  $I$  respectively the level of the capital stock and gross investment. As is standard in capital accumulation models, the firm's capital stock  $K$  evolves according to:

$$dK_t = (I_t - \delta K_t) dt, \quad t \geq 0, \tag{1}$$

where  $\delta \geq 0$  is the rate of depreciation.

The firm's operating revenue at time  $t$  is proportional to its capital stock  $K_t$ , and is given by  $K_t dA_t$ , where  $dA_t$  is the firm's revenue (or productivity) shock over time increment  $dt$ . After risk adjustment (i.e. under the risk-neutral probability measure), the firm's revenue  $dA_t$  over time period  $dt$  are given by

$$dA_t = \mu(s_t) dt + \sigma(s_t) dZ_t,$$

where  $Z_t$  is a standard Brownian motion, while  $\mu(s_t)$  and  $\sigma(s_t)$  denote the expected return on capital and its volatility in regime  $s_t$ .

The firm's incremental operating profit  $dY_t$  over time increment  $dt$  is then given by:

$$dY_t = K_t dA_t - I_t dt - \Gamma(I_t, K_t, s_t) dt, \quad t \geq 0, \quad (2)$$

where  $I$  is the cost of the investment and  $\Gamma(I, K, s)$  is the additional adjustment cost that the firm incurs in the investment process if it is in regime  $s$ . Note that we allow the adjustment costs to be regime dependent. Intuitively, in expansion, the firm may be subject to lower transaction costs. Following the neoclassical investment literature (Hayashi (1982)), we assume that the firm's adjustment cost is homogeneous of degree one in  $I$  and  $K$ . In other words, the adjustment cost takes the homogeneous form  $\Gamma(I, K, s) = g_s(i)K$ , where  $i$  is the firm's investment capital ratio ( $i = I/K$ ), and  $g_s(i)$  is an regime-dependent increasing and convex function. Our analyses do not depend on the specific functional form of  $g_s(i)$ , and to simplify we assume that  $g_s(i)$  is quadratic:

$$g_s(i) = \frac{\theta_s(i - \nu_s)^2}{2}, \quad (3)$$

The firm can liquidate its assets at any time. The liquidation value  $L_t$  is proportional to the firm's capital at time  $t$  in regime  $s_t$ , i.e.  $L_t = l_s K_t$ , where  $l_s$  is the recovery per unit of capital in regime  $s$ .

## 2.2 Stochastic Financing Opportunities

Neoclassical investment models (à la Hayashi (1982)) assume that the firm faces frictionless capital markets and that the Modigliani and Miller (1958) theorem holds. However, in reality, firms face

important financing frictions for incentive, information asymmetry, and transaction cost reasons.<sup>2</sup> Our model incorporates a number of financing costs that firms face in practice and that empirical research has identified, while retaining an analytically tractable setting. The firm may choose to use external financing at any point in time. For the baseline model, we assume that the only source of external financing is equity. We leave the important extension of allowing the firm to also issue debt for future research.

We assume that the firm incurs a fixed and a variable cost of issuing external equity. The fixed cost is given by  $\phi_s K$ , where  $\phi_s$  is the fixed cost parameter in regime  $s = 1, 2$ . For tractability, we take the fixed cost to be proportional to the firm's capital stock  $K$ . Mainly, this assumption ensures that the firm does not *grow out of its fixed issuing costs*. The firm also incurs a proportional issuance cost  $\gamma_s$  for each unit of external funds it raises, which may also be regime dependent. That is, after paying the fixed cost, the firm pays  $\gamma_s > 0$  in regime  $s$  for each incremental dollar it raises. We denote by  $H$  the process for the firm's cumulative external financing, and hence by  $dH_t$  the incremental external financing over time  $dt$ , and by  $X$  the firm's cumulative issuance costs.

We also denote by  $W$  the process for the firm's cash stock. If the firm runs out of cash ( $W_t = 0$ ) it needs to raise external funds to continue operating or its assets will be liquidated. If the firm chooses to raise new external funds to continue operating, it must pay the financing costs specified above. The firm may prefer liquidation if the cost of financing is too high relative to the continuation value (e.g. when the firm is not productive; that is when  $\mu$  is low). We denote by  $\tau$  the firm's (stochastic) liquidation time, then  $\tau = \infty$  means that the firm never chooses to liquidate.

Next, we denote by  $U$  the firm's cumulative non-decreasing payout process to shareholders, and hence by  $dU_t$  the incremental payout over time  $dt$ . Distributing cash to shareholders may take the form of a special dividend or a share repurchase. The benefit of a payout is that shareholders can invest the funds they obtain at the risk-free rate. For simplicity, we assume for now that the risk-free rate is constant  $r$  at all times. Finally, we denote by  $\lambda$  the *carry cost of cash inside the firm*.

Combining cash flows from operations  $dY_t$  given in (2), with the firm's financing policy given by the cumulative payout process  $U$ , the cumulative external financing process  $H$ , and the firm's

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<sup>2</sup>See Jensen and Meckling (1976), Leland and Pyle (1977), and Myers and Majluf (1984), for example.

interest earnings minus cash carry cost from its cash inventory, then  $W$  evolves according to:

$$dW_t = [K_t dA_t - I_t dt - \Gamma(I_t, K_t, s_t)] dt + (r(s_t) - \lambda) W_t dt + dH_t - dU_t, \quad (4)$$

where, the second term is the interest income (net of the carry cost  $\lambda$ ), the third term  $dH_t$  is the cash inflow from external financing, and the last term  $dU_t$  is the cash outflow to investors, so that  $(dH_t - dU_t)$  is the net cash flow from financing. Note that this is a completely general financial accounting equation, where  $dH_t$  and  $dU_t$  are endogenously determined by the firm.

The homogeneity assumption embedded in the adjustment cost and the “ $AK$ ” production technology allows us to deliver our key results in a parsimonious and analytically tractable way. Adjustment costs may not always be convex and the production technology may exhibit long-run decreasing returns to scale in practice, but these functional forms substantially complicate the formal analysis (see Hennessy and Whited, 2005, 2007, for an analysis of a similar, but non-homogenous, model). As will become clear below, the homogeneity of our model in  $K$  allows us to reduce the dynamics to a one-dimensional equation, which is relatively straightforward to solve.

### 2.3 Firm optimality

The firm chooses its investment  $I$ , its cumulative payout policy  $U$ , its cumulative external financing  $H$ , and its liquidation time  $\tau$  to maximize firm value defined below:

$$\mathbb{E} \left[ \int_0^\tau e^{-\int_0^t r_s ds} (dU_t - dH_t - dX_t) + e^{-\int_0^\tau r_s ds} (L_\tau + W_\tau) \right]. \quad (5)$$

The expectation is taken under the risk-adjusted probability. The first term is the discounted value of payouts to shareholders and the second term is the discounted value upon liquidation. Note that optimality may imply that the firm never liquidates. In that case, we simply have  $\tau = \infty$ . We impose the usual regularity conditions to ensure that the optimization problem is well posed. See the appendix for details.

Let  $P(K, W, s)$  denote firm value, where  $s = 1, 2$ . We are interested in the impact of stochastic financing, investment opportunities on corporate decision making. In particular, we study how firms manage the risks of investment opportunities  $\mu(s)$ , volatility  $\sigma(s)$ , and costs of external financing  $(\phi_s, \gamma_s)$ .

### 3 Model Solution

#### 3.1 First-best Benchmark

We first summarize the solution for the neoclassical  $q$  theory of investment, in which the Modigliani-Miller theorem holds. We solve the firm's investment and firm value using backward induction.

The first-best Tobin's  $q$  and investment-capital ratio  $i_s^{FB}$  satisfy

$$r_s q_s^{FB} = \mu_s - i_s^{FB} - \frac{1}{2} \theta_s (i_s^{FB} - \nu_s)^2 + q_s^{FB} (i_s^{FB} - \delta) + \zeta_s (q_{s-}^{FB} - q_s^{FB}), \quad s = 1, 2 \quad (6)$$

and

$$q_s^{FB} = 1 + \theta_s (i_s^{FB} - \nu_s). \quad (7)$$

We provide the closed form solutions for  $i_s^{FB}$  and  $q_s^{FB}$  in Appendix A.

Due to the homogeneity property in production and frictionless capital markets, marginal  $q$  is equal to average (Tobin's)  $q$ , as in Hayashi (1982). Next, we analyze the problem of a financially constrained firm.

#### 3.2 Second-best Solution

The firm value  $P(K, W, s)$  ( $s = 1, 2$ ) satisfies the following system of Hamilton-Jacobi-Bellman (HJB) equations when its cash holding is above the financing/liquidation boundary and below the payout boundary for the current state:

$$\begin{aligned} r_s P(K, W, s) = & \max_I [(r_s - \lambda_s) W + \mu_s K - I - \Gamma(I, K, s)] P_W(K, W, s) \\ & + \frac{\sigma_s^2 K^2}{2} P_{WW}(K, W, s) + (I - \delta K) P_K(K, W, s) + \zeta_s (P(K, W, s_-) - P(K, W, s)), \end{aligned} \quad (8)$$

for  $\underline{W}_s \leq W \leq \overline{W}_s$ .

We conjecture that firm value is homogeneous of degree one in  $W$  and  $K$  in each state, so that

$$P(K, W, s) = p_s(w)K,$$

where  $p_s(w)$  solves the following system of ordinary differential equations (ODE):

$$r_s p_s(w) = \max_{i_s} [(r_s - \lambda_s) w + \mu_s - i_s - g_s(i_s)] p'_s(w) + \frac{\sigma_s^2}{2} p''_s(w) + (i_s - \delta) (p_s(w) - w p'_s(w)) + \zeta_s (p_{s-}(w) - p_s(w)). \quad (9)$$

The first-order condition (FOC) for the investment-capital ratio  $i(w)$  is then given by:

$$i_s(w) = \frac{1}{\theta_s} \left( \frac{p_s(w)}{p'_s(w)} - w - 1 \right) + \nu_s. \quad (10)$$

The implied investment response to changes in  $w$  is thus given by

$$i'_s(w) = -\frac{1}{\theta_s} \frac{p_s(w) p''_s(w)}{p'_s(w)^2}. \quad (11)$$

At the optimally chosen endogenous payout boundary  $\bar{w}_s$ , we have the following value matching condition:

$$p'_s(\bar{w}_s) = 1, \quad (12)$$

which states that the marginal value of cash is one when the firm chooses to pay out cash. Moreover, the optimality of a payout implies the following super contact condition (see, e.g., Dumas (1991)) holds:

$$p''_s(\bar{w}_s) = 0. \quad (13)$$

For the lower endogenous financing boundary, the economic tradeoff is significantly different from the single-regime solution as in BCW. Let  $\underline{w}_s$  denote the endogenous lower boundary for equity issuance in state  $s$ . Let  $m_s$  denote the “return target” financing level in state  $s$  per unit of capital. A key result in BCW is that the firm shall never raise external equity before it exhausts its cash because the firm always has the option to raise equity in the future and the financing term does not change over time (i.e. constant financing opportunity). That is, in the single-regime setting of BCW, the firm optimally chooses  $\underline{w} = 0$ . However, this is no longer necessarily optimal in our setting with stochastic financing opportunity. For example, the firm might issue equity before it runs out of cash due to the concern that financing costs could rise in the future.

