Financial Expertise as an Arms Race*

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Abstract

We propose a model in which firms involved in trading securities overinvest in financial expertise. Intermediaries or traders in the model meet and bargain over a financial asset. As in the bargaining model in Dang (2008), counterparties endogenously decide whether to acquire information, and improve their bargaining positions, even though the information creates adverse selection. We add to this setting the concept of "financial expertise" as resources invested to lower the cost of later acquiring information about the value of the asset being traded. These investments are made before the parties know about their role in the bargaining game, as proposer or responder, buyer or seller. A prisoner's dilemma arises because investments to lower information acquisition costs improve bargaining outcomes given the other party's information costs, even though the information has no social benefit. These investments lead to breakdowns in trade, or liquidity crises, in response to random but infrequent increases in asset volatility.

1 Introduction

Credit markets "froze up" in 2007-2008 following a drop in housing prices, an increase in uncertainty about the value of asset-backed securities, and the failures or government bailouts of several major financial institutions. Traditional lending relations were disrupted. Businesses, municipalities, and consumers around the world were unable to obtain credit. Economists and other informed observers agree that the negative consequences for real economic activity were dramatic and are likely to persist for some time.

Why were so many financial intermediaries unwilling to trade with each other, despite the apparent gains to trade? Financial economists explain these market failures as a consequence of adverse selection. Firms in the financial sector had invested vast resources transforming relatively straight-forward securities, such as residential mortgages and credit-card debt, into complex instruments through securitization. They had then created trillions of dollars worth of derivative contracts based on these asset-backed securities. To facilitate this, financial firms hired legions of highly trained and highly compensated experts to design, value, and hedge the complex securities and derivatives. Unfortunately, when housing prices fell and default rates rose, the complexity of the financial instruments, and the opacity of the over-the-counter markets where they traded, made it extremely difficult to identify where in the system the riskiest or most impaired liabilities were located. Estimates for the fundamental value of these financial instruments became highly uncertain and volatile. Indeed, the expertise firms had developed may have worked against them in the crises. Their relative advantage in valuing securities may have increased the asymmetric information they faced in dealing with relatively uninformed parties, who were in a position to supply liquidity.

The benign explanation for how the financial system reached this point is that such liquidity crises are an unfortunate consequence of increasing the efficient allocation of risk through more complex derivative securities. Individuals, firms, and regulators must learn about new financial innovations through experimentation, before their risks, and the limitations of models and systems that support their use, are fully understood. There may well be considerable truth to these arguments. We offer an interpretation of these phenomena that is less benign, but which may also help

us move to a more complete understanding of them.

We develop a model in which the investments by firms in financial expertise, such as hiring Ph.D. graduates to design and value financial instruments of ever increasing complexity, becomes an "arms race." By this phrase we mean two things:

- Investment in financial expertise confers an advantage on any one player in competing for a fixed surplus that is neutralized in equilibrium by similar investment by his opponents.
- Investment in financial expertise is dangerous, in that it creates a risk of destruction of the surplus itself when there is an exogenous shock.

Our model shows that financial firms involved in trading assets with uncertain value may find optimal to acquire socially undesirable levels of expertise and this might interfere with the efficient functioning of financial markets. In the model, traders (or financial intermediaries generally) acquire expertise in processing information about an asset. The resulting efficiency in acquiring information gives them an advantage in subsequent bargaining with competitors. Neither the information, nor the expertise in acquiring and evaluating it, has any social value in the model. Yet intermediaries build such expertise despite the knowledge that it may increase adverse selection in subsequent trading and cause breakdowns in liquidity.

A basic problem in viewing financial expertise as an arms race is addressing the more fundamental question of why anyone would acquire information about a common-value object when doing so creates adverse selection problems that limit gains to trade. In most models with adverse selection in finance, some party is exogenously asymmetrically informed. If they could (publicly) avoid becoming informed, they would do so.

For example, in the classic setting described in Myers and Majluf (1984), an owner-manager-entrepreneur wishes to finance investment in a new project by selling securities to outsiders who know less about the intrinsic value of his existing assets than he does. The positive Net Present Value of this new investment is common knowledge. The entrepreneur is assumed to have acquired his information through his past history managing the firm. This informational advantage, however, is an impediment to the entrepreneur in his dealings with the financial markets, as it costs him

gains to trade associated with the NPV of the new investment. If he could manage the firm's assets effectively without acquiring this information, he would do so in order to minimize frictions associated with financing. Similarly, used-car dealers would not choose to employ expert mechanics if they could manage the car dealership without them and thus avoid the costs of the lemons problem in dealing with customers.

Given the obvious value of precommitting *not* to acquire information, why do we see financial firms, whose major business is intermediating and facilitating trading, investing vast resources in expertise that speeds and improves their ability to acquire and process information about the assets they trade? Indeed, as these entities have hired more Ph.D. graduates in finance, economics, and mathematics, and built up the elaborate information systems required to support their activities, they have increased the complexity and opacity of the financial instruments the experts and expert-systems are used to evaluate. That this has a social cost in the form of adverse-selection problems is quite evident from the massive breakdowns in liquidity associated with the sub-prime mortgage crises.

A recent paper by Dang (2008) offers a model of endogenous information acquisition in alternating-offer bargaining. The model illustrates why traders might decide to acquire costly information about a common-value asset, even though the anticipation of that acquisition may lead trade to break down in equilibrium. In Dang's model, a buyer and a seller bargain over an asset with uncertain value and symmetric gains to trade. Each party has the opportunity to pay a cost, which is exogenous and symmetric across the buyer and seller, to determine the value of the asset. Such a model of the trading process is suited to an environment where traders become informed as a matter of choice, rather than as a necessary consequence of doing business. This abstracts from other reasons intermediaries might have private information about the value of the assets they trade, but doing so allows us to highlight incentives to over invest in both information and financial expertise. The bargaining mechanism employed in the model is simple and tractable, but it shares with more complex mechanisms the feature that adverse selection leads to breakdowns in trade.[1]

In our model, long-lived agents encounter randomly chosen counterparties each period, and

[1]More general bargaining mechanisms are discussed in more detail in the next section.

bargain over a financial asset in a setting similar to Dang's model.[2] At an initial date, however, agents can invest resources to lower the costs of acquiring information about the financial assets they expect to trade. We interpret these initial investments as building financial expertise. We show that intermediaries face a prisoner's dilemma that drives them to invest in expertise just to the point where, under normal circumstances, additional investments would lead trade to break down because of adverse selection. The resulting equilibrium is therefore delicate. Jumps in volatility lead to market failures. Even when occasional high-volatility regimes are anticipated ex ante, agents optimally overinvest in expertise even though this leads to market failures and a loss of gains to trade when volatility is high.

The model in our paper is naturally interpreted as trading in an over-the-counter market, since trade involves bilateral bargaining rather than intermediation through a specialist or an exchange. Most of the complex securities associated with high levels of financial expertise are traded over the counter—including mortgage- and asset-backed securities, collateralized debt obligations (CDOs), credit default swaps (CDSs), currencies, and fixed-income products such as treasury, sovereign, corporate, and municipal debt. Several models of over-the-counter trading have been proposed in the literature, such as Duffie, Garleanu and Pedersen (2005) and Duffie, Garleanu and Pedersen (2007). In these models search frictions and relative bargaining power are the sources of illiquidity. The search frictions are taken as exogenous. Investments in "expertise" that reduced search frictions would be welfare enhancing, and would lead to greater gains to trade. In contrast, adverse selection is the central friction in our model. Investments in expertise are socially wasteful and put gains to trade at risk.

Other models such as Carlin (2008) view financial complexity as increasing costs to counterparties. In Carlin (2008), however, the financial intermediary directly manipulates search costs to consumers, so these costs are most naturally interpreted as hidden fees for mutual funds, bank accounts, credit cards, and other consumer financial products. Our intent is to model an arms race among equals—intermediaries trading with each other in the financial markets. We interpret

[2]This portion of our analysis in Section 4 generalizes Dang's model to allow for asymmetric costs, as this is required to study the impact of investments that lower the costs of information acquisition. Dang's analysis of the situations where the first mover in the bargaining game is informed contains errors that alter the nature of the solution. We provide a more complete characterization of the equilibria, as well as correcting these errors.

financial expertise as a relative advantage in verifying the value of a common-value financial asset in an environment where the complexity of the security, or the opacity of the trading venue, makes this costly.

Economists since Hirshleifer (1971) have recognized that in a competitive equilibrium, private incentives may lead agents to overinvest in information gathering activities that have redistributive consequences but no social value. Our model captures, in addition, the potential these investments have to create adverse selection, and thus destroy value beyond the resources invested directly in acquiring information. In addition, agents in our model behave strategically, rather than competitively, so we can capture the prisoner's dilemma they face, which drives them to invest in expertise in gathering information.

The general notion that economic actors may over-invest in professional services that help them compete in a zero-sum game goes back at least to Ashenfelter and Bloom (1993), which empirically studies labor arbitration hearings and argues that outcomes are unaffected by legal representation, as long as both parties have lawyers. A party that is not represented, when his or her opponent has a lawyer, suffers from a significant disadvantage. In this setting, however, the investment in legal services is not destructive of value beyond the fees paid to the lawyers. In our setting, expertise in finance has the potential to cause breakdowns in trade since it creates adverse selection.

Baumol (1990) and Murphy, Shleifer, and Vishny (1991) draw parallels between legal and financial services in arguing that countries with large service sectors devoted to such "rent-seeking" activities grow less quickly than economies where talented individuals are attracted to more entrepreneurial careers. They do not directly model the source of rent extraction, as we do.

Hauswald and Marquez (2006) show that banks will overinvest in acquiring information, as they sometimes do in our model. The banks in their model, however, acquire information about the credit worthiness of borrowers because it softens price competition between the banks as they compete for market share. Information is socially useful in their setting in efficiently allocating credit. We model the interaction between financial intermediaries in their role as traders, where higher expertise facilitates the (inefficient) acquisition of information and leads to periodic break downs in trade that can naturally be interpreted as periods of illiquidity.

The paper is organized as follows. In the next section we describe the model. Section 3 informally describes the central tradeoffs that drive the results. Section 4 studies the equilibria of the subgame where financial firms meet and bargain over the price of an asset. In Section 5 we evaluate the decision to invest in financial expertise, and prove our main result. Section 6 concludes and outlines directions for extensions and additional research. Proofs of lemmas and propositions, along with some other technical derivations, are contained in the Appendix. A supplementary appendix, available from the authors or on their websites, provides a detailed analysis of the uniqueness and symmetry of the equilibrium in expertise.

2 Model

There is a continuum of risk-neutral and infinitely-lived financial intermediaries or traders. In each period t, $t = 1, \ldots, \infty$, trader i meets a random counterparty drawn from the set of potential traders, and they have the opportunity to exchange a financial security. When they meet, agent i is assigned the role of buyer or seller with equal probability, and his counterparty assumes the other role. At $t = 0$ trader i can invest resources, denoted e_i, in financial expertise. This serves to lower the cost of acquiring information about the values of the assets he will be bargaining over in future trading encounters.

The asset traded in any given encounter has a common-value component, v. Private values to the buyer and seller generate gains to trade. The buyer's valuation of the object is $v + \Delta$, and the seller's valuation is $v - \Delta$. The gains to trade are common knowledge and constant through time.

The common value is independently distributed through time. It can be high, $v = v_h$, or low, $v = v_l$, with equal probability. The distance between the two possible values is a measure of the uncertainty about that asset's value, or its volatility. We assume the common values are drawn from two possible regimes, high-volatility and low-volatility. The high-volatility regime is defined by more uncertainty concerning the common value—a larger distance between the possible outcomes for the common value, $(v_h - v_l)$. The high volatility regime occurs infrequently, with probability π, compared to the low-volatility regime. Traders know, when they engage in bargaining, whether they are in the high or the low volatility regime. They do not, however, know whether the value of

the asset is high or low unless they incur a cost to acquire that information. Our central result is that for any size jump in volatility there is a sufficiently small π such that it is an equilibrium for the traders to invest in expertise even though it results in liquidity breakdowns when the high-volatility regime occurs.

Suppose i is the seller and j is the buyer in a particular trading encounter. Then at a cost of $c_s = c(e_i)$ the seller can engage in research activities to evaluate the asset and determine its value. The cost of determining the value to the buyer is $c_b = c(e_j)$. We will refer to a decision to incur this cost as a decision to "investigate." The cost of investigation, $c(e)$, is positive, continuous, twice continuously differentiable, convex, and monotonically decreasing in the resources invested at date zero in building expertise ($c'(e) < 0, c''(e) > 0$). Expertise, which can be viewed as both human capital and the infrastructure to support it, allows a trader or an institution to more quickly and efficiently value a security. We assume that all agents who do not acquire expertise face the same investigation cost, $c(0) > 0$.