If the firm chooses to issue equity at  $\underline{w}_s$ , we have the following value matching and smooth

pasting conditions:

$$p_s(\underline{w}_s) = p_s(m_s) - \phi_s - (1 + \gamma_s)(m_s - \underline{w}_s), \quad (14)$$

$$p'_s(m_s) = 1 + \gamma_s. \quad (15)$$

If the firm uses external financing, it first pays the fixed equity issuance cost  $\phi_s$  per unit of capital and then incurs the marginal issuance cost  $\gamma_s$  for each unit of equity it raises. The condition (14) gives the accounting relation for firm value immediately before and after issuance. Next, because the firm optimally chooses its external financing at the margin, we have the marginal benefit of issuance  $p'_s(m_s)$  is equal to the marginal cost of doing so, which yields the condition (15).

To determine the lower issuance boundary  $\underline{w}_s$ , we use the following argument. First, suppose that optimal  $\underline{w}_s$  is interior, i.e.  $\underline{w}_s > 0$ . Then, the standard optimality condition implies that the derivatives of the left and the right sides of (14) with respect to  $\underline{w}_s$  should be equal. This argument gives the following condition:

$$p'_s(\underline{w}_s) = 1 + \gamma_s. \quad (16)$$

However, if there exists no  $\underline{w}_s$  such that the above condition holds, we have the corner solution, i.e.  $\underline{w}_s = 0$ .

Using the above procedure, we obtain the optimal lower boundary  $\underline{w}_s \geq 0$ . Then, we need to compare whether external equity issuance (the above strategy) or liquidation is optimal. The firm has an option to liquidate. The firm's capital is productive and thus its going-concern value is higher than liquidation. Intuitively, the firm never chooses to exercise its liquidation option before it runs out cash. Therefore, under liquidation, we have

$$p_s(0) = l_s. \quad (17)$$

Therefore, the firm chooses equity issuance provided the equilibrium firm value  $p_s(0)$  is greater than  $l_s$ .

Finally, we specify the value function outside of the financing and payout boundary. If the firm has too much cash in regime  $s$  (i.e.  $w > \bar{w}_s$ ), it will reduce its cash holding to  $\bar{w}_s$  immediately by

making a lump-sum payout. That is, we have

$$p_s(w) = p_s(\bar{w}_s) + (\bar{w}_s - w). \quad (18)$$

This scenario is possible when the firm with high cash holding moves into a state with a lower payout boundary. Similarly, it is possible that  $w$  falls below the financing boundary  $\underline{w}_s$  when the firm with low cash holding moves into a state with a higher financing boundary. In this case, the firm will immediately issue external equity and restore the cash balance to the target level  $m_s$ ,

$$p_s(w) = p_s(m_s) - \phi_s - (1 + \gamma_s)(m_s - w). \quad (19)$$

In the remainder of the paper, we explore the model's predictions by working out a few different economic settings. We first consider two special cases, one where the firm is trying to survive a temporary shutdown of the market for external financing, the other where the firm is anticipating a financial crisis. Then we examine the effects of hedging, as well as the matching/mis-matching of investment and financing opportunities.

## 4 Fighting for Survival in a Crisis

We begin our analysis by focusing on the impact of stochastic financing opportunities on investment and risk management. First, we examine how firms manage to survive in a financial crisis. Specifically, we refer to state 1 and 2 as  $G$  and  $B$ . State  $G$  is the normal state, where external financing is available. We set the fixed cost of equity issuance to be 1% of the firm's capital stock,  $\phi_G = 1\%$ , and the marginal cost of issuance  $\gamma_G = 6\%$ . We also set the liquidation value of assets  $l_G = 1$ . State  $B$  is a state of financial crisis, where the market for external financing is shut down. Thus, should the firm run out of cash it would then be forced into liquidation. In addition, during a financial crisis, few investors have the deep pockets or risk appetite or even incentives to acquire assets. This often leads to fire sales of assets and low liquidation value of capital in crisis (Shleifer and Vishny (1992)). Thus, we set  $l_B = 0.7$ .

The rest of the parameters remain the same in the two states: the riskfree rate is  $r = 4.34\%$ , the risk-adjusted mean and volatility of return on capital are  $\mu = 21.2\%$  and  $\sigma = 20\%$ , the rate of

depreciation of capital is  $\delta = 15\%$ , the adjustment cost parameters are  $\theta = 6.902$  and  $\nu = 12\%$ .<sup>3</sup> Finally, the cash-carrying cost is  $\lambda = 1.5\%$ . Although in reality these parameter values clearly change with the state of nature, we keep them fixed under this scenario so as to isolate the effects of changes in external financing costs  $\phi$  and  $\gamma$ . To further simplify our analysis, we assume that the firm is currently in a financial crisis (state  $B$ ), while the normal state  $G$  is absorbing. That is, once the firm reaches state  $G$ , it remains there permanently. The firm exits the crisis state with transition probability  $\zeta\Delta t$  over time period  $\Delta t$ , and as a benchmark we set  $\zeta = 0.3$ . Whenever applicable, the parameter values are at the annual frequency.

The firm's behavior in the absorbing state  $G$  will be identical to that in a model with constant financing opportunities (see BCW). Figure 1 plots the average  $q$  and its derivative, as well as the investment-capital ratio  $i(w)$  and its derivative in this state. The average  $q$  is a natural measure of the value of capital. It is defined as the ratio between the firm's *enterprise value*,  $P(K, W, s) - W$ , and its capital stock:

$$q_s(w) = \frac{P(K, W, s) - W}{K} = p_s(w) - w. \quad (20)$$

The sensitivity of average  $q$  to changes in cash holdings is thus given by

$$q'_s(w) = p'_s(w) - 1. \quad (21)$$

We may interpret  $q'_s(w)$  as the (net) marginal value of cash, as it measures how much the firm's *enterprise value* increases with an extra dollar of cash. The firm's investment-capital ratio  $i_s(w)$  and investment-cash sensitivity  $i'_s(w)$  in each state are given by equations (10) and (11), respectively.

After reaching the permanent and absorbing state  $G$ , the firm's financing follows pecking order in that it always first uses internal funds before tapping external funds. The payout boundary is  $\bar{w}_G = 0.49$ , and the return target for equity issuance is  $m_G = 0.17$  (marked by the two vertical lines on the graphs). As the cash-capital ratio  $w$  rises, the financing constraint gets relaxed. As a result, both the average  $q$  and investment rise with  $w$ , while the net marginal value of cash and the investment-cash sensitivity fall with  $w$ . The transition intensity  $\zeta$  naturally has no impact on the results in the absorbing state.

Next, we turn to the crisis state  $B$ . In this state, while the firm's overriding concern is *survival*

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<sup>3</sup>Other than volatility  $\sigma$ , we choose the technology parameters by following Eberly, Rebelo, and Vincent (2009).

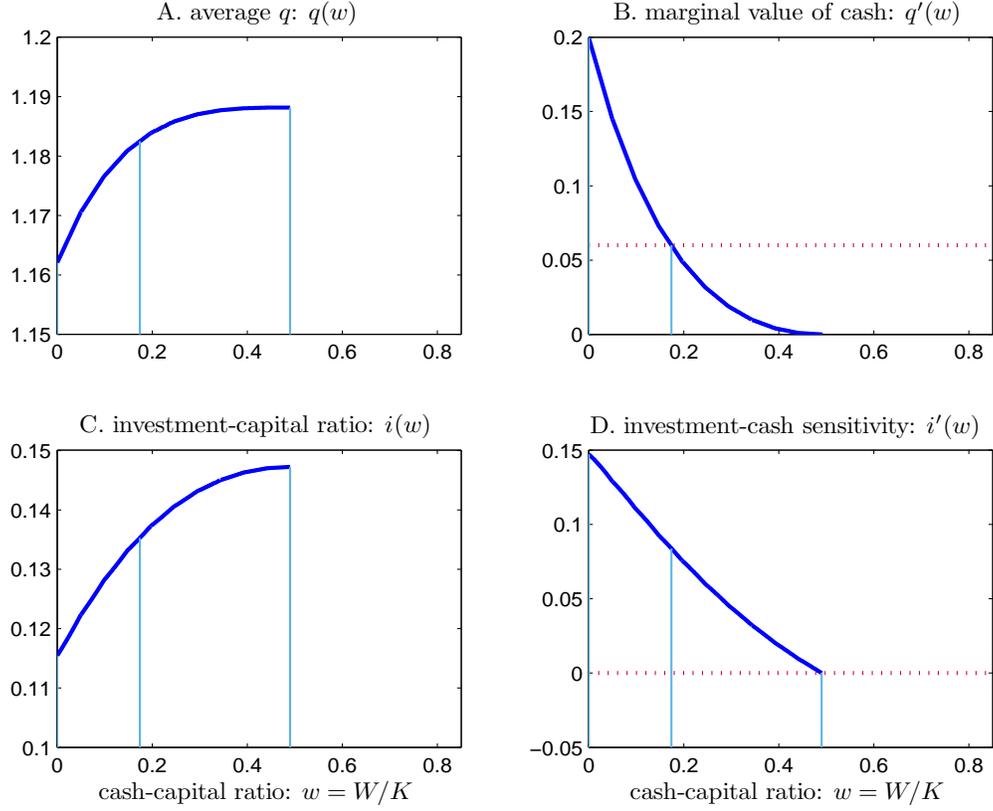


Figure 1: **Firm value and investment in (absorbing) state  $G$ .** This figure plots the average  $q$  and investment in state  $G$  for the case where  $G$  is absorbing. Costly external financing is available in state  $G$ , but not in  $B$ . All parameter values are given in Section 4.

due to the lack of any external financing, it also anticipates an improvement in the financing opportunities. Thus, a rise in the probability of leaving the crisis state can have two effects. First, it might induce the firm to behave opportunistically when choosing its optimal policies with the hope that external financing will become available soon. Second, it also raises the continuation value for the firm, which makes the firm place extra weight on survival in order to preserving its going concern value. The tradeoff between these two effects determines how the firm times (i.e. delay) payout and investment when it is in the crisis state.

Figure 2 plots the average  $q$  and investment in state  $B$ . Panel A plots  $q_B(w)$  and gives the optimal payout boundary  $\bar{w}_B$  in the transitory state  $B$ . Regardless of the transition probability, the average  $q$  always starts at  $l_B = 0.7$  due to liquidation at  $w = 0$ . When the probability of exiting a crisis increases, the firm responds by reducing its cash holding. The payout boundary  $\bar{w}_B$  falls

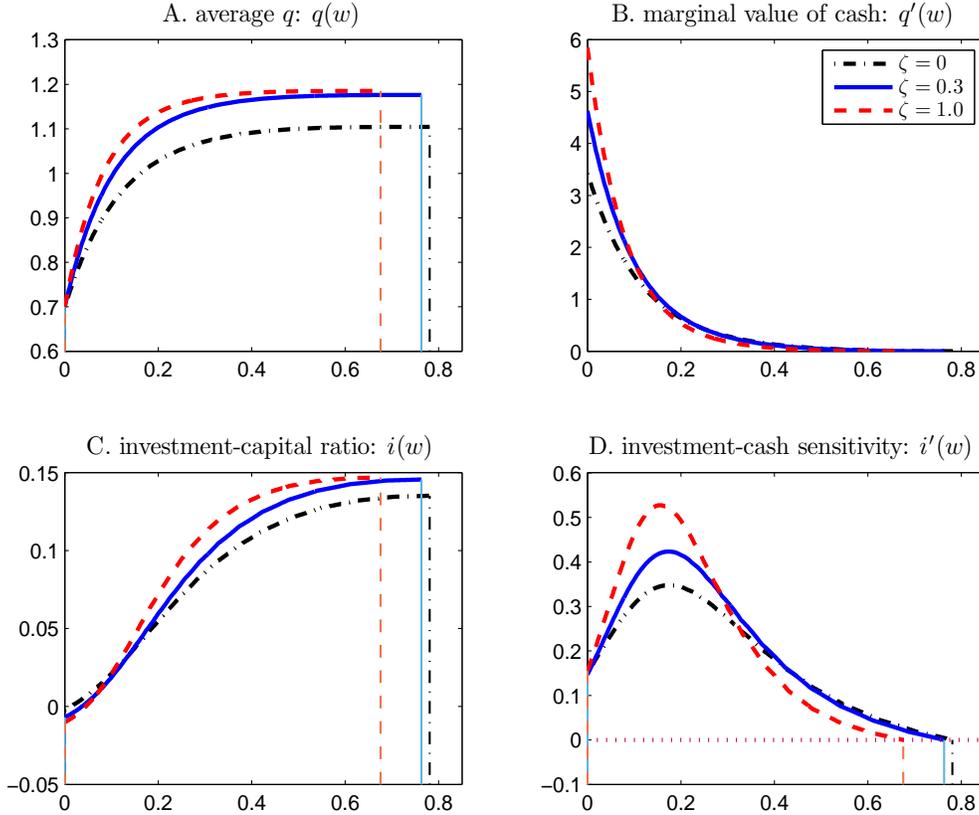


Figure 2: **Firm value and investment in (transitory) state  $B$ .** This figure plots the average  $q$  and investment in state  $B$  for the case where  $G$  is absorbing. Costly external financing is available in state  $G$ , but not in  $B$ . We consider three transition probabilities  $\zeta = 0, 0.3, 1.0$ . All other parameter values are given in Section 4.

from 0.78 to 0.76 and then to 0.68 as  $\zeta$  rises from 0 to 0.3 and then to 1.0. Moreover, while moving from state  $B$  to  $G$  can increase firm value significantly when the cash holding is low, the effect is much smaller when the cash holding is high. This difference reflects the fact that the firm uses precautionary savings to cushion the impact of severe financing constraints.

Panel B plots the net marginal value of cash  $q'_B(w)$  in state  $B$ . As  $w$  approaches 0, the marginal value of cash rises significantly because an extra dollar of cash can reduce the chance of costly liquidation. Moreover, when the cash holding is low, the marginal value of cash increases with the transition probability  $\zeta$ . This result might appear counter-intuitive because a higher probability of ending the crisis ought to help relax the financial constraint the firm is facing. However, the marginal value of cash not only reflects how close the firm is to running out of cash, but also the

*going concern value* for the firm. Since the firm value is higher in state  $G$ , a higher probability of transiting out of state  $B$  increases the continuation value of the firm, which tends to raise the marginal value of cash. This second effect is especially strong when  $w$  is small, which explains the increase in  $q'_B(w)$  with  $\zeta$ . On the other end (i.e. for sufficiently high  $w$ ), the effect from the continuation value is small. Thus, the marginal value of cash decreases with  $\zeta$  because the prospective of leaving the crisis lowers the firm's precautionary demand for cash.

It is interesting to compare the payout boundary  $\bar{w}_G$  with the payout boundaries  $\bar{w}_B$  in state  $B$  for different values of transition intensity  $\zeta$ . Not surprisingly, in all three cases the firm holds more cash compared to in state  $G$ . Also, the net marginal value of cash is substantially smaller in state  $G$ , as it reaches at most \$0.2 as  $w \rightarrow 0$ , as opposed to as high as \$6 in state  $B$ . Again, this is due to the fact that the firm has access to external equity markets once entering into state  $G$ .

Panel C and D plot the investment-capital ratio  $i(w)$  and investment-cash sensitivity  $i'(w)$  in state  $B$  for the same transition probabilities  $\zeta = 0, 0.3, 1.0$ . With sufficiently high  $w$ , investment increases with  $\zeta$ . This is because the firm that is more likely to leave the crisis state has lower marginal valuation of cash and hence invests more and is less concerned with preserving cash, *ceteris paribus*. However, on the other end, as the firm depletes its cash (i.e.  $w \rightarrow 0$ ), investment  $i(w)$  is *lower* for the firm with a *higher* probability of exiting the crisis state  $\zeta$ . This result is closely related to the result on marginal value of cash in Panel B. Underinvestment acts as a risk management device. The value of managing risk in constrained periods (low  $w$ ) is greater and hence underinvestment is more severe when the *going concern value* is higher, i.e. when  $\zeta$  is higher. The different behavior at the lower and higher ends of  $w$  highlights the importance of dynamic risk management. Panel D shows that the investment-cash sensitivity  $i'(w)$  is positive but non-monotonic in  $w$ . Kaplan and Zingales (1997) show that investment increases with net worth (i.e.  $i'(w) > 0$ ) but cannot sign  $i''(w)$  in static settings. In the special case that we illustrate here, the sensitivity  $i'(w)$  is positive, and indeed can be either increasing or decreasing in  $w$ .

In summary, when current financing is impossible but may improve in the future, the potential change of financing terms in the future affects the firm's payout and investment policies. When the firm is less likely to exit the crisis state it hoards more cash on average. However, the firm's investment policy is cash dependent and not monotonic in probability of leaving the crisis state.

## 5 Market timing: building a war-chest when financing is cheap

The previous section focuses on survival in a crisis. By assumption, the firm cannot issue equity in bad times and can *always* issue equity at any time in the good state. Therefore, the option value to “time” the equity market, i.e. lumpy equity issuance before the firm runs out of its cash and absolutely has to raise cash for survival, is zero.

In this section, we analyze the case where market timing naturally arises. Suppose that the firm is currently in state  $G$ , where the firm has access to external financing, but this state is transitory. The firm leaves the current state  $G$  with probability  $\zeta \Delta t$  over period  $\Delta t$  and enters state  $B$ , i.e. the crisis state, where the firm cannot access external financing and can only survive on internal funds. Therefore, external financing window only exists in the state  $G$  and hence has limited duration. The predictable worsening of stochastic financing conditions generates option value for the firm to “time” the market. The intuition is as follows. On one hand, tapping external equity markets while the firm still can build war chest for future investment needs. On the other hand, deferring external financing saves financing costs and the subsequent cash carry costs. The firm chooses its equity issuance together with its capital expenditure and payout policies to tradeoff these margins to maximize its value.

The firm’s behavior in the absorbing state  $B$  is essentially the same as in BCW. Figure 3 plots the average  $q$  and its derivative, as well as the investment-capital ratio  $i(w)$  and its derivative in the absorbing state  $B$ . Clearly, the transition likelihood  $\zeta$  into state  $B$  has no role on average  $q$  and investment  $i(w)$ . If the firm runs out of cash in state  $B$ , inability to raise external funds leads to immediate liquidation. Average  $q$  thus is equal to the liquidation value  $l_B = 0.7$  at  $w = 0$ . Average  $q$  is concave in  $w$  (as in BCW). The net marginal value of cash  $q'(w)$  decreases from 3.5 to 0 monotonically as we increase  $w$  from zero. As in BCW, investment is increasing but is not necessarily concave in cash, i.e.  $i'(w)$  is positive but not monotonic (see Panels C and D).

Now we turn to the transitory state  $G$ . Figure 4 plots firm value, investment, and their sensitivities in state  $G$ . Panel A plots average  $q$  for three levels of transition intensity  $\zeta = 0, 0.3, 1.0$  out of state  $G$ . It shows that the more likely the firm leaves state  $G$  for state  $B$  (a higher  $\zeta$ ), the lower firm value is.