Trading is an ultimatum game as in Dang (2008). The buyer decides, first, whether or not to pay the cost c_b to learn the asset's value, and then makes a take-it-or-leave-it offer to the seller to exchange the asset at price p. The seller decides whether to pay the cost c_s, and determine the value of the asset, and then whether to accept the offer. The seller's investigation decision is, thus, conditioned on the buyer's offer. If the buyer's price is accepted, trade occurs and the buyer receives a payoff of

$$v + \Delta - p$$

while the seller gets

$$p - (v - \Delta).$$

This trading game is a relatively simple mechanism, in which the consequences of adverse selection are stark and (relatively) straightforward to characterize. The resulting tractability allows us to endogenize the acquisition of information, and the investment in expertise. We can then highlight the tradeoff between bargaining power gained with expertise and the increased risk of illiquidity, which is our central focus. The effects adverse selection has on trading outcomes in this

setting, however, are similar to those in more complex and general mechanisms. Trade "breaks down" when parties bargaining are asymmetrically informed about valuations, even if it is common knowledge that there are gains to trade. For example, Myerson and Satterthwaite (1983) demonstrate that no bilateral trading mechanism (without external subsidies) achieves efficient ex-post outcomes. Efficient mechanisms all involve mixed strategies that with non-zero probability lead to inefficient allocations. Samuelson (1984) shows that when only the responder is informed, exchange occurs if and only if the proposer can successfully make a take-or-leave it offer, as we assume he can in our model. Admati and Perry (1987) show in pure strategy bargaining games that asymmetric information results in costly delays in bargaining. Thus, illiquidity, or the loss of gains to trade in some circumstances, is a general feature of bilateral bargaining with asymmetric information. It is in no way unique to our setting.

We assume that all random variables are drawn independently across time, and that the trading histories of firms are not observable, consistent with the opacity of OTC markets. Levels of expertise, which are the result of investments made at $t = 0$, are known to all counterparties. These assumptions ensure that agent i plays the same trading game in each period, conditional on the expertise of his counterparty.

Reversing the role of the buyer and seller, as proposer or responder, has no effect on our results beyond notational complexities, since we assume that agents do not know who will be the buyer and who will be the seller until they meet. They invest in expertise before this, at $t = 0$. We assume what is predetermined is who moves first, buyer or seller, and then allow uncertainty over who becomes buyer and who becomes seller. Alternatively, we could assume that it is predetermined which agent is buyer or seller, and that who moves first is uncertain. This would add notational complexity, since the two cases (buyer moves first or seller moves first) would need to be tracked through the subgame. What is important for our results is that, at the point when they decide to invest in expertise, the agents face symmetry with respect to their chances of being the proposer or responder in the ultimatum trading game.

Information acquired by paying c_s or c_b has no social value in this model. It simply serves to increase one's share of a fixed pie, unless it destroys value by shutting down trade due to

adverse selection. Similarly, investments in expertise, since they only serve to alter the information acquisition costs, are also socially wasteful. We are abstracting from any broader benefits to expertise and information acquisition, such as improved risk sharing or better coordination of real investment due to more informative prices. This highlights the incentives to engage in an arms race in expertise, despite the costs of adverse selection it engenders. In a more general model, where information and expertise in acquiring and efficiently processing it do add value, the incentives we highlight would lead to *over*investment in financial expertise.

3 Outline of Central Tradeoffs

In each bargaining subgame, the gains to trade are 2Δ. Under symmetric information, since each party moves first half the time, the expected surplus to each is Δ.

Consider first the two extreme cases.

With $c_s = c_b = 0$ it is simple to show that acquiring information is always a best response for the responder. Given this, the first-mover will also acquire information. Trade occurs at two fully informed prices of $v_h - \Delta$ and $v_l - \Delta$ and all gains to trade are realized.

Similarly, if the costs are very high regardless of expertise, then in equilibrium neither party will acquire information, trade occurs at $p = E(v) - \Delta$, and the buyer extracts all the gains to trade.

Most of the intuition driving our results can be illustrated by considering the effect of the seller's information cost on the strategies and payoffs around the point where those costs are so high that the buyer can extract the full surplus. The critical level of the investigation cost for the responder that precludes information acquisition is evident from the analysis in Dang's paper. It is

$$\frac{1}{4}(v_h - v_l),$$

which Dang (2008) refers to as the "gains from speculation." The quantity $(v_h - v_l)$ also has a natural interpretation in financial settings as a measure of volatility, or uncertainty about the asset's fundamental value. Thus, the incentive to investigate, which is socially wasteful, is more

difficult to resist when volatility is high. Learning whether an asset is worth 50 or 150 is more valuable than learning whether it is worth 99 or 101.

Assume the buyer does not acquire information, and attempts to extract all the surplus by offering $p = E(v) - \Delta$. If the seller accepts the offer, he gets an expected payoff of zero. If he pays the cost to investigate and determine the value, he will only sell if $v = v_l$. Trade will occur with probability 0.5, and the seller's expected payoff at the time he must decide whether to pay the cost will be

$$\begin{aligned} \frac{1}{2}\left[p - (v_l - \Delta)\right] - c_s &= \frac{1}{2}\left[(E(v) - \Delta) - (v_l - \Delta)\right] - c_s \\ &= \frac{1}{4}(v_h - v_l) - c_s. \end{aligned} \tag{1}$$

The seller will not investigate when gains from speculation, net of the cost of investigation, are negative. Thus, if the seller's cost is sufficiently high, the buyer can make a take-it-or-leave-it offer, and extract the full surplus of 2Δ. The buyer will do this irrespective of his own information acquisition cost, since he cannot do better than an expected payoff of 2Δ.

Now suppose c_s is slightly lower than $\frac{1}{4}(v_h - v_l)$, and let

$$\varepsilon \equiv \frac{1}{4}(v_h - v_l) - c_s.$$

If the buyer offers the seller's unconditional reservation price in this situation, he knows the seller will investigate and only sell if the value is low. This subjects the buyer to adverse selection, and occasions a loss of the gains to trade half of the time. By offering a slightly higher price,

$$p = E(v) - \Delta + \eta,$$

where $\eta > 0$, the buyer may be able to "bribe" the seller to accept the offer unconditionally. If the seller investigates he will have to pay c_s. He will only trade, and collect the "bribe," half of the time. If the seller accepts unconditionally, he earns the full price premium, η, and saves the information acquisition cost.

The unconditional-trade outcome is obviously better for the buyer. If the seller accepts unconditionally he gains the full surplus of 2Δ, less the price premium of η, while if the seller investigates the gains to trade are lost half of the time and, worse, when trade does occur he buys at an average price and loses money because the asset's value is low.

How big must the price premium η be? If the seller investigates, his payoff is

$$\begin{aligned} \frac{1}{2}\left[(E(v)-\Delta+\eta)-(v_l-\Delta)\right]-c_s &= \frac{1}{4}(v_h-v_l)-c_s+\frac{1}{2}\eta \\ &= \varepsilon+\frac{1}{2}\eta. \end{aligned} \tag{2}$$

If he accepts unconditionally, the seller receives η. For the latter to be attractive to the seller, we must have $\eta \geq 2\varepsilon$. The buyer wants to pay as little as possible, and so the equilibrium price will be:

$$\begin{aligned} p &= E(v)-\Delta+2\varepsilon \\ &= E(v)-\Delta+2\left[\frac{1}{4}(v_h-v_l)-c_s\right] \\ &= v_h-\Delta-2c_s. \end{aligned} \tag{3}$$

The seller receives an expected surplus of

$$2\varepsilon=\frac{1}{2}(v_h-v_l)-2c_s$$

while the buyer earns

$$2\Delta-2\varepsilon=2\Delta-\frac{1}{2}(v_h-v_l)+2c_s.$$

Notice the the payoffs of the buyer and seller are symmetric and linear in the seller's cost and do not depend on the buyer's cost. The lower the seller's cost, the more credible the seller's threat to investigate, which would create adverse selection and destroy gains to trade, and the higher the price premium the buyer must offer to prevent this.

Now consider the consequences of allowing the traders to invest resources at an initial date—

to build financial expertise that lowers the information acquisition costs they face in subsequent trading. At date zero, when investments are made in financial expertise, an agent has an equal chance of becoming the buyer or seller on encountering a counterparty. Suppose all agents invest zero in expertise, $e_i = e_j = 0$, and that at zero investment the cost of investigation rules out information acquisition by the seller, $c(0) > \frac{1}{4}(v_h - v_l)$. In equilibrium at these costs, the buyer makes an unconditional offer that extracts the full surplus from the seller, $p = E(v) - \Delta$, and earns a surplus of 2Δ, while the seller earns no surplus.

This gives the traders a powerful incentive to lower their respective investigation costs. If agent i can invest to the point where $c(e_i) < \frac{1}{4}(v_h - v_l)$ he is pushed into the region where, whenever he is the seller, he captures part of the surplus. The price the buyer pays him increases linearly as his cost declines. By lowering his costs agent i protects himself in subsequent bargaining whenever he turns out to be the seller. This costs him nothing when he turns out to be the buyer, because the division of surplus depends only on the seller's costs. When agent i is the buyer, his opponent's costs are the result of his opponent's investment choice, which is taken as given in a Nash equilibrium. As long as the resources invested in expertise, e_i, are less than the discounted value of half the seller's surplus in the subgame at the lower cost, it is a dominant strategy to make the investment in expertise.

By symmetry the same arguments apply to any other agent j, so the resulting prisoner's dilemma will force the agents to invest in expertise to lower investigation costs that, in that particular subgame, they never pay anyway.

Facing agent j in a particular trading encounter, agent i receives a surplus of $\frac{1}{2}(v_h - v_l) - 2c(e_i)$ as the seller, and a surplus of $-\frac{1}{2}(v_h - v_l) + 2\Delta + 2c(e_j)$ as the buyer. Agent i's expected surplus across the two possibilities is

$$\Delta - [c(e_i) - c(e_j)]. \tag{4}$$

Note that this is half the total surplus of 2Δ, less the excess of his cost over that of the other agent. Taking his counterparty's expertise as given, each agent has an incentive to increase his own expertise, as it helps him when he is the responding party in bargaining, but has no effect on his payoffs when he is the proposer.

Where will the traders stop investing? What, beyond the curvature of the function $c(\cdot)$, limits the arms race? One possibility is that the price premium paid by the proposing buyer to the responding seller increases to the point where the seller is absorbing the entire surplus. Then any additional investment will make any offer by the buyer unprofitable, and trade will simply break down. Another possibility is that the buyer's share of the surplus will decline to the point where he will be better off acquiring information himself. The trading game then becomes a signalling game, in which all equilibria involve mixed strategies that rule out fully efficient trade. The analysis in the next section is devoted to delineating these boundaries formally.

In either case, the players will have incentives to keep investing until their costs reach such a boundary. At that point any decrease in their cost, or, alternatively, an increase in the volatility from $(v_h - v_l) = \sigma$ to $(v_h - v_l) = \theta\sigma$, where $\theta > 1$, will lead to breakdowns in trade. In Section 5 we demonstrate this formally by showing that for any $\theta > 1$, if the probability of the high-volatility regime, π, is low enough, then traders will invest in expertise until they reach the boundary for efficient trade in the low volatility regime. In such an equilibrium, as is clear from expression (4), the investigation costs have no effect on the efficiency of trade under normal conditions of low volatility. These costs are not actually paid in equilibrium. The ex-ante expected surplus from the trading game is the same as when costs exceed $\frac{1}{4}(v_h - v_l)$, because the gains one obtains as a seller are offset by the higher price paid as a buyer. The deadweight loss is limited to the resources expended building expertise, e_i. If volatility randomly increases trade breaks down due to adverse selection. That is, along with an "arms race," we occasionally have a "war" in which the surplus the players are competing for is either partly or entirely destroyed.

Under normal conditions of low volatility in our model, financial expertise serves as a threat in bargaining with counterparties, but it is not actually used in equilibrium. We do not mean to imply that highly trained and compensated financial professionals literally "do nothing" for their pay. Rather, these arguments illustrate that part of their value to their firms, and thus of their compensation, is due to their ability to deter others from opportunistic behavior. From a social perspective, they might be viewed as overqualified for the routine activities associated with their work. By analogy, the most highly paid divorce lawyers might well neutralize each other's impact

on the division of the divorcing couple's assets. This does not imply that they are twiddling their thumbs during their billable hours. In equilibrium, the tasks they perform might be performed as competently by lawyers with less experience, expertise, and reputation who would charge less, but those lawyers would not serve to deter the other party's more expensive counsel.

4 The trading subgame

For certain regions of the parameters and investigation costs, the first mover in the trading game may choose to become informed. This creates a signaling game, in which the proposer's offered price can reveal information to the responder. Agents will generally employ mixed strategies, creating an endogenous positive probability of trade breakdown. We will first analyze the subgame assuming the buyer, who is the first mover or proposer in the trading game, never investigates, and then study the subgame where the first mover is informed. The final subsection then evaluates when the buyer will choose to become informed, comparing his payoffs in the various subgames. We assume throughout the analysis that when indifferent between multiple responses that yield the same expected payoff, the responder (or seller) chooses the one that is best for the proposer (or buyer).

4.1 Subgame with uninformed buyer

Assume the buyer, who makes the take-it-or-leave-it offer, does not acquire information. In this subgame, the equilibrium price will depend only on the seller's investigation cost, c_s.

First, note that we can ignore prices $p < v_l - \Delta$, where the seller would not trade regardless of his information, or $p > E(v) + \Delta$, where the buyer would never trade given that he is uninformed.

For any p between these bounds, if the seller investigates, he sells only if $v = v_l$. The seller's expected surplus at the point he decides to investigate is then

$$\frac{1}{2}[p - (v_l - \Delta)] - c_s \tag{5}$$

and the buyer's expected surplus is

$$\frac{1}{2}(v_l + \Delta - p).$$

If the seller does not investigate, and accepts the offered price, the seller's surplus is

$$p - [E(v) - \Delta] \tag{6}$$

while the buyer receives

$$E(v) + \Delta - p.$$

If the seller does not investigate and refuses the offer both parties get zero.