Importantly, firm value is no longer globally decreasing in  $w$ . Financial constraints induce the

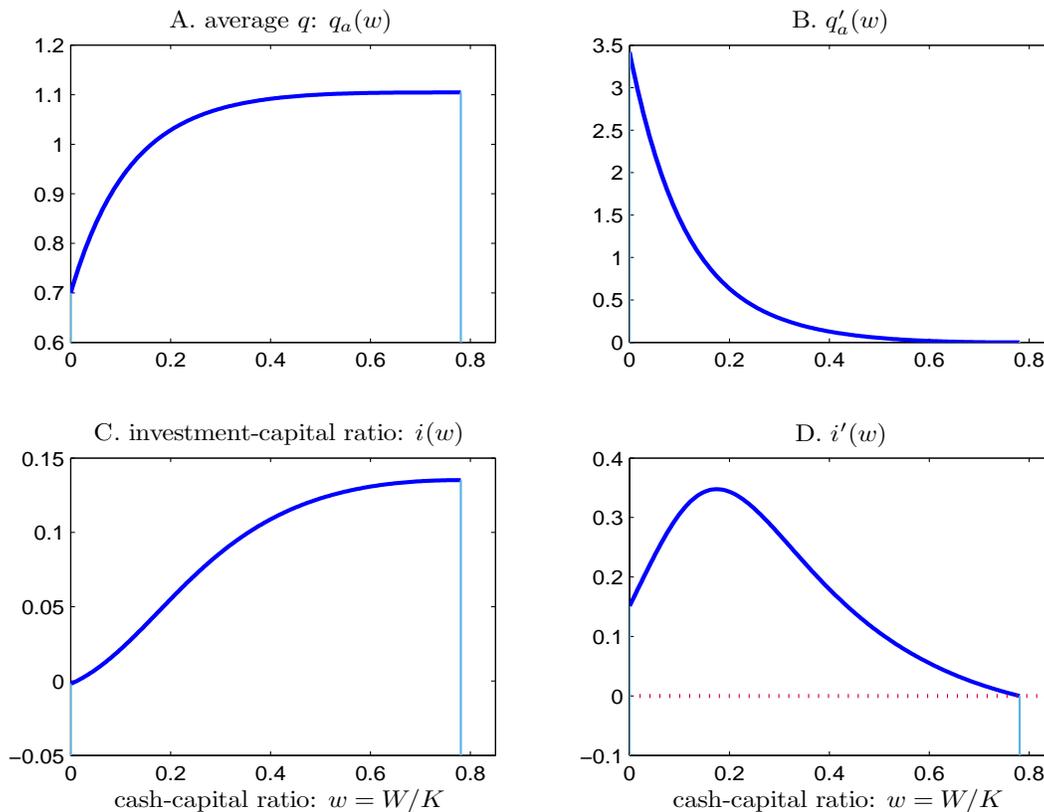


Figure 3: **Firm value and investment in (absorbing) state  $B$ .** This figure plots the average  $q$  and investment in state  $B$  for the case where  $B$  is absorbing. Costly external financing is not available in state  $B$ . All parameter values are given in Section 4.

firm to hoard cash for precautionary motives. As a result, in almost all models featuring financial constraints, firm value is increasing and concave in financial slack. In our model, the firm also has precautionary motive. Unlike other models, financing conditions are stochastic and external financing windows may have finite durations.

We also show that investment does not necessarily increase with  $w$ . That is,  $i'(w)$  can be negative. These results are fundamentally different from the standard results in the financial constraint literature. In almost all models with financial constraints, investment increases with financial slack. The debate in the investment/financial constraint literature has primarily centered around the economic effects of change in financial slack on investment sensitivity  $i'(w)$ , i.e. it is about the second order derivative  $i''(w)$  as between Kaplan and Zingales (KZ) and FHP.<sup>4</sup>

<sup>4</sup>See Stein (2003) for a summary on the issue of investment/cash sensitivity.

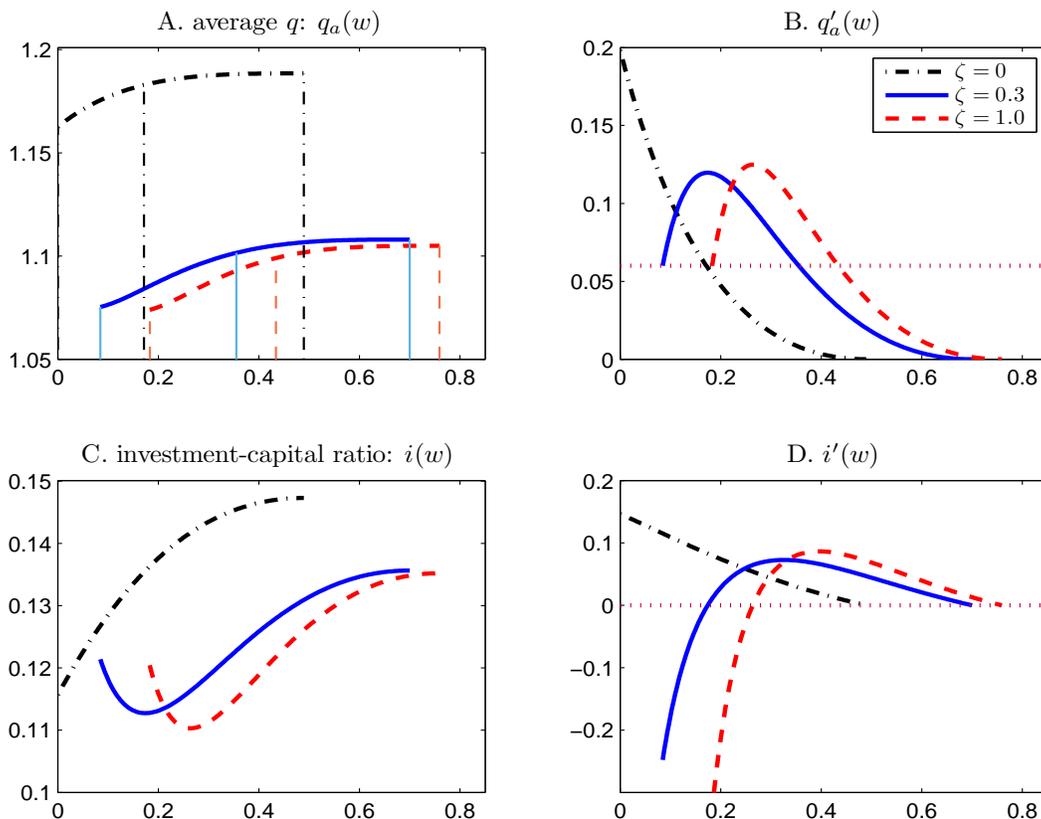


Figure 4: **Firm value and investment in (transitory) state  $G$ .** This figure plots the average  $q$  and investment in state  $G$  for the case where  $G$  is transitory. Costly external financing is available in state  $G$ , but not in  $B$ . We consider three transition probabilities  $\zeta = 0, 0.3, 1.0$ . All other parameter values are given in Section 4.

Figure 4 shows that for sufficiently high  $w$  (e.g.  $w \geq 0.17$  with  $\zeta = 0.3$  and  $w \geq 0.26$  with  $\zeta = 1$ ),  $q(w)$  is concave. This is intuitive. When the firm has sufficient amount of cash, its cash hoarding is primarily for precautionary motive. Therefore, the marginal value of cash falls with cash. The firm's equity issuance decision is quite distant and hence equity timing option is out of money. Recall that the sign of  $i'(w)$  is determined by  $p''(w)$ . In that region, investment responds positively to cash because  $q'_G(w) < 0$ . See Panels C and D of Figure 4. To sum up, in our model, when the firm has sufficient financial slack, the firm behaves effectively in the same way as in standard models with financial constraints.

However, when  $w$  is sufficiently low (e.g.  $w \leq 0.17$  with  $\zeta = 0.3$  and  $w \leq 0.26$  with  $\zeta = 1$ ), the firm has more longer-term financing needs. Because external financing option is not always around,

the firm may issue equity before it exhausts cash. The finite duration of the option to time the equity market and the fixed cost of external finance together make firm value potentially convex in cash. With relatively low current cash holding (close to equity issuance boundary  $\underline{w}$ ), the expected cash holding in the next period is high (due to the likely lumpy equity issuance in the near future). This anticipation effect of higher expected cash in the next period coupled with need to speed up financing (otherwise, the option may disappear) substantially weakens the firm's incentive to distort investment to replenish cash. Anticipating that a sizable amount of cash is coming in the near future, the firm increases its investment when it gets closer to issuance boundary. That is,  $i'(w) < 0$  as  $w$  approaches its issuance boundary  $\underline{w}$ . Instead, the firm wants to invest in productive capital and to prepare its self for its much higher *expected* cash holding. This forward looking and anticipated discrete change of cash holding is the key to drive this non-monotonic result.

Consider the limiting case when state  $G$  is absorbing (i.e.  $\zeta = 0$ ). The firm issues equity only when it runs out of its cash (i.e.  $\underline{w} = 0$ ). To capitalize on the fixed cost, the firm issues a lumpy amount  $m = 0.17$ . There is no market timing in issuance if financing opportunity is available at all times even with fixed financing costs. This implies that firm value  $q(w)$  is globally concave in  $w$  and  $i(w)$  increases with  $w$  at all times. Anticipating that cash holding increases by a sizable amount itself is not sufficient to make the net marginal value of cash  $q'(w)$  and investment  $i(w)$  decreasing in cash  $w$ . Fixed external financing cost itself is not sufficient to generate market timing. The finite duration of (relatively cheap) financing opportunities (as captured by state  $G$ ) is important.

One potential empirically testable prediction which differentiates our model from other market timing models is to analyze the joint predictions of equity issuance and corporate investment. Our model predicts that underinvestment problem is substantially mitigated when the firm is close to equity financing, even when equity financing is driven by the firm's motive to replenish its cash reserve as in the baseline model of this section. The positive correlation between investment and equity issuance in our model is not due to good investment opportunities (the real side of the economy is held constant across the two states). In the section with recurrent states, we allow for both stochastic investment and financing opportunities.

The firm chooses to time the market earlier if the transition into the crisis is more likely. Intuitively, the shorter the financing window will be open, the more valuable it is to time the financing. Therefore, the lower issuance boundary  $\underline{w}$  increases with  $\zeta$ . The firm also chooses to

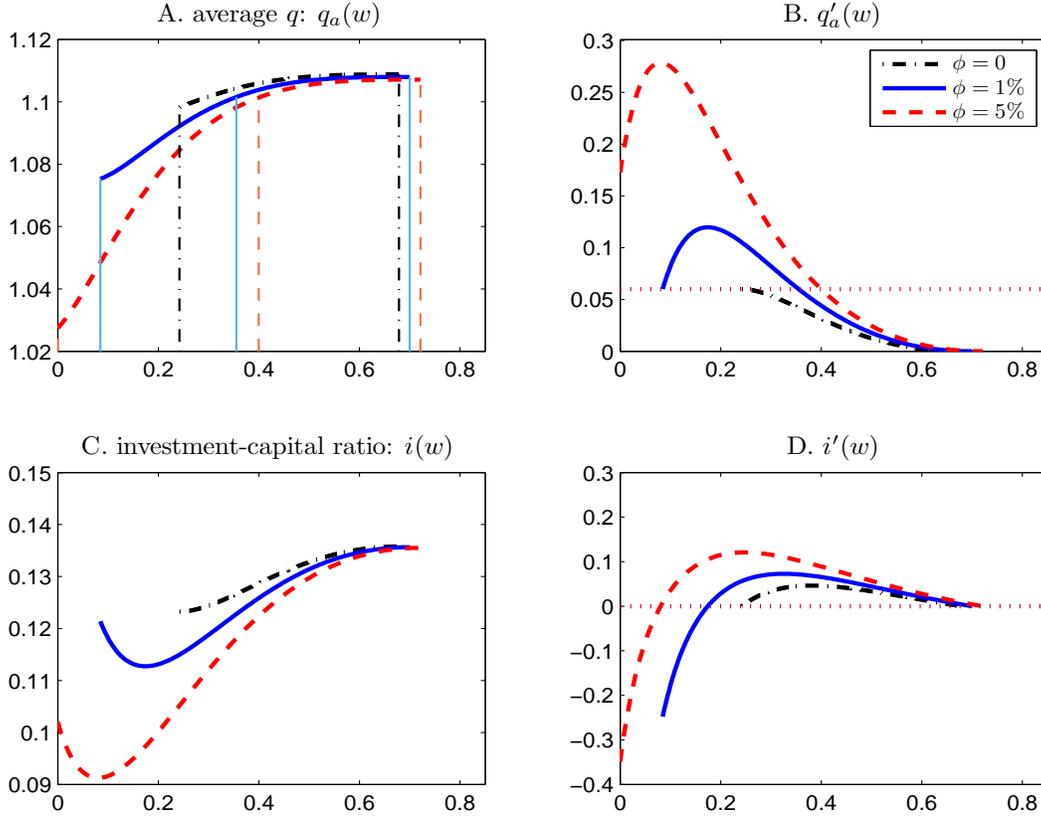


Figure 5: **The effects of financing costs on firm value and investment in (transitory) state  $G$ .** This figure plots firm value and investment in state  $G$  for the case where  $G$  is transitory. We consider three levels of fixed costs of equity financing in state  $G$ :  $\phi_G = 0, 0.01, 0.05$ . The transition intensity is  $\zeta = 0.3$ . All other parameter values are given in Section 4.

preserve more cash in response to a greater crisis risk by postponing payments to shareholders for a longer period. This can be seen by the shift to the right in the payout boundary  $\bar{w}$  as  $\zeta$  rises. In sum, Panel A shows that through a combination of market timing and reduced payout, the firm optimally responds to a greater crisis risk by holding more cash on average.

**The effects of fixed costs  $\phi$  on market timing.** In Figure 5, we plot average  $q$  and  $i(w)$  and the sensitivities with respect to cash,  $q'(w)$  and  $i'(w)$ , in the transitory state  $G$  for three levels of the fixed costs of equity financing  $\phi = 0, 1\%, 5\%$  with transition probability  $\zeta = 0.3$ .

First, the lower the fixed cost parameter  $\phi$  is, the earlier the firm issues equity. Intuitively, the firm exercises the financing option earlier if the cost of doing so is lower. In Panel A, as  $\phi$

drops from 5% to 1% and then to 0, the financing boundary  $\underline{w}$  rises from 0 to 0.08 and then to 0.24. Second, without the fixed cost (i.e.  $\phi = 0$ ), the firm issues just enough amount of equity to stay away from its optimally chosen financing boundary  $\underline{w}$ . Therefore, the net marginal value of cash cannot be higher than the marginal cost of financing,  $\gamma$ . In this case, the marginal value of cash  $q'_a(w)$  is monotonically decreasing in  $w$  as seen from Panel B, hence the firm value is globally concave in  $w$  even with market timing. **Thus, stochastic financing costs and fixed costs are both necessary to generate convexity in our model.**

When the fixed cost of issuing equity is positive but not very high, e.g.  $\phi = 1\%$ , the marginal value of cash is not monotonic in  $w$ . The firm issues equity when  $w$  reaches its endogenous issuance boundary  $\underline{w} = 0.08$ . The firm issues a lumpy amount  $m - \underline{w} = 0.27$  to capitalize on the fixed financing cost. Firm investment *increases* as the firm's cash *decreases* for  $w \leq 0.17$ . Although its cash holding is decreasing, the firm anticipates that its *expected* cash holding will be much higher after equity issuance and also his financing opportunity is not permanently available, the firm rationally times the market and increases its investment as its cash holding falls.

Finally, when the fixed cost of issuing equity is very high (e.g.  $\phi = 0.05$ ), the firm does not time the market because issuance is too expensive. As a result, the optimal equity issuance boundary is zero. However, the fixed financing cost  $\phi$  may still make the net marginal value of cash  $p'(w)$  and investment not monotonic in  $w$ . That is, the option value of timing the market is zero. However, the fixed cost of issuance may still imply that the marginal value of cash increases with  $w$  and investment decreases with  $w$  for sufficiently low  $w$ . The finite duration of financing opportunities makes the marginal value of cash lower when the firm is closer to issuance.

Our finding is yet another warning for empirical studies on the link between corporate investment and cash. A simple linear regression linking capital expenditure to average Q and cash will be misspecified and produce estimated coefficients on the cash variable that are impossible to interpret.

## 6 Market Timing and Dynamic Hedging

We have thus far restricted the firm to only use internal funds and external equity to manage risk. We now extend the model in the previous section to address the effect of hedging via derivatives such as market index futures and/or options on market timing and firm financing/investment decisions.

How does market timing affect the firm's other risk management decisions such as hedging? How does the firm's dynamic hedging strategy affect its market timing behavior?

For concreteness, we consider the setting where the firm has the option to use index futures contracts to manage its exposure to systematic risk. We denote by  $F$  the index futures price for the market portfolio. After risk adjustment (i.e. under the risk-neutral probability measure), the future prices  $F$  evolves according to:

$$dF_t = \sigma_m F_t dB_t, \quad (22)$$

where  $\sigma_m$  is the volatility of the market index portfolio, and  $\{B_t : t \geq 0\}$  is a standard Brownian motion that is correlated with the firm's productivity shock  $\{Z_t : t \geq 0\}$  with a constant correlation coefficient  $\rho$ . Note that futures price  $F$  is a martingale after risk adjustment. The interesting case is one where index futures is imperfectly correlated with the firm's productivity shock.

We denote by  $\psi_t$  the fraction of the firm's total cash  $W_t$  that it invests in the futures contract. Futures contracts require that investors hold cash in a *margin account*. Let  $\kappa_t \in [0, 1]$  denote the fraction of the firm's total cash  $W_t$  held in the margin account; cash held in this margin account incurs a flow unit cost  $\epsilon \geq 0$ . Futures market regulations typically require that an investor's futures position (in absolute value) cannot exceed a multiple  $\pi$  of the amount of cash  $\kappa_t W_t$  put in the margin account. We let this multiple  $\pi$  to be state dependent, i.e.  $\pi(s_t)$ . We therefore impose the following margin requirement:  $|\psi_t| \leq \pi(s_t) \kappa_t$ . As the firm can costlessly reallocate cash between the margin account and its regular interest-bearing account, it optimally holds the minimum amount of cash necessary in the margin account when  $\epsilon > 0$ . BCW analyze this general hedging case but under constant financing opportunities. For simplicity, we ignore this haircut on the margin account and assume that  $\epsilon = 0$ . Under this assumption, the firm allocates all cash in the margin account. Therefore, we can set  $\kappa_t = 1$ . The firm's cash holding thus evolves as follows:

$$dW_t = K_t [\mu(s_t) dA_t + \sigma(s_t) dZ_t] - (I_t + \Gamma_t) dt + dH_t - dU_t + [r(s_t) - \lambda(s_t)] W_t dt + \psi_t W_t \sigma_m dB_t, \quad (23)$$

where  $|\psi_t| \leq \pi(s_t)$ . To avoid unnecessary repetition, we only consider the case with positive correlation, i.e.  $\rho > 0$ .

We consider the more interesting case where the absorbing state is the crisis state and the firm

is currently in the transitory state  $G$ .

## 6.1 Risk management and risk taking

We first summarize the risk management rules in the absorbing state, effectively the results from BCW. Then, we analyze the hedge ratio in the transitory state  $G$ .

**In the absorbing state  $B$ .** After reaching the absorbing state, the firm faces the same decision problem as the firm in BCW does. For simplicity, in the crisis state, as in the previous section, the firm has no external financing but can enter index futures contract.

BCW show that the optimal hedge ratio (with constant financing opportunities) is given by<sup>5</sup>

$$\psi_B^*(w) = \max \left\{ \frac{-\rho\sigma_B}{w\sigma_m}, -\pi_B \right\}. \quad (24)$$

Intuitively, the firm wants to choose the hedge ratio  $\psi$  such that the firm only faces idiosyncratic volatility after hedging. The hedge ratio that achieves this objective is  $-\rho\sigma_B\sigma_m^{-1}/w$ . However, if the firm has too little cash, this hedge ratio is not attainable due to the collateral requirement. In that case, the firm chooses the maximally admissible hedge ratio  $\psi_B^*(w) = -\pi_B$ . Equation (24) captures the effect of margin constraints on hedging. Because there is no hair cut (i.e.  $\epsilon = 0$ ), the hedge ratio  $\psi$  given in (24) is independent of firm value and only depends on  $w$ . We next turn to the focus of this section: hedging in the transitory state  $G$ .

**In the transitory state  $G$ .** Before entering the crisis state, the firm has external financing opportunity. Moreover, the margin requirement may be different (i.e.  $\pi_G > \pi_B$ ). In the transitory state  $G$ , the firm chooses its investment policy  $I$  and its index futures position  $\psi W$  to maximize firm value  $P(K, W, G)$  by solving the following HJB equation:

$$\begin{aligned} r_G P(K, W, G) &= \max_{I, \psi} [(r_G - \lambda_G) W + \mu_G K - I - \Gamma(I, K, G)] P_W + (I - \delta K) P_K \\ &\quad + \frac{1}{2} (\sigma_G^2 K^2 + \psi^2 \sigma_m^2 W^2 + 2\rho\sigma_m\sigma_G\psi WK) P_{WW} + \zeta [P(K, W, G) - P(K, W, B)]. \end{aligned} \quad (25)$$

subject to  $|\psi| \leq \pi_G$ .