In any situation where there is trade, regardless of the seller's response, the buyer's payoff decreases in the price. For any given response by the seller, therefore, the buyer will offer the lowest price that sustains that response. Accordingly, we need only consider three candidate prices. There is a minimum price that would cause a seller to reject the offer rather than investigating. This price, p_1^*, sets (5) equal to zero and is given by

$$p_1^* = v_l - \Delta + 2c_s. \tag{7}$$

There is a unique price at which the seller is indifferent between investigating and trading without information. This price sets (5) equal to (6), and is given by

$$p_2^* = v_h - \Delta - 2c_s. \tag{8}$$

Finally, there is the lowest price that keeps an uninformed seller from rejecting the offer and earning zero. This sets (6) equal to zero and is given by

$$p_3^* = E(v) - \Delta. \tag{9}$$

We present four lemmas that exhaust the possible outcomes to the subgame when the buyer is uninformed. The logic of the proofs, which are in the Appendix, generally follows Dang (2008),

allowing for asymmetric costs. The first result shows that if costs are high enough, efficient trade always occurs.

Lemma 1 *When the buyer does not acquire information and $c_s > \frac{1}{4}(v_h - v_l)$, then in the unique Perfect Bayesian Equilibrium to the trading subgame:*

1. *The seller does not investigate.*
2. *Trade always takes place.*
3. *The price is $p_3^* = E(v) - \Delta$.*
4. *The buyer's expected surplus is 2Δ.*
5. *The seller's expected surplus is zero.*

The next result details two sets of conditions when the surplus is split between the two counterparties, and plays a central role in our subsequent analysis. In these regions, the payoffs to the seller are decreasing in his costs of investigation, providing him with incentives to improve his bargaining position by investing in expertise and lowering these costs.

Lemma 2 *When the buyer does not acquire information and either or both of the following conditions hold*

(a.) $\max\{\Delta, \frac{1}{4}(v_h - v_l) - \Delta\} < c_s \leq \frac{1}{4}(v_h - v_l)$,

(b.) $\frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta < c_s \leq \min\{\Delta, \frac{1}{4}(v_h - v_l)\}$,

then in the unique Perfect Bayesian Equilibrium to the trading subgame:

1. *The seller does not investigate.*
2. *Trade always takes place.*
3. *The price is $p_2^* = v_h - \Delta - 2c_s$.*
4. *The buyer's expected surplus is*
$$-\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s.$$
5. *The seller's expected surplus is*
$$\frac{1}{2}(v_h - v_l) - 2c_s.$$

The next result shows that when costs are sufficiently low, the seller investigates and some gains to trade are lost when value is high.

Lemma 3 *When the buyer does not acquire information and* $c_s \leq \min\{\Delta, \frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta\}$*, then in the unique Perfect Bayesian Equilibrium to the trading subgame:*

1. *The seller investigates and trades only if* $v = v_l$*.*
2. *The price is* $p_1^* = v_l - \Delta + 2c_s$*.*
3. *The buyer's expected surplus is*
$$\Delta - c_s.$$
4. *The seller's expected surplus is zero.*

Finally, we must consider what happens when the buyer cannot make a profitable trade, either by making an offer that leads the seller to investigate or by making an offer that prevents the seller from investigating.

Lemma 4 *When the buyer does not gather information and* $\Delta < c_s < \frac{1}{4}(v_h - v_l) - \Delta$*, then in the set of perfect Bayesian Equilibria to the trading subgame:*

1. *The seller does not investigate.*
2. *The buyer offers a price* $p < v_l - \Delta + 2c_s$*.*
3. *Trade never occurs.*
4. *The buyer's expected surplus is zero.*
5. *The seller's expected surplus is zero.*

Figure 1 illustrates the full set of possible outcomes associated with the four lemmas above. While there are five possible cases, depending on the relative magnitudes of the gains to trade, Δ, and the volatility, $v_h - v_l$, all of them share the following structure. For high levels of the seller's information acquisition cost, $c_s \geq \frac{1}{4}(v_h - v_l)$, trade takes place with probability one, and the buyer, as first mover, extracts the full surplus. Below this region is an area described by Lemma 2, where

trade takes place with certainty, but the buyer and seller split the surplus. The buyer must share enough of the surplus with the seller to keep him from investigating, in order to preserve the gains to trade when value is high. As is clear from Lemma 2, this payment increases as the seller's cost declines and the threat to investigate becomes more credible. The seller's share of the surplus is maximized at the lower boundary of this region, which depends on the relative magnitudes of the gains to trade, Δ, and the volatility, $(v_h - v_l)$. Below the region covered by Lemma 2, trade breaks down in whole or in part. In Cases 2-4, for low values of the seller's cost of investigation, the seller investigates and declines the buyer's offer when the value is high. In all these situations the seller earns zero surplus (see Lemma 3). In Case 1 there is also an intermediate region where the surplus the buyer would need to pay the seller to discourage investigation is too high for the buyer to break even, and trade breaks down entirely as shown in Lemma 4.

4.2 The trading game with an informed first mover

In this section, we analyze the game under the assumption that the buyer has acquired information. Our analysis of this situation differs substantially from the results in Dang (2008).[3]

When the first mover in the trading game becomes informed with positive probability, then the trading game becomes a signaling game. We begin the analysis of the buyer's decision about whether to become informed by noting two simple features of the model.

First, if the seller's investigation cost is high enough to discourage him from ever investigating, $c_s > \frac{1}{4}(v_h - v_l)$ as in Lemma 1, then it would never pay for the buyer to become informed about the asset if $c_b > 0$. Since the buyer earns the entire surplus of 2Δ in this situation, he cannot improve his bargaining power in any way by investigating, and investigating involves a positive cost. Thus, outcomes are first-best if the seller's costs are high enough. For this case we have fully characterized the equilibrium.

[3]There are several algebraic mistakes in the proofs of Dang (2008) for this situation, which we note in our analysis below and in the Appendix. As a result our expressions for the equilibrium prices are quite different than his, and the equilibrium that is best for the buyer will in some circumstances not exist. In addition, we note that Dang (2008) treats each price offered by the informed first-mover as initiating a proper subgame. In fact, there is a continuum of equilibria that are perfect Bayesian.

Lemma 5 *If $c_s > \frac{1}{4}(v_h - v_l)$, then for any value of $c_b > 0$, the buyer chooses not to become informed about the value of the asset and the equilibrium quantities are given in Lemma 1.*

Second, as Dang (2008) shows, once the buyer does become informed, there are no best responses in pure strategies, so any equilibrium to the trading subgame must involve mixed strategies. If the buyer learns v, and truthfully offers $p_l = v_l - \Delta$ or $p_h = v_h - \Delta$, the seller would always accept without investigating. But given that strategy for the seller, the buyer could, with positive probability, offer p_l when $v = v_h$ and be better off. If, on the other hand, the seller refuses any offer less than $p = v_h - \Delta$, the buyer should always offer that price when $v = v_h$. But given a truthful offer when value is high, the seller should accept a price of $v_l - \Delta$ when value is low, as it reveals the truth. Thus, truthful offers are not an equilibrium, and neither is always offering a high price.

Similar arguments also imply that the seller will never investigate with probability one when the buyer is informed. If the seller always investigates, he will reject any offer that is not truthful. This implies, however, that the buyer will never make a misleading offer, as it costs him gains to trade. If the buyer is always truthful, then the seller has no incentive to incur the costs of investigation.

We now introduce two new critical prices. The first is the lowest price at which a seller will not expect to lose money regardless of his beliefs:

$$p_h^* = v_h - \Delta.$$

The second critical price is :

$$p_l^* = v_l - \Delta + z^*,$$

where

$$z^* \equiv \frac{1}{2}\left[(v_h - v_l) - \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s}\right].$$

This will turn out to be the equilibrium price in the best equilibrium for the buyer when the buyer investigates and the value is low *or* when the value is high and the buyer attempts to exploit his private information by offering a price that will give the seller a negative payoff if he accepts.[4]

[4] Our expression for z^* is completely different than that in Step 3d in the Appendix of Dang (2008).

It is immediate that in any perfect Bayesian equilibrium no price above p_h^* will ever be offered. As is typical in signaling games, where the informed party moves first, there are many perfect Bayesian equilibria, reflecting the many possible equilibrium beliefs the responding party might have about the buyer's type as a consequence of the offered price. We will focus on the equilibrium most favorable to the buyer. This is equivalent to assuming that the buyer can announce and commit to a finite set of prices he will charge before observing the true value.

We show that in the subgames where the buyer investigates the equilibria all involve two possible prices. When the buyer observes a low value, he always offers p_l^*. When he observes a high value, however, he mixes between this price and offering the seller p_h^*. When the seller accepts the low price in the high-value state, without investigating, then the buyer is earning information rents. Our proof of the two lemmas below shows that the buyer never mixes over more than two prices. In some circumstances, described by Lemma 7, there is a continuum of equilibria, each involving a different p_l^*, but all of them have the structure that the buyer offers that price when value is low, and mixes across it and p_h^* when value is high.

The following two lemmas describe the optimal equilibrium for the buyer in the subgame in which he decides to become informed.

Lemma 6 *When $c_s \leq \frac{2\Delta(v_h-v_l)^2}{(v_h-v_l+2\Delta)^2}$ and the buyer acquires information, then in the optimal Perfect Bayesian Equilibrium from the perspective of the buyer:*

1. *If the buyer observes $v = v_l$, the buyer offers the price p_l^*.*
2. *Using $z^* = \frac{1}{2}\left[(v_h - v_l) - \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s}\right]$, if the buyer observes $v = v_h$, he offers p_l^* with probability*
$$\frac{c_s}{2z^*}$$
and p_h^* otherwise.
3. *The seller buys if the buyer offers p_h^*.*
4. *If the buyer offers p_l^*, the seller investigates with probability*
$$\frac{2\Delta}{v_h - v_l + 2\Delta - c_s}$$
and sells only if the value is low.
5. *If the seller does not investigate, he always sells.*

6. *The buyer obtains a surplus of*

$$2\Delta - c_b - \frac{1}{2}z^*.$$

7. *The seller receives zero surplus.*

Lemma 7 *When* $\frac{1}{4}(v_h - v_l) > c_s > \frac{2\Delta(v_h - v_l)^2}{(v_h - v_l + 2\Delta)^2}$, *the least upper bound of the payoffs to the buyer in a perfect Bayesian equilibrium is given by*

$$\Delta - c_b + \frac{2\Delta^2}{v_h - v_l + 2\Delta},$$

while the greatest lower bound for the seller's payoff is zero. In any sequence of Perfect Bayesian Equilibria whose payoff to the buyer converges to the least upper bound:

1. *The price offered by the buyer when* $v = v_l$ *converges to* $v_l - \Delta$.
2. *The probability that the price offered by the buyer when* $v = v_h$ *is* p_h^* *converges to 1.*
3. *The probability that the seller sells after seeing an offer of* p_l *converges to*

$$\frac{2\Delta}{v_h - v_l + 2\Delta}.$$

4. *The probability that the seller investigates converges to 0.*

Lemma 7 describes the limit of the equilibria that are best for the buyer when the seller's costs lie in an intermediate range. Equilibria exist in which the buyer can drive the seller arbitrarily close to his participation constraint, or a zero expected payoff. He would achieve this in an equilibrium where his offered price always revealed the truth, where the seller accepted unconditionally, and where the price when value is low was $v_l - \Delta$ (i.e., $z^* = 0$). We know, however, from the arguments at the beginning of this section, that truth telling is not an equilibrium. The buyer must, with some positive probability, offer a low price when the value is high, but some surplus will accrue to the seller by making that offered price higher. Hence, the buyer will find optimal to keep the probability of offering a low price when he knows the value is high as small as possible without reaching zero.

4.3 The first mover's information acquisition decision

Given the costs of investigation for the buyer and seller, the buyer will decide whether or not to investigate the value of the asset by comparing his payoffs in the various subgames when he is informed (Lemmas 6 and 7) and when he is not (Lemmas 1-4). A formal derivation of the boundaries between the various lemmas is provided in Part B of the Appendix.

Figure 2 illustrates the set of choices for the buyer and seller described by various lemmas. Across the four panels, we vary the ratio of volatility to gains from trade, $\frac{(v_h - v_l)}{\Delta}$. (For lower values of this ratio, the resulting figures all resemble Panel (a), while for higher values they resemble Panel (d).) In each panel the cost pairs are divided into sections, numbered according to the lemma that describes play in the corresponding subgame. The dashed 45-degree line describes the set of symmetric cost pairs that could result from a symmetric equilibrium in financial expertise, where the potential counterparties' investments at date zero determine their investigation costs. As we show in the next section, the only equilibria that involve pure strategies in the choice of expertise are symmetric ones. The heavy lines separate regions where the first mover, or buyer, investigates. The boundary is derived simply by comparing the payoffs to the buyer from when he does not acquire information (Lemma 1 to Lemma 4, depending on the value of c_s) to the value when he does acquire information (Lemma 6 or Lemma 7). In each panel the lower left-hand corner, where the costs of both the proposer and responder are low, correspond to Lemmas 6 and 7. To the right of and above these cost pairs, the proposer does not pay for information and makes an uninformed offer. That region, in turn, is divided into areas corresponding to Lemma 1, where high costs for the seller and buyer lead to efficient bargaining outcomes, through Lemma 4, where no trade occurs. Note that the buyer's decision to investigate depends on the seller's costs, because he anticipates that party's response. The cost at which he decides not to become informed first rises and then falls in the seller's costs. The rate at which the buyer's payoff changes in the seller's costs varies, as we move between the regions in the figure, leading to this reversal.

Note that in every parameterization in Figure 2 the 45-degree line does not pass through the area associated with Lemma 7. This rules out symmetric equilibria in expertise with subgames in

the region where the optimal equilibrium for the buyer does not exist.[5] Rather, the choices over expertise in symmetric equilibria will put the agents at the boundaries between areas governed by Lemmas 2 and 4, where the buyer is uninformed, or Lemmas 2 and 6, where the buyer investigates before making an offer.

5 Investing in expertise

In this section we consider the equilibrium choices of investment in expertise, and show that an arms race occurs. We will show that in equilibrium, for most parameter values and cost functions, $c(e)$, investments in expertise reduce investigation costs to the lower boundary of Lemma 2.

We assume the common values are drawn from two possible regimes, high-volatility and low-volatility. In the normal, or low-volatility regime, $v_h - v_l = \sigma$. This regime occurs with probability $1 - \pi$. The high-volatility regime occurs infrequently, with probability π. The two possible values are then further apart: $v_h - v_l = \theta\sigma$, where $\theta > 1$. Traders know, when they engage in bargaining, whether they are in the high or the low volatility regime.

We focus on equilibria that are symmetric in the choice of expertise. These equilibria are a natural focal point, since it can be shown that with $\pi = 0$ there are no asymmetric, pure-strategy equilibria in expertise.[6] Our main result shows that the equilibria that prevail when there is only one, low-volatility regime ($\pi = 0$) survive when π is positive but sufficiently small.

We will begin by outlining the intuition behind the arms race as an equilibrium when $\pi = 0$, so that the agents anticipate only one possible value for volatility. The proposition that follows then formally proves this is an equilibrium, and further that the same equilibrium survives when $\pi > 0$, but small.

Accordingly, consider candidate equilibria on the dashed 45-degree lines in Figure 2. In Panel (c),where $\Delta = \frac{1}{9}(v_h - v_l)$, the boundary between the regions described by Lemmas 2, 4, and 6 intersect the 45-degree line through the origin, where investigation costs are symmetric. The costs

[5] From the expression for the boundary of the buyer's information acquisition decision it is straightforward to verify this for all $\frac{v_h - v_l}{\Delta} > 0$.

[6] A formal proof of this is in a supplementary appendix, available on the authors' web sites or on request from the authors.

at this point define a level of expertise $e^* = c^{-1}(\frac{5}{4}\Delta)$. When volatility is lower (Figure 2, panels (a) and (b)), the diagonal crosses the boundary between Lemmas 6 and 2, while if volatility is higher (Panel (d)) the diagonal passes through the Lemma 4 region, where trade does not occur.

If we consider, first, the case in Panel (d), where volatility is high, it is immediately obvious that no symmetric cost pair in which play proceeds according to Lemma 4 can be an equilibrium with positive investment in expertise. The agents would be investing resources to build expertise but not enjoying any benefit from their investment. Since investment in expertise is costly, they could unilaterally reduce this investment at date zero at no cost to their payoffs in subsequent trading.

The boundary between Lemmas 2 and 4, however, is a natural candidate for an equilibrium. Here each agent gains the full surplus as a seller. Deviations that involve additional investment in expertise will move the agents into the region governed by Lemma 4, where both parties earn zero. Even higher levels of investment will move agents into the region governed by Lemma 6. There, the additional expertise benefits the agent when he is the buyer, but because mixed strategies imply some loss in gains to trade, these benefits must be less than the loss of the full surplus of 2Δ he earns at the lower, equilibrium expertise when he is the seller.

A similar logic governs the choices when volatility is lower. Then the 45-degree line crosses the boundary between Lemmas 2 and 6. At that point, he does not earn zero as a buyer and 2Δ as a seller. The boundary in this case is such that the buyer's surplus in the subgame where he investigates is equal to his surplus when he does not and play proceeds according to Lemma 2. His expected surplus in Lemma 2 across his roles as both buyer and seller equals the full gains to trade. Under Lemma 6, in contrast, he earns zero as a seller and less than 2Δ as a buyer, because the mixed strategies necessarily imply loss of gains to trade.

Indeed, in all situations where investing in expertise provides a sufficient decrease in costs, both agents will invest in expertise up to the point where any additional investment in expertise would move them out of the Lemma 2 region.

Now suppose there is some small probability, $\pi > 0$, that volatility will jump to $\theta\sigma$. This moves the boundaries in Figure 2 to the upper right, or northeast. At the equilibrium point described above, the traders then find themselves in the interior of the no-trade or the mixed-strategy regions

in the figure, with a consequent loss of gains to trade. They can avoid this by reducing their investment in expertise. Since the jump in volatility is discrete, however, the decrease in expertise must be non-infinitesimal as well, leading to a discrete drop in the expected benefits to the trader in the low-volatility regime. For a sufficiently small π, the loss in benefits in the low-volatility state will exceed the gains to reduced expertise in the high-volatility regime. The traders will be better off remaining at the high expertise they would choose if $\pi = 0$. Occasional jumps in volatility thus lead to breakdowns in liquidity—an unwillingness to trade due to adverse selection and despite gains to trade that are common knowledge.

The next proposition, which is the central result of the paper, formalizes these arguments. Recall that in the low-volatility regime, $v_h - v_l = \sigma$, and in the high-volatility regime $v_h - v_l = \theta\sigma$, where $\theta > 1$.

Proposition 1 *Define the level of expertise e^* as:*

$$e^* = \begin{cases} c^{-1}\left(\frac{1}{4}\sigma - \Delta\right) & \text{if } \frac{\sigma}{\Delta} \geq 9 \\ c^{-1}\left(\frac{5}{36}\sigma\right) & \text{if } \frac{\sigma}{\Delta} \leq 9 \end{cases}$$

(A). For any $\frac{\sigma}{\Delta} > 0$, when $\pi = 0$ and positive investment in expertise is an equilibrium, there exists an equilibrium in which both agents invest in expertise at the level e^* if and only if $c'(e^*) \leq -(1-\delta)$. If $c'(e^*) \in [-2(1-\delta), -(1-\delta)]$, then e^* is a unique equilibrium level of expertise. If instead $c'(e^*) > -(1-\delta)$, the level of expertise e' $(< e^*)$ such that $c'(e') = -(1-\delta)$ is the unique equilibrium.

(B). For any $\frac{\sigma}{\Delta} > 0$ and $\theta > 1$, if e^* is the equilibrium in (A), there is a $\pi^\theta > 0$ such that, for all $\pi < \pi^\theta$, there exists a perfect Bayesian equilibrium where both agents invest in expertise up to e^* and play proceeds according to Lemma 2 in the low-volatility regime. However, in the high-volatility regime, trade breaks down with positive probability as play proceeds according to Lemma 4 if $\frac{\sigma}{\Delta} \geq \frac{28}{3}$ or if $\frac{\sigma}{\Delta} \in (9, \frac{28}{3})$ while $\theta \leq \frac{[\sigma-12\Delta]^2}{\sigma[28\Delta-3\sigma]}$ and according to Lemma 6 if $\frac{\sigma}{\Delta} \leq 9$ or if $\frac{\sigma}{\Delta} \in (9, \frac{28}{3})$ while $\theta \geq \frac{[\sigma-12\Delta]^2}{\sigma[28\Delta-3\sigma]}$.

In the equilibrium with $e = e^*$ agents invest to the point at which any further investment would move them out of Lemma 2 when they are the seller. In the case where $\frac{\sigma}{\Delta} > 9$, the seller receives 2Δ and the buyer receives 0. For $\frac{\sigma}{\Delta} < 9$, the situation is slightly different. Agents locate at the boundary between Lemmas 2 and 6, but this is *not* the point associated with the full surplus for

the seller. As can be seen from Figure 2, costs in this equilibrium are such that the seller's cost exceeds the threshold at which he receives the full surplus, and the surplus is divided between the buyer and the seller. Play in the low-volatility subgame is efficient in this equilibrium, but the investment in expertise is wasteful.

As the proposition makes clear, for a wide range of cost functions, e^* will be the unique equilibrium that involves pure strategy investment in expertise. However, when the effect of expertise on costs remains very strong until costs are very low, there may exist[7] another symmetric equilibrium that involves even greater investment in expertise, with a consequent loss in gains to trade, even without shifts in the volatility. This equilibrium has play proceeding according to Lemma 6 and is Pareto dominated by the equilibrium at e^*.

This can be seen in Figure 3, where we parameterize the model and numerically study the optimal investment in expertise (Panel A) and the resulting costs of investigation (Panel B). The cost of information acquisition, as a function of investment in expertise, e, and of the initial cost of investigation, C_0, is

$$c(e) = C_0 \exp\{-\frac{\lambda e}{C_0}\},$$

where we set $\lambda = 1$.

The parameterized function, $c(e)$, is decreasing and strictly convex and satisfies $\lim_{e\to+\infty} c(e) = 0$. This function also has the convenient property that

$$c'(e) = -\lambda \frac{c(e)}{C_0}.$$

This property implies that, at $e = 0$, the "marginal benefit" of expertise, $-c'(e)$, is λ for all possible starting values of C_0. It also implies that the marginal benefit of expertise at a point $c(e)$ is proportional to the ratio of the current cost $c(e)$ to the initial cost C_0. The farther the current investigation cost is from its starting point, the more expensive it will be to further reduce that cost.

In the Figure, we set the discount factor, $\delta = 0.9$. We normalize $\Delta = 1$ and set $\sigma = 10$. The

[7] Much weaker conditions for uniqueness are available, but the derivation of these is omitted for the sake of space. Details are available in the supplementary material posted on the authors' websites and available on request.

probability of the high-volatility regime, π, is 5%, and the jump in volatility is 10% ($\theta = 1.1$).

The upper panel in Figure 3 shows the best responses in the level of expertise given the initial cost of investigation, C_0, and the cost level of potential counterparties. The bottom panel shows the costs that result from optimal investment in expertise, given the same variables. The highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that constitute symmetric pure-strategy Nash equilibria in financial expertise. Notice that the costs resulting from optimal investment in expertise are flat, except for very low levels of either the agent's cost or his opponent's cost. The flat level of costs corresponds to $1.5 = c(e^*) = \frac{1}{4}(v_h - v_l) - \Delta$, the lower boundary of the region given in Lemma 2. That this is the cost associated with optimal expertise in equilibrium, for a wide range of initial costs, is evident in the fact that this level of cost results from a best response when the opponent's cost is also 1.5 (the highlighted line where the surface folds up in the figure). For an agent with a starting cost between 1.5 and 6.69, the best response to an opponent's cost of 1.5 is to acquire expertise until his investigation cost is also 1.5. Hence, when both agents have initial costs between 1.5 and 6.69, acquiring expertise until investigation cost is 1.5 is a symmetric pure-strategy Nash equilibrium.

This equilibrium is, however, delicate. Since both agents' costs reach the edge of Lemma 2, even very small increases in volatility will lead to breakdowns in trade. For this set of parameters, when the high-volatility regime occurs, both agents will end up playing according to Lemma 4 instead. Trade will therefore break down. In the high volatility state, the buyer prefers not to trade rather than purchasing a "lemon" from an informed seller. Hence, all the gains to trade are lost in the high-volatility regime. Note that if we had parameterized asset volatility to be lower, both agents could end up playing according to Lemma 6 in the high-volatility regime instead of Lemma 4, as stated in Proposition 1. The probability of trade breaking down would then be lower than 5% since trade occurs with positive probability in Lemma 6 compared to with zero probability in Lemma 4.

Figure 4 illustrates that for some initial cost values multiple symmetric pure-strategy equilibria will exist. The figure shows what panel (b) in Figure 3 would look like from above and the highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that represent symmetric pure-strategy Nash equilibria in financial expertise. At very low levels of initial costs,

i.e., when initial cost is below 1.89, we can see that a (Pareto-dominated) second pure-strategy equilibrium arises. In such equilibrium, the buyer acquires information before making an offer, and adverse selection eliminates some gains to trade. As Proposition 1 makes clear, the symmetric equilibrium at e^* with $c(e^*) = 1.5$ will be unique at least for the region of initial costs where $-2(1-\delta) < c'(e^*) < -(1-\delta)$. In the current parameterization, that equilibrium is in fact unique for the larger region where initial costs are between 1.89 and 6.69.[8]

Figure 5 makes clear that expertise acquisition leading to a delicate trading equilibrium is not unique to a particular parameterization. It shows the best response level of investment in expertise, and resulting costs of information acquisition, for parameterizations corresponding to each of Cases 2-5 in Figure 1. We use the same cost function and discount rate and set $\Delta = 1$ as before, but allow volatility σ to be 8 (Case 2), 6 (Case 3), 4 (Case 4), and 2 (Case 5). In each case, as in Figure 3, optimal investment in expertise generally pushes the agent to a flat level of costs. The exception occurs when the opponents start with very low costs of information acquisition, where investment in expertise spikes and costs drop.

The flat region where the final cost reaches one of the lower bounds defined in Lemma 2 does not allow for a symmetric equilibrium in expertise when $\frac{\sigma}{\Delta} < 9$. Instead, when initial costs are not very low nor very high, the equilibrium in expertise has the final costs of both agents reaching the level where Lemma 2 meets Lemma 6. In all cases, there are symmetric equilibria at such level of costs. Thus, equilibrium investment in expertise is pushing the agents to the boundaries of the areas in Figures 1 and 2, and outcomes will be sensitive to slight increases in volatility. In all cases, there is also an arms race in expertise for very low levels of initial costs. Finally, Lemma 1 prevails when initial costs are sufficiently high, and there is no point in investing in expertise. All the upper portion of the surface are equilibrium outcomes, since no investment in expertise occurs.