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<sup>5</sup>The hedge ratio given in (24) is a special case of BCW with  $\epsilon = 0$  and  $\kappa = 1$ .

When firm value is concave in cash (i.e.  $P_{WW}(K, W, G) < 0$ ), we have the same solution as in the absorbing state with margin  $\pi_G$ , i.e.  $\psi_G^*(w) = \max \{-\rho\sigma_G\sigma_m^{-1}/w, -\pi_G\}$ . However, market timing opportunities combined with fixed costs of equity issuance imply that firm value may be convex in cash, i.e.  $P_{WW}(K, W, G) > 0$  for certain regions of  $w = W/K$ . With convexity, the firm naturally *speculates* in derivatives markets. Given the margin requirement, the firm takes the maximally allowed futures position, i.e. the corner solution  $\psi_G(w) = \pi_G$ . Note that the firm is long in futures despite positive correlation between its productivity shock and the index futures. Let  $\hat{w}_G$  denote the endogenously chosen point at which  $P_{WW}(K, W, G) = 0$ , or  $p_G''(\hat{w}_G) = 0$ . We now summarize the firm's futures position in the transitory state as follows:

$$\psi_G^*(w) = \begin{cases} \max \{-\rho\sigma_G\sigma_m^{-1}/w, -\pi_G\}, & \text{for } w \geq \hat{w}_G, \\ \pi_G, & \text{for } \underline{w}_G \leq w \leq \hat{w}_G. \end{cases} \quad (26)$$

Note the discontinuity of the hedge ratio  $\psi_G^*(w)$  in  $w$ . The firm switches from a hedger to a speculator when its cash-capital ratio  $w$  falls below  $\hat{w}_G$ .

## 6.2 Quantitative Analysis

We choose the correlation between index futures and the firm's productivity shock to be  $\rho = 0.4$ . The market return volatility is  $\sigma_m = 20\%$ . The margin requirements in good and bad states are  $\pi_G = 5$  and  $\pi_B = 2$ . For all other parameter values, we use the same ones as in the previous section.

**Optimal hedge ratios  $\psi(w)$ .** Figure 6 plots the optimal hedge ratios in both states:  $\psi_G^*(w)$  and  $\psi_B^*(w)$ . First, we note that for sufficiently high  $w$ , the firm hedges in the same way in both states because hedging is costless and the firm therefore chooses its hedge ratio,  $-\rho\sigma\sigma_m^{-1}/w$ , to eliminate the idiosyncratic volatility of the productivity shock. This explains the concave and overlapping part of the hedging policies in Figure 6.

Now we turn to the region where hedging strategies differ in the two states. In state  $B$ , for  $w \leq 0.2$ , the hedge ratio hits the constraint  $-\pi_B = -2$ . In the transitory state  $G$ , firm value turns from concave to convex when  $w$  is sufficiently low. In our example, at the point  $\hat{w}_G = 0.17$ , we have  $p''(\hat{w}_G) = 0$ . For  $w \in (\underline{w}_G, \hat{w}_G)$ , firm value is convex in cash. Therefore, the firm engages in maximally allowed speculation, i.e.  $\psi_B^*(w) = \pi_G = 5$  for  $w \in (0.06, 0.17)$ . The firm seeks to load

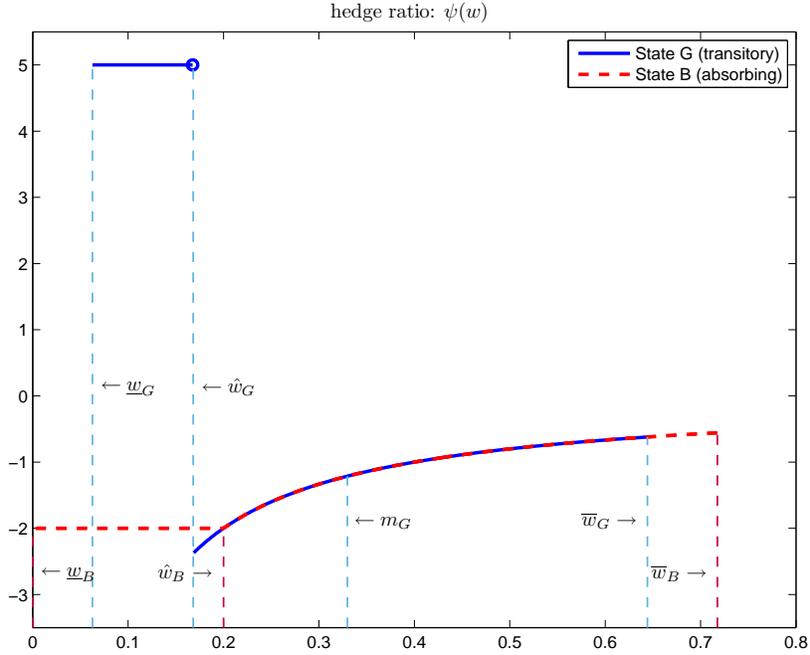


Figure 6: **Optimal hedge ratios  $\psi^*(w)$  in states  $G$  and  $B$  for the case where state  $B$  is absorbing.** The parameter values are: market volatility  $\sigma_m = 20\%$ , correlation coefficient  $\rho = 0.4$ , margin requirements  $\pi_G = 5$  and  $\pi_B = 2$ . All other parameter values are given in Section 4.

up on risk as much as possible.

**Hedging and investment in the absorbing state  $B$ .** Figure 7 plots firm value and investment-capital ratio as functions of  $w$  in the absorbing state  $B$ . For simplicity, there is no external financing opportunities in state  $B$ . We compare two cases with and without hedging opportunities in state  $B$ . As in BCW, we have the intuitive result that firm value,  $q_a(w)$ , is higher with hedging opportunities than without (Panel A). Also intuitively, when  $w$  is sufficiently high, the net marginal value of cash  $q'_a(w)$  is higher for firms without hedging opportunities than with. This is because cash plays a more important role in risk management and hence greater marginal value when the firm does not have other instruments such as index futures to manage risk. However, when cash is low, marginal value of cash is higher when the firm has hedging opportunities than not. This is due to the fact that firm is more valuable with hedging opportunities in the future than without. Hence, the marginal value of cash is greater for firms with better future prospects.

Similarly, investment-capital ratios on average are higher with hedging opportunities and also

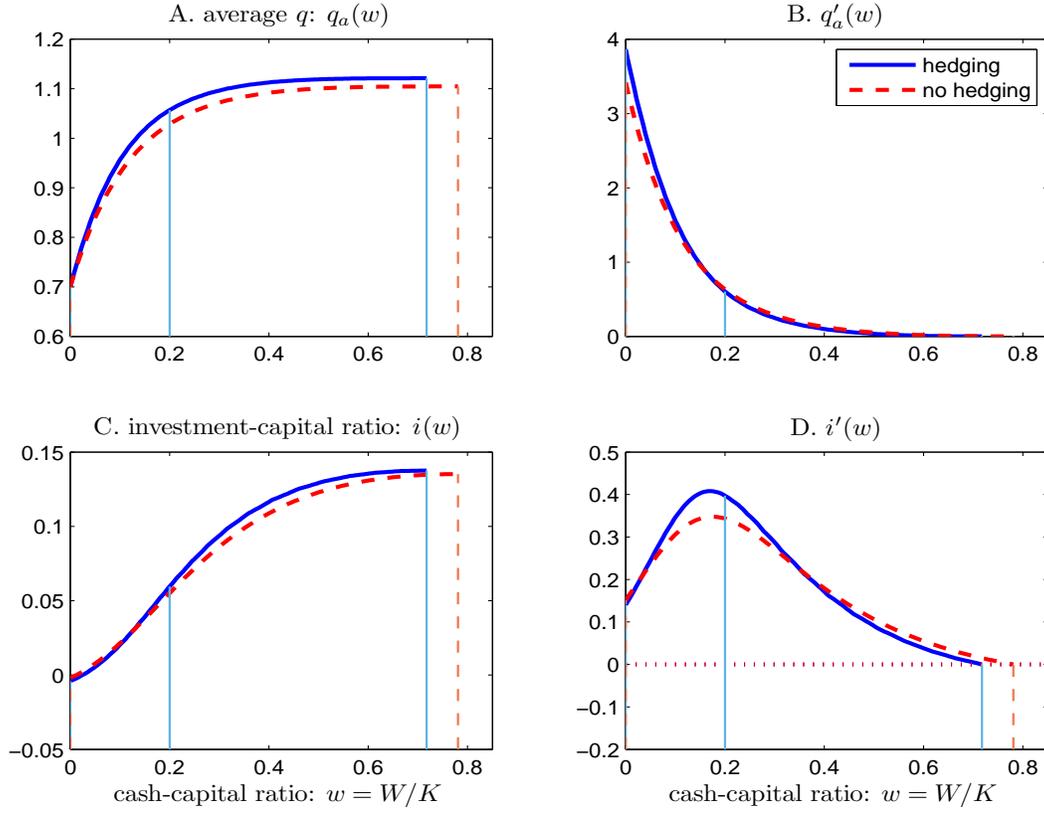


Figure 7: **Firm value and investment in (absorbing) state  $B$ .** This figure plots the average  $q$  and investment in state  $B$  with and without hedging opportunities. Costly external financing is available in state  $G$ , but not in  $B$ . For the hedging case, we set market volatility  $\sigma_m = 20\%$ , correlation coefficient  $\rho = 0.4$ , margin requirements  $\pi_G = 5$  and  $\pi_B = 2$ . All other parameter values are given in Section 4.

for firms with sufficiently high cash so that constraints are not severe. However, when cash is low, disinvestment becomes a risk management tool. By reducing investment in productive capital, while costly, the firm replenishes cash and potentially lowers other costs of risk management. The end result is higher firm value. That is, risk management in the long run is to mitigate underinvestment, however, in the short run, underinvestment can be an effective tool for risk management. This logic shows the importance of analyzing an intertemporal model with firms have more room to manage its financial slack in a state contingent way.

For both settings with and without hedging, investment increases with cash, but its sensitivity  $i'(w)$  does not necessarily increase with  $w$ . That is, in the absence of market timing, there is no non-monotonic relation between investment and cash holding.