Our model predicts that, in some circumstances, financial intermediaries might find optimal to acquire expertise even though it makes trade fragile when the volatility in fundamental value increases. Investing in expertise makes it easier for an intermediary to subsequently acquire information about an asset's value and it amplifies the possibility of an adverse selection problem.

[8] We are able to characterize the conditions under which this second symmetric equilibrium will exist. A full analysis in a supplementary appendix is available from the authors upon request or from the authors' web sites.

6 Conclusion and Extensions

The model in this paper illustrates the incentives for financial market participants to overinvest in financial expertise. Expertise in finance increases the speed and efficiency with which traders and intermediaries can determine the value of assets when they are negotiating with potential counterparties. The lower costs give them advantages in negotiation, even when the information acquisition has no value to society, and even when it can create adverse selection that disrupts trade if uncertainty about the volatility of fundamental values increases too quickly or unexpectedly to allow intermediaries to adjust or scale back their investment in expertise. When volatility jumps, the increased likelihood of facing an informed counterparty may force an intermediary to acquire information before actually proposing a trade. Anticipation of this cost can make the whole idea of a trade unattractive to that intermediary. If the probability of the high-volatility regime is small enough, the gains to trade lost in the high-volatility regime will not be as important as the increase in profits that added expertise, and the ensuing improved bargaining position, bring in the low-volatility regime. The intermediary will find optimal to acquire expertise that increases expected profits in the more probable low-volatility regime, even though it decreases profits because of trade breakdowns in the less probable high-volatility regime.

Some extensions to the model may warrant additional research. Financial expertise might also allow intermediaries to increase the costs of information acquisition for their counterparties, as well as lowering their own costs. Investment in expertise permits firms to create, and make markets in, more complex financial instruments. In our notation, we can view the cost of acquiring information for agent i as $c(e_i, e_j)$, which decreases in i's own expertise and increases in that of his counterparty. The logic of our analysis suggests firms benefit from increasing the relative costs of their counterparties. The tension between the incentives to raise others' costs, which would reduce adverse selection, and lower one's own costs, which increases it, may help us better understand innovation and evolution in financial markets.

In our model, intermediaries invest in expertise only once, and the volatility states are drawn independently through time. This illustrates the consequences shocks to volatility have for liquidity. If volatility is persistent through time, and intermediaries can adjust, with some cost, their level of

expertise in response to changing volatility, then shocks to volatility will still lead to breakdowns in liquidity, but they will also trigger contractions in "expertise" which can be interpreted as employment of financial professionals. Such a model might be informative about the nature of employment cycles in financial services.

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Appendix A Proofs of Lemmas and Propositions

Proof of Lemma 1: We first show that if $c_s > \frac{1}{4}(v_h - v_l)$, then the best response of the seller at p_1^* or p_3^* is to accept the offered price without investigating. (At p_2^* the seller is by definition indifferent between accepting without investigating and investigating and accepting if $v = v_l$.)

At an offered price of p_1^* the seller's surplus from accepting unconditionally is

$$\begin{aligned} p_1^* - (E(v) - \Delta) &= v_l - \Delta + 2c_s - (\frac{1}{2}v_h + \frac{1}{2}v_l - \Delta) \\ &= 2c_s - \frac{1}{2}(v_h - v_l). \end{aligned}$$

This is positive as long as $c_s > \frac{1}{4}(v_h - v_l)$. The seller's expected surplus from investigating is, by construction, zero at p_1^*, so accepting unconditionally is a best response.

At an offered price of p_3^* the seller's surplus from accepting unconditionally is zero by construction. The payoff from investigating is

$$\begin{aligned} \frac{1}{2}[p_3^* - (v_l - \Delta)] - c_s &= \frac{1}{2}\left[\frac{1}{2}v_h + \frac{1}{2}v_l - \Delta - (v_l - \Delta)\right] - c_s \\ &= \frac{1}{4}(v_h - v_l) - c_s. \end{aligned}$$

This is negative as long as $c_s > \frac{1}{4}(v_h - v_l)$, so accepting unconditionally is a best response.

We know that at p_2^*, the seller is indifferent between investigating and accepting unconditionally. However, the seller's surplus at this price is negative if $c_s > \frac{1}{4}(v_h - v_l)$. By accepting unconditionally, the seller gets:

$$\begin{aligned} p_2^* - (E(v) - \Delta) &= v_h - \Delta - 2c_s - (\frac{1}{2}v_h + \frac{1}{2}v_l - \Delta) \\ &= \frac{1}{2}(v_h - v_l) - 2c_s \\ &< 0. \end{aligned}$$

Therefore, if offered p_2^*, the seller would decline and earn zero, yielding a zero payoff for the buyer as well. Thus, we need only compare the buyer's payoff at p_1^* and p_3^*, where the seller's best response is unconditional acceptance.

At p_3^*, since the seller receives his reservation price, the buyer collects the gains to trade of 2Δ. At p_1^* the buyer receives:

$$\begin{aligned} E(v) + \Delta - p_1^* &= \frac{1}{2}v_h + \frac{1}{2}v_l + \Delta - (v_l - \Delta + 2c_s) \\ &= \frac{1}{2}(v_h - v_l) - 2c_s + 2\Delta \end{aligned}$$

which is less than 2Δ if $c_s > \frac{1}{4}(v_h - v_l)$.■

Proof of Lemma 2: The same steps that showed unconditional acceptance to be a best response of the seller to offers of p_1^* and p_3^* when $c_s > \frac{1}{4}(v_h - v_l)$ imply investigating and accepting the offer only when $v = v_l$ is a best response when $c_s \leq \frac{1}{4}(v_h - v_l)$, which is the case under both (a.) and (b.). Similarly, we showed that the seller's surplus was negative when offered p_2^*, which makes him indifferent to acquiring information, as long as $c_s > \frac{1}{4}(v_h - v_l)$. It will be positive when this inequality is reversed.

At p_2^*, the seller is indifferent between investigating and unconditional acceptance, but the buyer prefers the latter. To see this, compare the buyer's surplus when the seller is informed

$$\frac{1}{2}(v_l + \Delta - p_2^*) = -\frac{1}{2}(v_h - v_l) + \Delta + c_s \tag{1}$$

to that when the seller accepts unconditionally,

$$E(v) + \Delta - p_2^* = -\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s. \tag{2}$$

The latter is obviously larger by $\Delta + c_s > 0$. We also know this expected surplus for the buyer is positive, since $c_s > \frac{1}{4}(v_h - v_l) - \Delta$ under condition (a.). Under the condition in (b.), note that $\Delta > \frac{1}{8}(v_h - v_l)$. Therefore,

$$\begin{aligned} -\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s &\geq -\frac{1}{2}(v_h - v_l) + 2\Delta + 2(\frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta) \\ &= -\frac{1}{6}(v_h - v_l) + \frac{4}{3}\Delta \\ &\geq -\frac{1}{6}(v_h - v_l) + \frac{4}{3}\frac{1}{8}(v_h - v_l) \\ &= 0 \end{aligned} \tag{3}$$

where the first inequality follows by substituting the lower bound on c_s from condition (b.).

We now need to evaluate the buyer's expected payoff at p_1^* and p_3^*. At p_1^*, given the seller's best response, the buyer gets

$$\frac{1}{2}(v_l + \Delta - p_1^*) = \Delta - c_s, \tag{4}$$

which is negative since $\Delta < c_s$ under condition (a.), but may be positive under (b.). Under condition (b.), the buyer's surplus at p_2^* is greater than that at p_1^* if

$$-\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s > \Delta - c_s$$

or if

$$-\frac{1}{6}(v_h - v_l) + \frac{1}{3}\Delta + c_s > 0,$$

which follows from the lower bound on c_s in (b.) We can therefore eliminate p_1^* as a candidate equilibrium price.

At p_3^*, where the seller's best response is investigation:

$$\frac{1}{2}(v_l + \Delta - p_3^*) = -\frac{1}{4}(v_h - v_l) + \Delta. \tag{5}$$

The right-hand side is negative under (a.), since $\Delta < c_s \leq \frac{1}{4}(v_h - v_l)$, but it may be positive under (b.). The buyer's surplus at p_2^* is greater than that at p_3^* if

$$-\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s > -\frac{1}{4}(v_h - v_l) + \Delta,$$

or if

$$-\frac{1}{4}(v_h - v_l) + \Delta + 2c_s > 0.$$

Using the lower bound on c_s from (b.),

$$\begin{aligned} -\frac{1}{4}(v_h - v_l) + \Delta + 2c_s &\geq -\frac{1}{4}(v_h - v_l) + \Delta + \frac{1}{3}(v_h - v_l) - \frac{2}{3}\Delta \\ &= \frac{1}{12}(v_h - v_l) + \frac{1}{3}\Delta \\ &> 0. \end{aligned}$$

This eliminates p_3^* as a candidate equilibrium.■

Proof of Lemma 3: We have already established that with $c_s < \frac{1}{4}(v_h - v_l)$, which is true here, the seller's best response at p_1^* and p_3^* is to investigate and sell only if $v = v_l$. The buyer's surplus at p_1^* is

$$\frac{1}{2}(v_l + \Delta - p_1^*) = \Delta - c_s,$$

which is positive since $c_s < \Delta$.

At p_3^* the buyer's surplus is

$$\frac{1}{2}(v_l + \Delta - p_3^*) = \Delta - \frac{1}{4}(v_h - v_l).$$

Since $c_s < \frac{1}{4}(v_h - v_l)$, the buyer prefers p_1^*, eliminating p_3^* as a candidate for an equilibrium.

At p_2^*, we know the seller is indifferent between investigating and accepting unconditionally, but that the buyer prefers unconditional acceptance, from the comparison of equations (1) and (2). The buyer's surplus from offering p_2^* is given by equation 2 and exceeds the payoff to offering p_1^*, $\Delta - c_s$, only if

$$c_s > \frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta. \tag{6}$$

Note also that this surplus must be positive and therefore preferred to no trade, since it exceeds $\Delta - c_s$, which in turn exceeds 0 by the assumption that $c_s < \Delta$.■

Proof of Lemma 4: From the analysis associated with equations (4) and (5) we know that if $c_s > \Delta$, the best payoff that the buyer can achieve when inducing the seller to investigate is less than zero. But, from (2), we can also see that the buyer can achieve a positive profit when preventing the seller from acquiring information only if

$$c_s > \frac{1}{4}(v_h - v_l) - \Delta.$$

But, the bounds on c_s imply that $\Delta < \frac{1}{4}(v_h - v_l) - \Delta$, which in turn implies the existence of a region where trade is not profitable either with or without the seller gathering information.■

Proof of Lemma 5: The arguments in the text immediately preceding the statement of the lemma prove the result.■

Proof of Lemmas 6 and 7: We prove these two lemmas together. We first posit a two-price equilibrium in which the buyer offers one price, p_l, if the value is low and mixes between p_l and another price, p_h, if the value is high. We will then derive the payoff from the optimal equilibrium in the case of Lemma 6 and the supremum of the payoffs in the case of Lemma 7, from the perspective of the buyer under this assumption. We will then demonstrate that the buyer will not do better in any equilibrium that uses more than two prices.

Assuming that there are two prices, it is immediate that the higher price must never be offered by the buyer after observing the low value. If it were, this would imply that both prices are offered with positive probability for both types, which would imply that the buyer is indifferent between the two prices for both realizations of the value, which is impossible.[9] We can also conclude that in any Perfect Bayesian equilibrium, the high price must be $v_h - \Delta$, since any lower price would admit a profitable deviation to a higher price when $v = v_h$. Finally, we can rule out any pure strategy equilibria; the buyer must mix between the high and the low price when the value is high, and the seller must reject the low price with positive probability.[10] The seller will reject the low price under two circumstances. Either he does not investigate and rejects the low price anyway, or he

[9] For the buyer to be indifferent between the two prices following both realizations of the value, we require that the probability of trade, conditional on the price, times the expected payoff to the buyer conditional on trade occuring, be equal. This implies the ratio of expected payoffs, given trade, in each state be equal to the ratio of the probability of trade, so that:

$$\frac{v_h + \Delta - p_l}{v_h + \Delta - p_h} = \frac{v_l + \Delta - p_l}{v_l + \Delta - p_h},$$

which is clearly false for $p_h \neq p_l$.

[10] If the seller always accepts the low price, the buyer will always offer the low price and the seller will have negative expected profits. If the seller never accepts the low price, the buyer will never have an incentive to offer the low price when the true value is high (since he can obtain 2Δ by offering a price of $v_h - \Delta$), and the seller then must put probability one on the value being low following a low offer and therefore must accept with probability 1.

investigates and rejects the low price when the true value is high. We define the mixed strategies as follows. Let the probability that the buyer offers the low price when $v = v_h$ be α, let the probability that the seller does *not* investigate following a low offer be β, and let the probability that an uninformed seller sells at the low price be γ. We will also define $p_l = v_l - \Delta + z$ and $p_h = v_h - \Delta$ for convenience.

To construct the equilibrium, we consider two situations. First, if $z < c_s$, the seller will never investigate since his payoff to investigating is negative regardless of his beliefs.[11] Since the seller must reject the low price with positive probability, the seller must mix between unconditional acceptance and rejecting without investigation when the offer is low. The requirement for this to happen is then:

$$\frac{1}{2}z + \frac{1}{2}\alpha(v_l - v_h + z) = 0$$

where the left hand side is the payoff to the strategy of accepting either p_l or p_h and the right hand side is the payoff for only accepting p_h. This gives:

$$\alpha = \frac{z}{v_h - v_l - z} \in (0, \frac{c_s}{v_h - v_l - c_s}) \subset (0, \frac{1}{3}).$$

The subset restriction comes from the assumption that $c_s < \frac{1}{4}(v_h - v_l)$ and guarantees that we do not need to check whether the conditions for the seller to mix exist.

In order for the buyer to mix, we must have:

$$2\Delta = \gamma(v_h - v_l + 2\Delta - z)$$

which gives:

$$\gamma = \frac{2\Delta}{v_h - v_l + 2\Delta - z}$$

which will also fall in the interval $(0, 1)$ for all parameters under consideration, i.e., for $p_l < p_h$.

[11] In step 3c of Dang (2008), the claim is that the seller never chooses to acquire information if (in our notiation) $z < 2c_s$. This is based on an incorrect calculation of the expected payoff to the seller for using a strategy of investigating following an offer of p_l. The calculation fails to take into account that the seller will not pay the costs of investigating when v is high and the buyer offers p_h, an event that occurs with probability $\frac{1}{2}(1 - \alpha)$. Specifically, Dang (2008) states that the payoff to the seller of investigating when $p = p_l$ is:

$$\frac{1}{2}z - c_s,$$

whereas the correct value is

$$\frac{1}{2}z - \frac{1}{2}(1 + \alpha)c_s.$$

The payoff to the buyer in an equilibrium with a given z can be written as:

$$\begin{aligned}
& \frac{1}{2}(1-\alpha)2\Delta + \frac{1}{2}\alpha\gamma[v_h + \Delta - (v_l - \Delta + z)] + \frac{1}{2}\gamma[v_l + \Delta - (v_l - \Delta + z)] + \frac{1}{2}(1-\gamma)0 - c_b \\
= \; & \frac{1}{2}(1-\alpha)2\Delta + \frac{1}{2}\alpha 2\Delta + \frac{1}{2}\gamma(2\Delta - z) - c_b \\
= \; & \Delta - c_b + \frac{1}{2}\gamma(2\Delta - z).
\end{aligned}$$

This expression becomes

$$\Delta - c_b + \frac{\Delta(2\Delta - z)}{v_h - v_l + 2\Delta - z},$$

which is maximized at $z = 0$. But, $z = 0 \Rightarrow \alpha = 0$, which is not an equilibrium. We can thus conclude that there does not exist an optimal equilibrium from the perspective of the buyer with $z < c_s$, and we can also conclude that the sup of the payoffs to equilibria with $z < c_s$ is

$$\Delta - c_b + \frac{2\Delta^2}{v_h - v_l + 2\Delta} \tag{7}$$

and that the buyer always prefers a smaller z and consequently must choose α very small. The limiting value for γ is also immediate, but note that it does *not* go to zero as z disappears. We have now described all of the characteristics of the equilibria described in Lemma 7.

Alternatively, the buyer could offer a price that could induce the seller to investigate with positive probability. This price must have $z \geq c_s$, but while the offer price is higher, and therefore worse for the buyer, the equilibrium may be preferable because if the seller investigates with positive probability trade may occur more frequently when $v = v_l$. In this situation, we require that

$$\beta\gamma = \frac{2\Delta}{v_h - v_l + 2\Delta - z}$$

where $\beta\gamma$ is the probability that the seller does not investigate following an offer of p_l *and* the seller buys anyway. The payoff to the buyer is then:

$$\Delta - c_b + \frac{\Delta(2\Delta - z)}{v_h - v_l + 2\Delta - z} + \frac{1}{2}(2\Delta - z)(1-\beta).$$

The final term is the contribution to the payoff of the event that $v = v_l$ and the seller investigates, necessarily buying after discovering v.[12] It is straightforward to verify that this expression is strictly decreasing in both β and z. However, we face several additional constraints in order for z and β to

[12]This term is one of the elements missing from the analysis in Dang (2008) (Step 3d. of the appendix). As a consequence of including it, β and γ are not separately identified by the conditions ensuring indifference in a mixed-strategy equilibrium.

constitute a mixed strategy equilibrium. First, we know that:

$$\beta\gamma = \frac{2\Delta}{v_h - v_l + 2\Delta - z}.$$

Since γ does not enter the equilibrium payoff separately from $\beta\gamma$, it is immediate that at the equilibrium that provides the highest payoff to the buyer we will have $\gamma^* = 1$. This condition will allow β to be set as small as possible, but we still must have $\beta \geq \frac{2\Delta}{v_h - v_l + 2\Delta - z}$ when $\gamma \leq 1$. Note that this constraint depends on the value of z, but decreasing z relaxes the constraint (i.e. allows a smaller β to be chosen). Thus, the payoff to the buyer will be maximized by setting z as low as possible given the other constraints. The constraints of concern are the following:

$$\frac{1}{2}(z - c_s) + \frac{1}{2}\alpha(-c_s) \geq 0 \tag{8}$$

$$\frac{1}{2}(z - c_s) + \frac{1}{2}\alpha(-c_s) \geq \frac{1}{2}z + \frac{1}{2}\alpha(v_l - v_h + z). \tag{9}$$

These constraints guarantee that the seller at least weakly prefers to use a strategy calling for investigation after an offer of p_l over a strategy calling for rejecting p_l and over a strategy calling for accepting p_l without investigation, respectively. In any equilibrium, at least one of these constraints must bind in order for the seller to mix as required. In the equilibrium we wish to construct, with $\beta \in (0, 1)$, the second inequality must bind. The first inequality may not bind as we have shown that $\gamma^* = 1$ and therefore the seller may strictly prefer investigating to not investigating and refusing to sell at p_l, but it will turn out that in the buyer's preferred equilibrium both inequalities will bind.

Inequality (8) reduces to

$$\alpha \leq \frac{z - c_s}{c_s}.$$

Inequality (9) reduces to

$$\alpha(v_h - v_l - c_s - z) \geq c_s.$$

Now suppose that $v_l - v_h + z > -c_s$. The payoff to being lied to when not investigating and the true value is high is not as negative as the cost of investigating, so investigation is dominated and could not be an equilibrium when $v_h - v_l - c_s - z$ is negative. Consequently, we have that $v_h - v_l - c_s - z > 0$ and inequality (9) becomes:

$$\alpha \geq \frac{c_s}{v_h - v_l - c_s - z},$$

which tells us:

$$\frac{c_s}{v_h - v_l - c_s - z} \leq \alpha \leq \frac{z - c_s}{c_s}.$$

In order for an equilibrium to call for mixing on the part of the seller, we must then have that:

$$\frac{c_s}{v_h - v_l - c_s - z} \leq \frac{z - c_s}{c_s},$$

which, noting that $z > c_s$, becomes:

$$z^2 - (v_h - v_l)(z - c_s) \leq 0.$$

Therefore, to maximize the payoff to the buyer, we minimize z subject to the above constraint. The constraint is clearly violated at $z = c_s$ and for very large or very negative z, and there can be no root of the right hand side expression for z between 0 and c_s. The admissible z's, if any, must then fall between the roots of $z^2 - (v_h - v_l)(z - c_s) = 0$, and therefore z is minimized subject to the constraint at

$$z^* = \frac{1}{2}\left[(v_h - v_l) - \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s}\right]$$

whenever

$$c_s \leq \frac{1}{4}(v_h - v_l).$$

Again, the buyer can always obtain the full surplus 2Δ without investigating when $c_s \geq \frac{1}{4}(v_h - v_l)$, so we can confine attention to those situations where c_s is smaller.

To recap, if $c_s < \frac{1}{4}(v_h - v_l)$, z^* is the smallest value for z such that the seller could, for some range of α, weakly prefer investigating following p_l to both buying outright and refusing to buy at all. Choosing this z and letting:

$$\alpha^* = \frac{c_s}{v_h - v_l - c_s - z^*},$$

we have the equilibrium preferred by the buyer, assuming he gathers information. Note that $\alpha^* > 0$ since $c_s + z < v_h - v_l$.

But, we can now solve for:

$$\alpha^* = \frac{2c_s}{(v_h - v_l) + \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s} - 2c_s}.$$

It is immediate to verify that, for $c_s < \frac{1}{4}(v_h - v_l)$, $\alpha^* \in (0, 1)$. We can also obtain

$$\beta^* = \frac{4\Delta}{v_h - v_l + \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s} + 4\Delta}.$$

The payoff to the buyer is then:

$$\Delta - c_b + \frac{1}{2}(2\Delta - z^*)$$

since the seller always either buys outright or investigates.

We can ignore equilibria in which the seller mixes between investigating and rejecting without investigation. These equilibria provide payoffs of:

$$\Delta - c_b + \frac{1}{2}\beta\gamma(2\Delta - z),$$

with $z > z^*$ and $\beta\gamma < 1$, so they are clearly dominated from the perspective of the buyer by the equilibrium with mixing between investigation and outright purchase. A three-way mix can occur only at $z = z^*$ and is clearly worse than the equilibrium with mixing only between acceptance and investigation.

So, the maximum payoff to the buyer is given by:

$$2\Delta - c_b - \frac{1}{4}\left[(v_h - v_l) - \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s}\right]. \tag{10}$$

Note finally that for $c_s \in (0, \frac{1}{4}(v_h - v_l))$, this expression decreases from $2\Delta - c_b$ to $2\Delta - c_b - \frac{1}{4}(v_h - v_l)$.

The seller receives zero surplus since he is indifferent among all alternatives at z^*, α^*.

Finally, we can compare the payoff to the equilibrium in which the seller obtains information with positive probability to the upper bound on the payoffs when the seller never obtains information to find the separation point between the two lemmas. As long as $\Delta < \frac{1}{2}(v_h - v_l)$ (leading any positive z to be accepted), the condition for the seller not to gather information with positive probability becomes:

$$c_s > \frac{2\Delta(v_h - v_l)^2}{(v_h - v_l + 2\Delta)^2}.$$

Note that the inequality is strict because the buyer cannot obtain the sup of the payoffs associated with equilibria that involve no investigation by the seller.

To complete the proof, we demonstrate that no PBE can have the buyer mixing over more than two prices.

Suppose the buyer is informed and plays a mixed strategy with K prices when $v = v_h$. Given the gains to trade, it would be suboptimal for the buyer to offer a price that would be accepted by the seller with zero probability. Under the same logic as in the two-price equilibrium (see footnote 9), the buyer cannot be indifferent between a particular pair of the K prices, simultaneously when $v = v_h$ and when $v = v_l$. Hence, only one price among the K prices can be chosen when $v = v_l$. Let p_l denote this price. In a PBE, any of the $K - 1$ prices not chosen when $v = v_l$ will be interpreted by an uninformed seller as a perfect signal that $v = v_h$. Therefore, the seller will accept with probability one any price that is higher than or equal to $v_h - \Delta$. The buyer prefers the lowest price possible, and $v_h - \Delta$ will be the only price chosen when $v = v_h$, other than p_l. Hence, in a PBE, the buyer will mix over no more than two prices when $v = v_h$. The only remaining way to have a PBE with more than two prices, therefore, is to have the buyer playing a mixed strategy when

$v = v_l$ involving prices other those played when $v = v_h$. Any prices the buyer choses when $v = v_l$ that are not p_l, an uninformed seller will interpret as a perfect signal that $v = v_l$. As in the case with $v = v_h$, only one price other than p_l can be chosen by the buyer with positive probability, and accepted by the seller with positive probability, when $v = v_l$ in a PBE. That price is $v_l - \Delta$, which is the minimum price the seller would accept in the model. At that price, the seller will know for sure that $v = v_l$ and will accept the offer. Since we need $p_l > v_l - \Delta$ to have mixing over more than two prices and acceptance by the seller with positive probability, the buyer will find optimal to deviate and always offer $v_l - \Delta$, rather than p_l, which rules out the possibility of another price coexisting with p_l in the low-value state. Hence, only two-price PBEs will exist.∎

Proof of Proposition 1

Part A: $\pi = 0$:

We will first consider the case when $\Delta \leq \frac{1}{9}\sigma$.

The proposed equilibrium places the agents at the intersection of Lemma 2 (part a) and Lemma 4, and, for $\frac{\sigma}{\Delta} = 9$, at the intersection of these two areas with that governed by Lemma 6. Any deviation to a higher investment in expertise reduces payoffs to the deviator when he is the seller from 2Δ to 0. While payoffs increase when the deviator is the buyer (from 0), 2Δ is an upper bound[13] on the increase. Therefore, since the additional expertise investment is costly, the deviation cannot be profitable. If $c'(e^*) \leq -(1-\delta)$, the payoffs given by Lemma 2(a), and the convexity of the cost function imply that deviations to a lower level of expertise investment are unprofitable. Then, no $e < e^*$ can be an equilibrium because there would be a profitable local deviation to a higher level of expertise as such an equilibrium would mean the agents were in the interior of the Lemma 2(a) region.

To see that these choices are unique under the restrictions on the marginal cost, suppose an agent chooses $e > e^*$. Then there is a profitable deviation to less expertise since $c'(e) > -2(1-\delta)$ for all $e > e^*$, because the payoff to the seller is invariant in his own costs in the Lemma 6 region while the payoff to the seller is decreasing linearly with slope $-\frac{1}{1-\delta}$ in his own costs.