**Hedging, investment, and market timing in the transitory state  $G$ .** We now analyze the firm's behavior in the transitory state  $G$ . Figure 8 plots firm value, investment-capital ratio, and their sensitivities as functions of  $w$  in the transitory state  $G$ . We compare the cases with and without hedging strategies. First, hedging significantly increases firm value (see Panel A). This value gain due to hedging is much greater than that in the absorbing state. Second, similar to the results in the absorbing state, the marginal value of cash is lower for firms with hedging opportunities provided that the firm is not too constrained (i.e. with enough cash). However, for cash strapped firms, the marginal value of cash is higher for firms with hedging opportunities because of the value effect, i.e. firm value is greater with hedging than without. Third, the firm issues equity later with hedging opportunities than without, i.e.  $\underline{w}$  is lower with hedging. This is also due to the fact that the marginal value of cash is higher for firms with hedging opportunities for sufficiently low cash, i.e. the value effect. Hedging lowers the volatility of the firm's productivity shock on average. With hedging, the firm hoards less cash, pays out to shareholders earlier, and raises equity later (lower issuance boundary  $\underline{w}$ ).

Now we turn the comparison between investment policies for firms with and without hedging opportunities. First, investment is on average higher with hedging than without hedging. This is essentially the observation that hedging increases firm value by mitigating its underinvestment problem as pointed out by Froot, Scharfstein, and Stein (1993). Second, while hedging mitigates underinvestment holds for most values of  $w$ , however, for sufficiently low  $w$ , investment is lower for firms with hedging than without hedging opportunities. The intuition is similar to the one in the absorbing state and also in BCW. While risk management mitigates underinvestment, sometimes, underinvestment is more efficient ways to manage risk especially when the firm is short on cash and near external financing boundary. To further increase underinvestment when firms are cash strapped, the firm manages risk better and mitigates underinvestment in the long run and hence is value enhancing. Note that with hedging, investment remains non-monotonic in cash.

Consider next the firm's conditional investment policy  $i(w)$  and the cash sensitivity of investment  $i'(w)$ . Figure 8 plots  $i(w)$  and  $i'(w)$  in state  $G$  with and without hedging opportunities.

It is interesting to note from Panel A, which plots  $i(w)$  in state  $G$ , that although *gambling* in the region for  $w \in [\underline{w}_G, \hat{w}_G]$  tends to smooth out  $p_G(w)$  this only has a marginal effect on investment close to the external financing boundary  $\underline{w}_G$ . In contrast, the effect of *hedging* on investment and

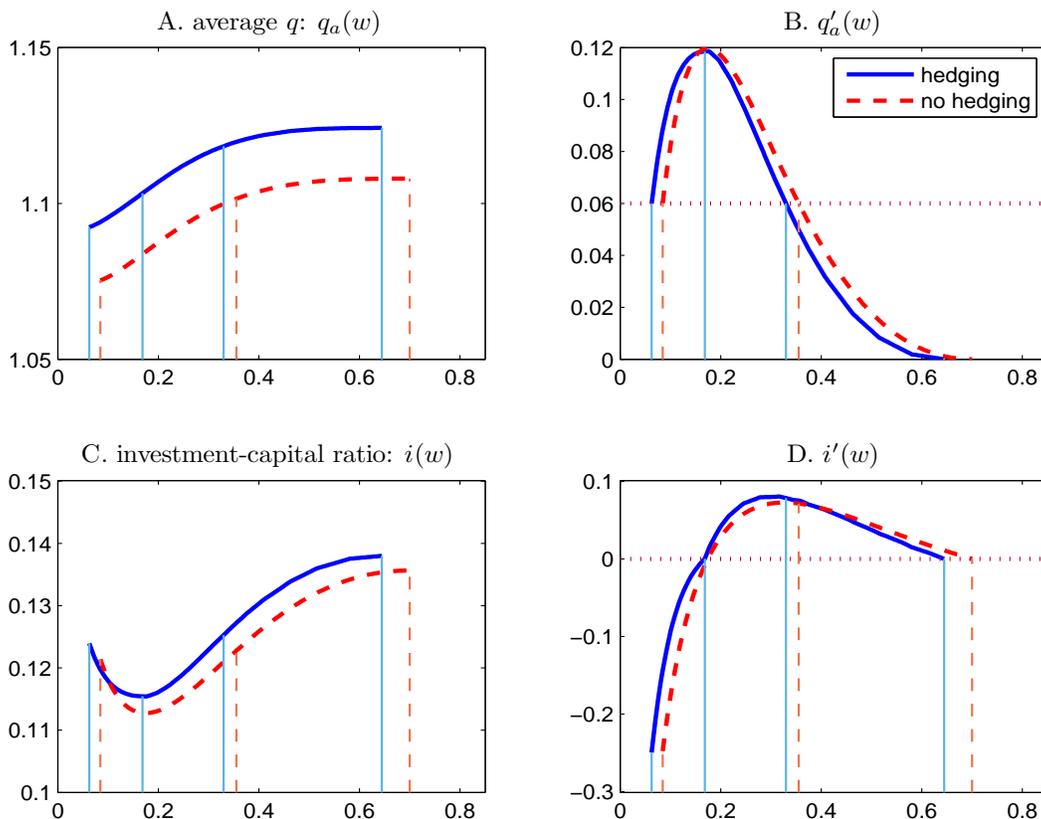


Figure 8: **Firm value and investment in (transitory) state  $G$ .** This figure plots the average  $q$  and investment in state  $G$  with and without hedging opportunities. Costly external financing is available in state  $G$ , but not in  $B$ . For the hedging case, we set market volatility  $\sigma_m = 20\%$ , correlation coefficient  $\rho = 0.4$ , margin requirements  $\pi_G = 5$  and  $\pi_B = 2$ . All other parameter values are given in Section 4.

payout for higher values of  $w$  is much more pronounced. Note also from Panel B that the effects of dynamic hedging on the cash sensitivity of investment is negligible. If anything the firm's gambling incentives close to the external financing boundary enhance the non-monotonicity of investment.

## 7 A recurrent two-state model

By assuming that one of the states is absorbing, we are able to isolate the effects of different financing conditions on the firm. In reality, firms face constantly changing financing opportunities, and it is important to see whether the main results we uncover in the previous sections are robust in such a setting. In this section, we turn to the more general recurrent two-state model.

Table 1: MODEL PARAMETERS.

This table summarizes the parameters that change over time in the recurrent two-state model. The rest of the parameters are constant:  $r_G = r_B = 0.0434$ ,  $\sigma_G = \sigma_B = 0.2$ ,  $\delta = 0.15$ ,  $\theta_G = \theta_B = 6.902$ ,  $\nu_G = \nu_B = 0.12$ .

	Technology		Financing		
	State G	State B	State G	State B	
$\mu$	0.212	0.212			
	0.242	0.182			
$l$	1	0.7	$\gamma$	0.06	0.06
$\zeta$	0.3	0.3	$\phi$	0.01	0.20

First, we consider a close extension of the settings considered in Section 4 and 5. We hold the investment opportunity constant by setting  $\mu_G = \mu_B$ , but allow financing costs to vary across the two states. Moreover, we assume that external equity financing is still available in state  $B$ , with the same proportional issuance cost ( $\gamma_G = \gamma_B$ ) but a significantly higher fixed cost ( $\phi_B > \phi_G$ ). Finally, the transition probabilities between the two states are the same ( $\zeta_G = \zeta_B$ ). Notice that these transition probabilities are under the risk-neutral measure, and can potentially differ from the actual probabilities due to the risk premium associated with each state.<sup>6</sup> Thus,  $\zeta_G = \zeta_B$  do not imply that the low/high financing costs states are equally likely over the long run. Table 1 summarizes the parameter values.

The “constant  $\mu$ ” case in Figures 9 and 10 correspond to the setting with constant investment opportunities. Qualitatively, the behavior of firm value, marginal value of cash, investment, and investment-cash sensitivity are all similar to what we see in the cases with absorbing states. The firm again times the market by issuing equity before exhausting its cash reserve in state  $G$ , with the financing boundary  $\underline{w}_G = 0.017$ . This is lower than the financing boundary in the case where state  $B$  is absorbing (see Figure 4 with  $\zeta = 0.3$ ). The payout boundary is also lower, with  $\bar{w}_G = 0.627$ . The fact that financing is still available in state  $B$  and that state  $B$  is now transitory makes the firm delay equity issuance and more willing to pay out in state  $G$ . The convexity of the value function in state  $G$  is also robust to the recurrent setting, and again we have the investment-capital ratio decreasing with the firm’s cash holding when cash is low.

<sup>6</sup>See Chen (2010) for details on how the stochastic discount factor determines the relation between the physical and risk-neutral transition probabilities of a Markov chain.

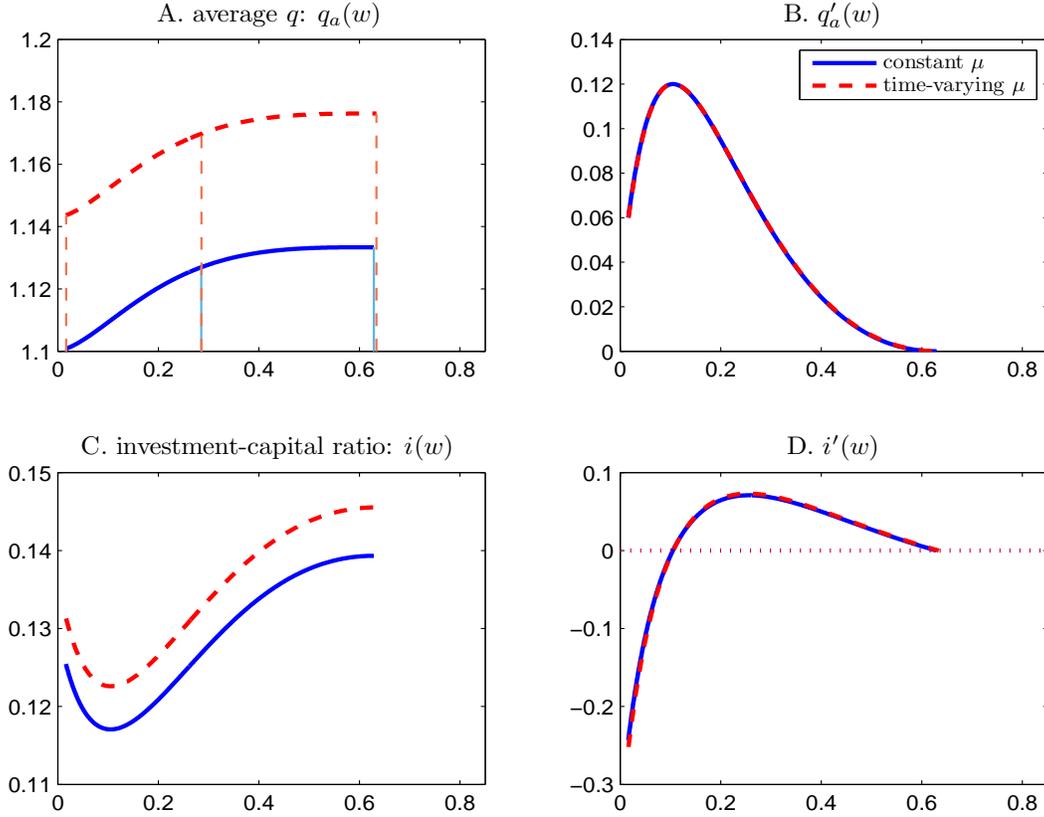


Figure 9: **Firm value and investment in state  $G$ .** This figure plots the average  $q$  and investment in the low financing cost state. We consider two settings for the investment opportunities, the first assuming constant investment opportunity, the second with high (low) productivity  $\mu$  in the state with low (high) financing costs. All parameter values are given in Table 1.