If $c'(e^*) > -(1-\delta)$, deviating to less expertise is profitable at e^*, and there is no profitable deviation to more expertise from $\hat{e}$. (If $c'(\hat{e}) = -(1-\delta) \Rightarrow c(\hat{e}) > \frac{1}{4}\frac{\sigma}{\Delta}$, there will be no investment in expertise, while if $c'(\hat{e}) = -1 \Rightarrow c(\hat{e}) < \frac{1}{4}\frac{\sigma}{\Delta}$, but $c(0) > \frac{1}{4}\frac{\sigma}{\Delta}$, the equilibrium *may* call for no investment in expertise since it may not be profitable to pay the fixed costs associated with increasing expertise up to $c(e) = \frac{1}{4}\frac{\sigma}{\Delta}$, the point at which expertise begins to pay off at the margin.)

The final step is to show that agents will always play according to Lemma 2(a) in the subgame. Play with the characteristics of Lemma 4 (and, for $\frac{\sigma}{\Delta} = 9$, Lemma 6) can also be supported in a subgame following investment in expertise of e^*. However, if play proceeds according to either Lemma 4 or Lemma 6 with positive probability there is a profitable deviation from e^*

[13] The value 2Δ is not the sup, but the fact that it is an upper bound is sufficient for our purposes.

since by unilaterally and infinitesimally reducing investment in expertise either party can move the agents into the interior of the Lemma 2(a) region. Since Lemma 2(a) provides efficient payoffs and Lemmas 4 and 6 do not, there would always be some small reduction in expertise investment that would be profitable.

We now consider the case where $\frac{\sigma}{\Delta} < 9$.

The discussion relating to $e < e^*$ from above applies effectively unchanged, so it remains only to show that there is no profitable increase in expertise when $c'(e^*) < -(1-\delta)$.

To show that there is no profitable increase in expertise, observe that $\{c(e^*), c(e^*)\}$ defines the symmetric cost point at which the buyer is just indifferent between gathering information and not. Any increase in expertise by one agent moves the game to either the Lemma 6 region or the Lemma 3 region when the deviator is the seller. This yields zero payoff to the seller in the subgame. The increase in the payoff when the deviator is the buyer is bounded above by $\left(2 - \frac{1}{12}\frac{\sigma}{\Delta}\right) - \left(2 - \frac{2}{9}\frac{\sigma}{\Delta}\right)$. The expression in the first set of parentheses is the payoff to adhering that accrues when the agent turns out to be the buyer, which involves positive surplus to the buyer because $\frac{5}{36}\frac{\sigma}{\Delta}$ exceeds the lower bound for the sellers cost for the Lemma 2 region when the buyer does not gather information whenever $\frac{\sigma}{\Delta} < 9$, as assumed. That is, the point at which Lemma 3 transitions to Lemma 2 occurs where the buyer and the seller are sharing the surplus, not where the seller gets all of the surplus. The expression in the second set of parentheses is the payoff when the deviator is the buyer, assuming he deviates to $c(e) = 0$, but does not accrue any additional expenses with respect to expertise (an upper bound on the payoff to the deviation. Direct calculation then shows that the payoff to a deviation is no greater than $-\frac{1}{24}\frac{\sigma}{\Delta}$, so there is no profitable deviation to more expertise.∎

Part B: $\pi > 0$:

In this proof it is convenient to introduce a normalization on the costs. We define $k_i \equiv \frac{c_i}{\Delta}$ for $i \in \{b, s\}$. All descriptions of boundaries for costs will be in terms of these normalized costs, and the ratio of volatility to gains to trade, denoted $A = \frac{\sigma}{\Delta}$.

First, we note the following facts:

The buyer's (normalized) cost, as a function of the seller's cost, that sets the buyer's payoff equal in Lemma 2 and in Lemma 6 is increasing in A when $k_s \in (0, \frac{1}{4}A]$. The derivative of that boundary with respect to A is:

$$\frac{A + \sqrt{A(A-4k_s)} - 2k_s}{4\sqrt{A(A-4k_s)}},$$

and is real and positive when $k_s < \frac{1}{4}A$, or equivalently when $c_s < \frac{1}{4}\sigma$. To see this, note that the buyer's payoffs for acquiring information are equal to the payoffs for not acquiring information when

$$-\frac{1}{2}\sigma + 2\Delta + 2c_s = 2\Delta - c_b - \frac{1}{4}\left(\sigma - \sqrt{\sigma(\sigma - 4c_s)}\right)$$

which gives

$$c_b = \frac{1}{4}\left(\sigma + \sqrt{\sigma(\sigma - 4c_s)} - 8c_s\right).$$

Dividing both sides by Δ gives the boundary in terms of normalized costs and A:

$$k_b = \frac{1}{4}\left(A + \sqrt{A(A - 4k_s)} - 8k_s\right).$$

Differentiating this expression with respect to A gives the desired expression. Note that in taking this derivative it is *not* appropriate to treat k_s as a function of Δ and therefore of A because we are considering where the buyer is indifferent for k_b, k_s pairs.

The buyer's (normalized) cost, as a function of the seller's cost, that sets the buyer's payoff equal in Lemma 4 and in Lemma 6 is also increasing in A. This boundary is given by:

$$k_b = \frac{1}{4}\left[-A + \sqrt{A(A - 4k_s)} + 8\right].$$

The derivative of that boundary with respect to A is:

$$\frac{A - \sqrt{A(A - 4k_s)} - 2k_s}{4\sqrt{A(A - 4k_s)}},$$

which is positive whenever $k_s \in (0, \frac{1}{4}A]$, or equivalently $c_s \in (0, \frac{1}{4}\sigma]$.

The seller's (normalized) cost that divides Lemma 2 and Lemma 4 is also increasing in A. This is obvious from Lemma 4, noting $c_s \leq \frac{1}{4}\sigma - \Delta$ is equivalent to $k_s \leq \frac{1}{4}A - 1$.

Now suppose that both agents invest in expertise up to $e^*(A)$, hence play proceeds according to Lemma 2 in the low-volatility regime.[14] For $A < 9$, $e^*(A)$ will be on the boundary between Lemmas 6 and 2. Thus, an increase in volatility to θA will move play into the Lemma 6 region, as is clear from the above three facts.

What happens when $A \geq 9$ is somewhat less obvious. When A increases, play can no longer remain in the Lemma 2 region because the seller's costs now imply that play proceeds according to Lemma 4 if the buyer is informed. However, since the boundary between Lemmas 4 and 6 boundary is increasing in A, we cannot immediately rule out the possibility that play proceeds according to Lemma 6, particularly for very large shocks. However, we observe, using l'Hospital rule, that

$$\lim_{A\to\infty} \frac{1}{4}\left(-A + \sqrt{A(A - 4k_s)} + 8\right) = \lim_{A\to\infty} \frac{1}{4}\left(-A + \sqrt{A(A - 4k_s)} + 8\right)\frac{1/A}{1/A} = -\frac{k_s}{2} + 2.$$

[14] Note that the condition for e^* from Proposition 1 can be rewritten in terms of k (with k, of course, defined as $\frac{c}{\Delta}$) as

$$e^* = \begin{cases} k^{-1}\left(\frac{1}{4}A - 1\right) & \text{if } A \geq 9 \\ k^{-1}\left(\frac{5}{36}A\right) & \text{if } A \leq 9 \end{cases},$$

which permits us to write e^* as a function of A.

That is, the function defining the boundary between Lemmas 4 and 6 will converge to a finite function. Since we know that, for $A \geq 9$, the symmetric equilibrium is where the diagonal intersects the vertical line at $\frac{1}{4}A - 1$, we know that play will always proceed according to Lemma 4 regardless of the magnitude of θ if and only if

$$-\frac{\frac{1}{4}A - 1}{2} + 2 < \frac{1}{4}A - 1,$$

which means that the limit, as A goes to $+\infty$, of the threshold on c_b for Lemma 4 is smaller than the smallest cost level in Lemma 2 and reduces to the condition $A > \frac{28}{3}$.

The magnitude of the shock required is given by finding the shock θ that moves the boundary for the buyer's decision to acquire information, evaluated at the normalized costs associated with $e^*(A)$ (which are given by $\frac{1}{4}A - 1$ from Proposition 1), to that same cost level. That is, we find the threshold for the shock to lead to play according to Lemma 6 by finding the increase in volatility that makes the buyer just indifferent between acquiring and not acquiring information at the equilibrium cost level when the seller's costs are the equilibrium costs.

$$\begin{aligned} \frac{1}{4}\left(-A\theta + \sqrt{A\theta(A\theta - 4\left(\frac{1}{4}A - 1\right)) + 8}\right) &= \frac{1}{4}A - 1 \\ \theta &= \frac{(A-12)^2}{A(28-3A)} \end{aligned}$$

Since the boundary is monotonically increasing in A (and therefore θ), this implies that we enter the Lemma 6 region if and only[15] if $\theta \geq \frac{(A-12)^2}{A(28-3A)}$. Otherwise, play proceeds according to Lemma 4.

We need to show that there does not exist a unilateral deviation in expertise that would be optimal for one agent. We first study what would happen if one agent were to increase expertise above e^* and then we study what would happen if one agent were to decrease expertise below e^*.

Suppose one agent were to increase his investment in expertise enough to move to the Lemma 6 region when he turns out to be the buyer. When he is the seller, the game would either remain in the region of Lemma 4 for small increases or it would move to Lemma 6 or Lemma 3 for larger increases. All of these regions would give him zero payoffs. The only gain to an increase in expertise would have to accrue when he is the buyer. But using Lemma 6 we can show that, keeping the seller's cost fixed and below $\frac{1}{2}\sigma$, the payoff to the buyer at any level of his own costs will be decreasing in A. Therefore, the buyer's payoff will be strictly lower in the high-volatility regime than in the low-volatility regime. So the benefits from increasing expertise in the stochastic-volatility setting will be smaller than the benefits from increasing expertise in the constant-volatility setting (where the low-volatility regime always occurs). Therefore, since increasing expertise above e^* is not a profitable unilateral deviation in the constant-volatility setting, such deviation will not be profitable when

[15] Since we assume the choice preferred by the buyer is made if the seller is indifferent.

volatility is stochastic either.

If $A \geq \frac{28}{3}$ or $9 < A < \frac{28}{3}$ and $\theta < \frac{(A-12)^2}{A(28-3A)}$, an agent could consider reducing his investment in expertise such that play is on the boundary between Lemma 4 and 2 and trade occurs with probability one in the high-volatility regime when he is the seller. This deviation clearly dominates all the other possible deviations where expertise is lower than e^*. Such deviation would be costly in the low-volatility regime because the deviator's payoffs would be reduced at a rate of $\frac{2}{1-\delta}$ as his cost increases, until he reaches the region of Lemma 1 where his payoff is zero. On the other hand, his payoff would increase to 2Δ in the high-volatility regime. A deviation to such a lower level of expertise (or anything lower) would not be profitable when π is low enough. To see this, let $\tilde{e}(\theta)$ be the level of expertise such that the seller gets a payoff of 2Δ in the high volatility state.

Then, the difference in payoffs from picking e^* rather than $\tilde{e}(\theta)$, when the probability of the high-volatility state is denoted π is given by:

$$\frac{\pi}{1-\delta}(-\Delta) + \frac{1-\pi}{1-\delta}[c(\tilde{e}(\theta)) - c(e^*)] - [e^* - \tilde{e}(\theta)] \tag{11}$$

If $\pi = 0$, then we get from property P2 that the expression in (11) is positive and e^* is the optimal level of expertise.

Now, if we take the derivative of the expression in (11) with respect to π, we get:

$$-\frac{1}{1-\delta}\Delta - \frac{1}{1-\delta}[c(\tilde{e}(\theta)) - c(e^*)],$$

which is bounded from below by $-\frac{1}{1-\delta}[\Delta + c(0)]$, which is finite. Thus, by continuity, for π close enough to zero, the expression in (11) is positive and e^* provides the seller with a higher expected payoff ex ante than any other level of expertise.

If $A \leq 9$ or $9 < A < \frac{28}{3}$ and $\theta \geq \frac{(A-12)^2}{A(28-3A)}$, the analogous reduction in expertise would lead play to the boundary between Lemmas 2 and 6 (or sometimes Lemma 7) in the high-volatility regime. In this case, the above discussion applies with the adjustment that 11 represents a lower bound on the excess value of choosing e^* over a lower level of expertise since payoffs to the seller will be less than 2Δ at $\tilde{e}(\theta)$ in the high volatility state and payoffs to the buyer at e^* are not zero in the high volatility state (since play proceeds according to Lemma 6 rather than Lemma 4), and in fact are higher at e^* than at $\tilde{e}(\theta) < e^*$ since the buyer's payoff in the subgame must be (weakly) decreasing in his own costs regardless of the costs of the seller. ∎

Appendix B Deriving the buyer's information acquisition boundary

Here, we the denote the ratio of volatility to gains to trade as $A = \frac{\sigma}{\Delta}$, and normalize $\Delta = 1$ without loss of generality.