In state  $B$ , the firm has no reason to issue equity early, so  $\underline{w}_B = 0$ . It pays out when cash-capital ratio reaches  $\bar{w}_B = 0.696$ , which is sooner than in the case where state  $G$  is absorbing (see Figure 2 with  $\zeta = 0.3$ ). The fact that state  $G$  is now transitory will tend to make the firm hold more cash in state  $B$ , but this effect is dominated by the effect of lower financing costs in state  $B$ .

Since both the financing and investment opportunities are likely to fluctuate with the aggregate economy, it is natural to ask how the two would interact. Next, we consider a mean-preserving spread in  $\mu$  across the two states, with  $\mu_G = 0.242$  and  $\mu_B = 0.182$ . Due to the constant-return-to-scale technology, under the first best, a higher  $\mu$  simply has a level effect in raising the firm value and investment. Interestingly, in state  $G$  where the firm is less financially constrained, this level effect appears to hold as well. Both the average  $q$  and investment-capital ratio appear to parallel

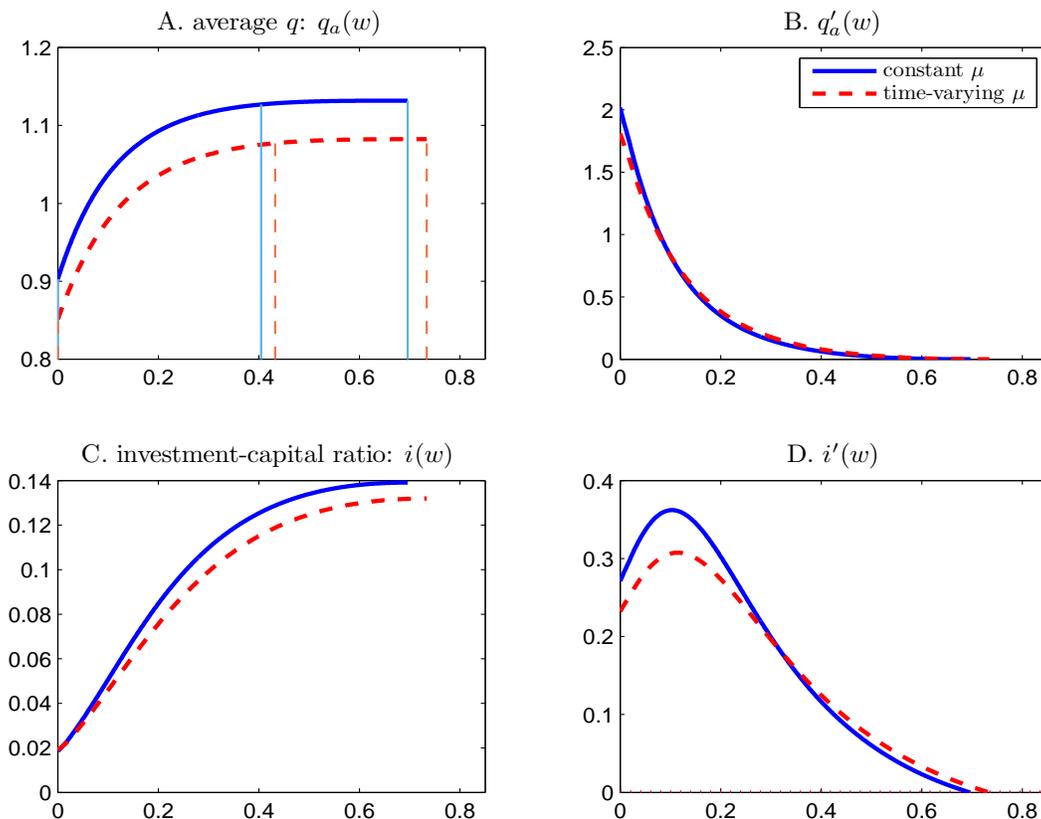


Figure 10: **Firm value and investment in state  $B$ .**

shift up from the case with constant  $\mu$ , while the net marginal value of cash and investment-cash sensitivity nearly remain the same.

In state  $B$ , financing costs start to have a bigger impact on the firm value and investment. While  $q_B(w)$  and  $i_B(w)$  are generally lower in the case with lower  $\mu_B$ , the difference becomes smaller when the cash holding is low, and the effect is especially visible for investment. As we have discussed earlier, firms underinvest more when the cash holding is low. The comparison between the case with constant and time-varying investment opportunity suggests that the effect of financing constraint on investment dominates that of the investment opportunity in the presence of large financing costs.

Finally, we see that the net marginal value of cash for the firm with time-varying investment opportunity is below that of the firm with constant  $\mu$  when  $w$  is small, but the opposite is true when  $w$  is high. The same result applies to the investment-cash sensitivity.

## 8 Conclusion

We provide a simple integrated framework of dynamic market timing, corporate investment, and risk management. Financing conditions and supply of external capital change stochastically over time. Firms anticipate the stochastic evolution of these financing opportunities and respond optimally. In particular, they optimally build war-chests by issuing equity and hoarding cash, when external financing is sufficiently cheap. For firms anticipating an equity issuance, investment may be decreasing in the firm's cash-to-asset ratio: when firms get closer to equity issuance their investment policy is less constrained by the availability of internal funds, as the firm anticipates that more cash will be raised through an equity issue in the near future. We also show that market timing is consistent with risk-seeking behavior by the firm. The key driving mechanism for these surprising dynamic implications is the convexity of firm value in its cash holdings.

While we provide the first dynamic framework to jointly study market timing, corporate investment, and risk management, our model is one with exogenous shifts of financing opportunities. It would clearly be desirable to consider a general equilibrium setting where the stochastic financing opportunities arise endogenously. We leave this for future research.

## Appendix

### A First Best Solution

Define  $y_1 = i_1 - v_1$ ,  $y_2 = i_2 - v_2$ , then (6) can be rewritten as

$$(r_1 - v_1 + \delta - y_1)(1 + \theta_1 y_1) = \mu_1 - v_1 - y_1 - \frac{1}{2}\theta_1 y_1^2 + \zeta_1(\theta_2 y_2 - \theta_1 y_1), \quad (27)$$

$$(r_2 - v_1 + \delta - y_2)(1 + \theta_2 y_2) = \mu_2 - v_1 - y_2 - \frac{1}{2}\theta_2 y_2^2 + \zeta_2(\theta_1 y_1 - \theta_2 y_2), \quad (28)$$

In general, a system of bi-variate quadratic equations

$$\alpha_{00} + \alpha_{10}x + \alpha_{01}y + \alpha_{20}x^2 + \alpha_{11}xy + \alpha_{02}y^2 = 0$$

$$\beta_{00} + \beta_{10}x + \beta_{01}y + \beta_{20}x^2 + \beta_{11}xy + \beta_{02}y^2 = 0$$

can be rewritten as

$$f(x) = \alpha_{20}x^2 + (\alpha_{10} + \alpha_{11}y)x + (\alpha_{00} + \alpha_{01}y + \alpha_{02}y^2) = a_2x^2 + a_1x + a_0 = 0 \quad (29)$$

$$g(x) = \beta_{20}x^2 + (\beta_{10} + \beta_{11}y)x + (\beta_{00} + \beta_{01}y + \beta_{02}y^2) = b_2x^2 + b_1x + b_0 = 0 \quad (30)$$

In the case where  $b_2 = 0$  and  $b_1 \neq 0$ , from (30) we get  $x = -b_0/b_1$ . Plugging  $x$  into (29) gives

$$a_2b_0^2 - a_1b_0b_1 + a_0b_1^2 = 0, \quad (31)$$

which is a quartic equation in  $y$ .

It is straightforward to map (27-28) into (29-30), where

$$a_2 = \frac{1}{2}\theta_1$$

$$a_1 = -[(r_1 - v_1 + \delta)\theta_1 + \zeta_1\theta_1]$$

$$a_0 = \zeta_1\theta_2y_2 + (\mu_1 - r_1 - \delta) = a_{00} + a_{01}y_2$$

$$b_2 = 0$$

$$b_1 = \zeta_2\theta_1$$

$$b_0 = \frac{1}{2}\theta_2y_2^2 - [(r_2 - v_2 + \delta)\theta_2 + \zeta_2\theta_2]y_2 + (\mu_2 - r_2 - \delta) = b_{00} + b_{01}y_2 + b_{02}y_2^2$$

Then we obtain a quartic equation in  $y_2$ :

$$\sum_{i=0}^4 c_i y_2^i = 0,$$

with

$$c_0 = a_2b_{00}^2 + a_{00}b_1^2 - a_1b_1b_{00}$$

$$c_1 = 2a_2b_{00}b_{01} - a_1b_1b_{01} + a_{01}b_1^2$$

$$c_2 = a_2b_{01}^2 + 2a_2b_{00}b_{02} - a_1b_1b_{02}$$

$$c_3 = 2a_2b_{01}b_{02}$$

$$c_4 = a_2b_{02}^2$$

The roots of the quartic equation are available in closed form (e.g. via Ferrari's method). When the solution exists, there will be a pair of real roots, from which we pick the one that is increasing in  $\mu$ . Finally, we have  $y_1 = -b_0/b_1$ .

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