To determine whether the buyer will acquire information for a particular cost pair $\{c_s, c_b\}$ at a particular A, we simply find the cost to the buyer that makes him indifferent between acquiring information and not acquiring information, exploiting the fact that his information acquisition decision induces a proper subgame. Thus, we can simply compare the payoffs to the buyer for a given A and c_s under the appropriate lemma when the buyer does and does not acquire information. These thresholds are all derived in the lemmas in the text and produce the following boundary segments:

For Lemma 3 versus 6, the buyer acquires information if his costs satisfy:

$$\begin{aligned} 1 - c_s &< \frac{1}{4}\left(-A + \sqrt{A(A-4c_s)}\right) + 2 - c_b \\ c_b &< \frac{1}{4}\left(-A + \sqrt{A(A-4c_s)} + 4c_s + 4\right). \end{aligned}$$

For Lemma 4 versus 6, the buyer acquires information if his costs satisfy:

$$\begin{aligned} 0 &< \frac{1}{4}\left(-A + \sqrt{A(A-4c_s)}\right) + 2 - c_b \\ c_b &< \frac{1}{4}\left(-A + \sqrt{A(A-4c_s)} + 8\right). \end{aligned}$$

For Lemma 2 versus 6, the buyer acquires information if his costs satisfy:

$$\begin{aligned} 2c_s + 2 - \frac{A}{2} &< \frac{1}{4}\left(-A + \sqrt{A(A-4c_s)}\right) + 2 - c_b \\ c_b &< \frac{1}{4}\left(-A + \sqrt{A(A-4c_s)} + 8c_s\right). \end{aligned}$$

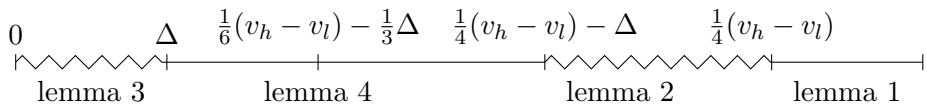
For Lemma 2 versus 7, the buyer acquires information if his costs satisfy:

$$\begin{aligned} 2c_s + 2 - \frac{A}{2} &\leq 1 - c_b + \frac{2}{A+2} \\ c_b &< \frac{A^2 - 4Ac_s - 8c_s}{2(A+2)}. \end{aligned}$$

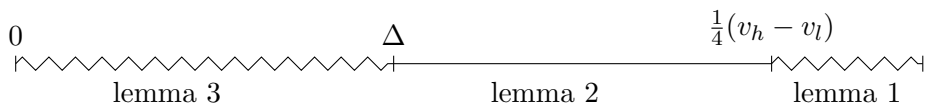
These four cases exhaust all possible relevant comparisons. If the game proceeds according to Lemma 1 when the buyer does not acquire information, he will clearly not acquire information if his costs are even weakly positive since he can capture the full surplus while remaining uninformed. Comparisons between Lemma 7 and any lemma other than Lemma 2 are irrelevant since $c_s > \frac{2A^2}{(A+2)^2} > 0$ (the threshold for Lemma 7 to be relevant) implies $c_s > \max\{\frac{1}{4}A - 1, \frac{1}{6}A - \frac{1}{3}\}$ (the threshold above which Lemma 2 becomes relevant when the buyer does not acquire information).

From the above 4 conditions, we can determine whether the buyer acquires information for any pair $\{c_s, A\}$. Of course for some values of A certain of the above conditions will be irrelevant for all values of c_s, as is obvious from Figure 1.

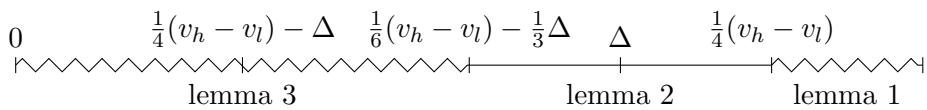
Case 1: $\Delta < \frac{1}{8}(v_h - v_l)$



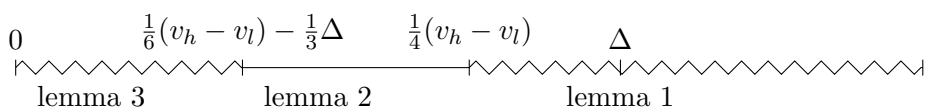
Case 2: $\Delta = \frac{1}{8}(v_h - v_l)$



Case 3: $\frac{1}{4}(v_h - v_l) > \Delta > \frac{1}{8}(v_h - v_l)$



Case 4: $\frac{1}{2}(v_h - v_l) > \Delta \geq \frac{1}{4}(v_h - v_l)$



Case 5: $\Delta \geq \frac{1}{2}(v_h - v_l)$

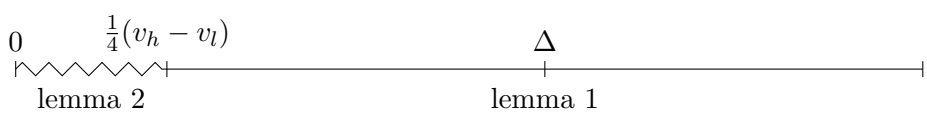
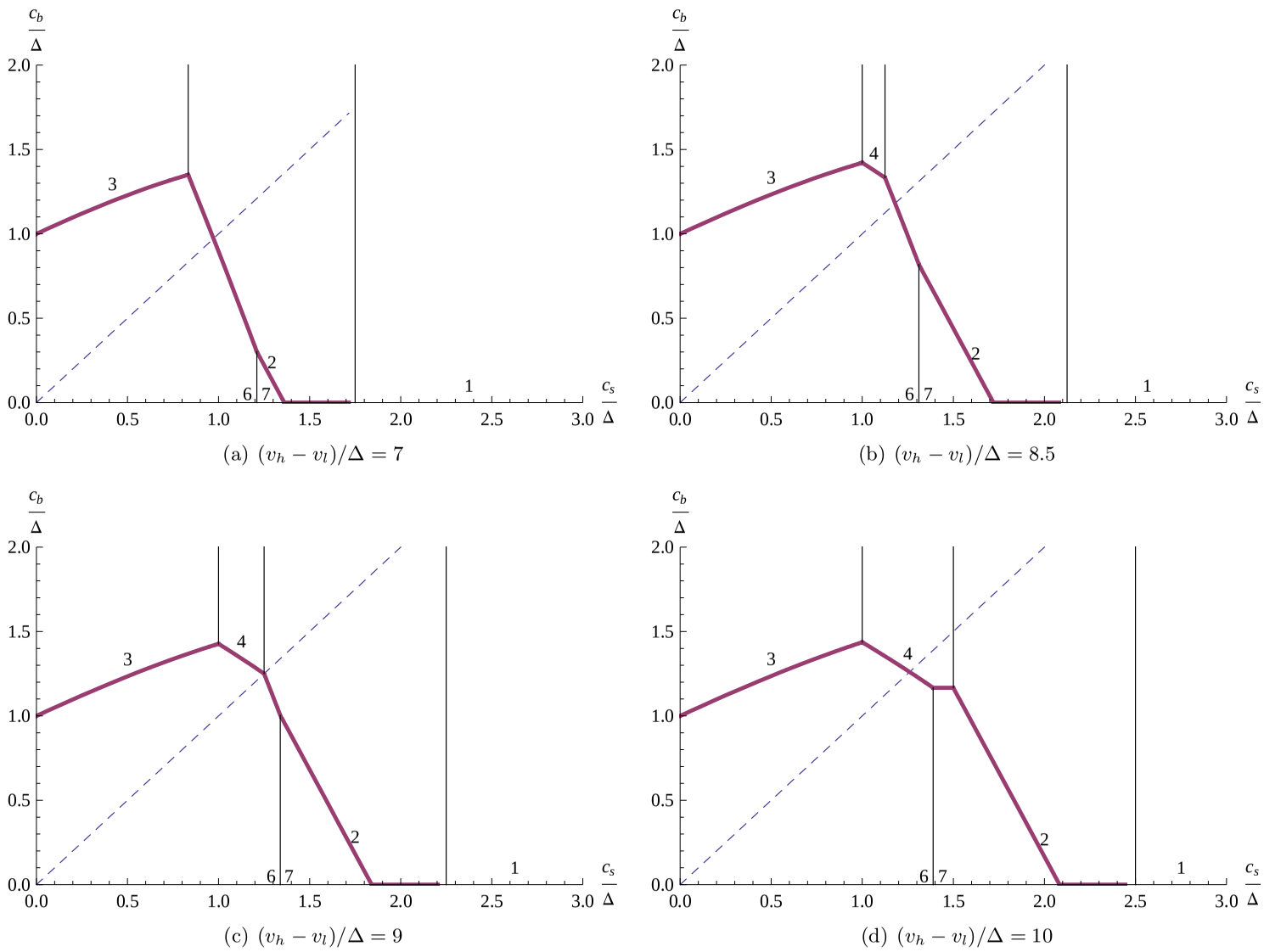


Figure 1: Regions of seller's costs where Lemmas 1-4 apply. The five cases depend on the relative magnitude of the gains to trade, Δ, and the volatility, $v_h - v_l$.



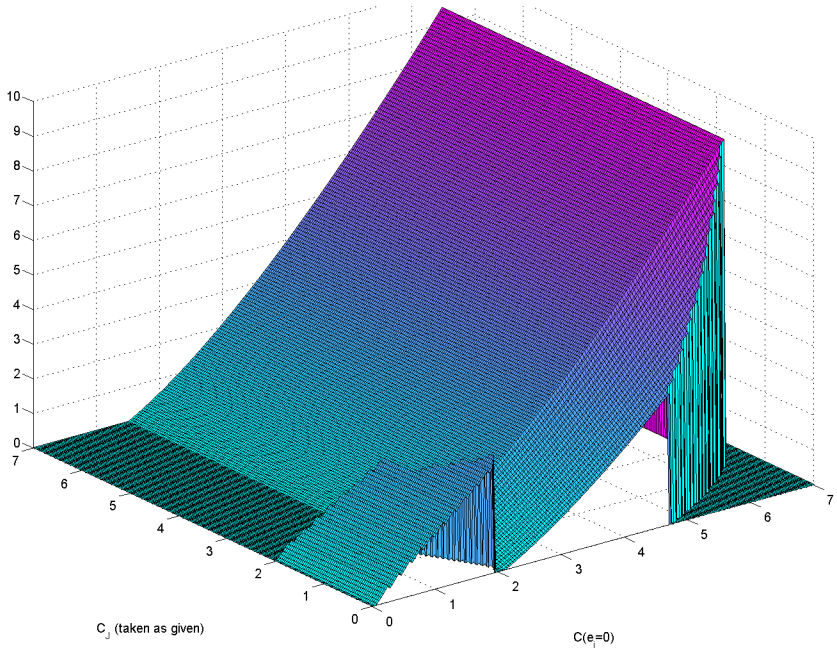
(a) $(v_h - v_l)/\Delta = 7$

(b) $(v_h - v_l)/\Delta = 8.5$

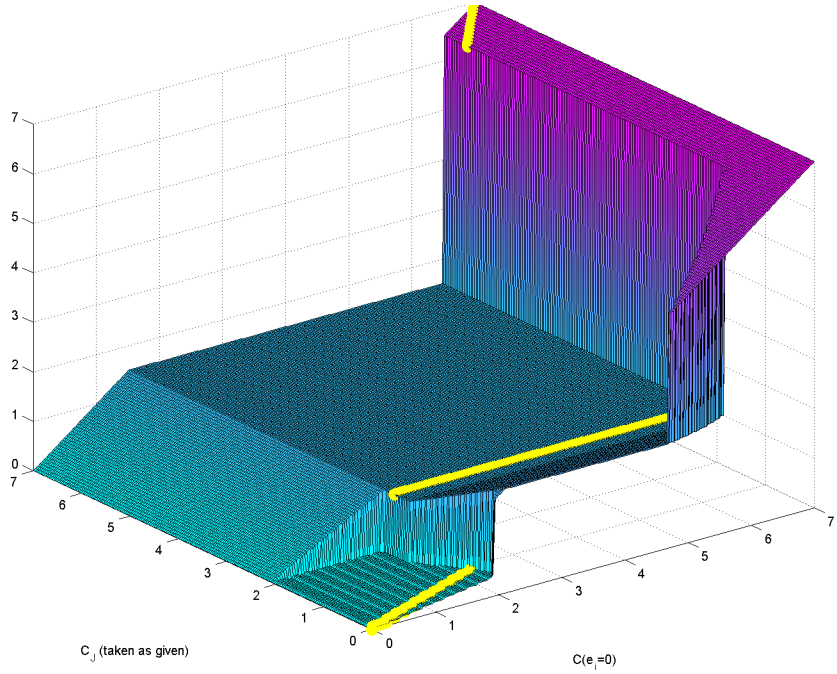
(c) $(v_h - v_l)/\Delta = 9$

(d) $(v_h - v_l)/\Delta = 10$

Figure 2: Costs, normalized by gains to trade, at which the seller and buyer will investigate as functions of the ratio of volatility to gains to trade: $(v_h - v_l)/\Delta$. The possible cost pairs are divided into areas covered by Lemmas 1-4 and 6-7. For cost pairs in the lower left-hand corner, the buyer investigates before making an offer to the seller (Lemmas 6 and 7). To the right and above that area, the buyer stays uninformed. The 45-degree dashed line illustrates possible symmetric equilibria in information acquisition.



(a) Expertise



(b) Cost

Figure 3: Response Surfaces when Volatility is Stochastic. Panel (a) shows the best responses in the level of expertise given the initial cost of investigation and the cost level of potential counterparties. Panel (b) shows the costs that result from optimal investment in expertise, given the same variables. The highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that lead to symmetric pure-strategy Nash equilibria in financial expertise. Figures are generated by setting: $\delta = 0.9$, $\lambda = 1$, $\Delta = 1$, $\theta = 1.1$, $\pi = 0.05$, and $\sigma = 10$.

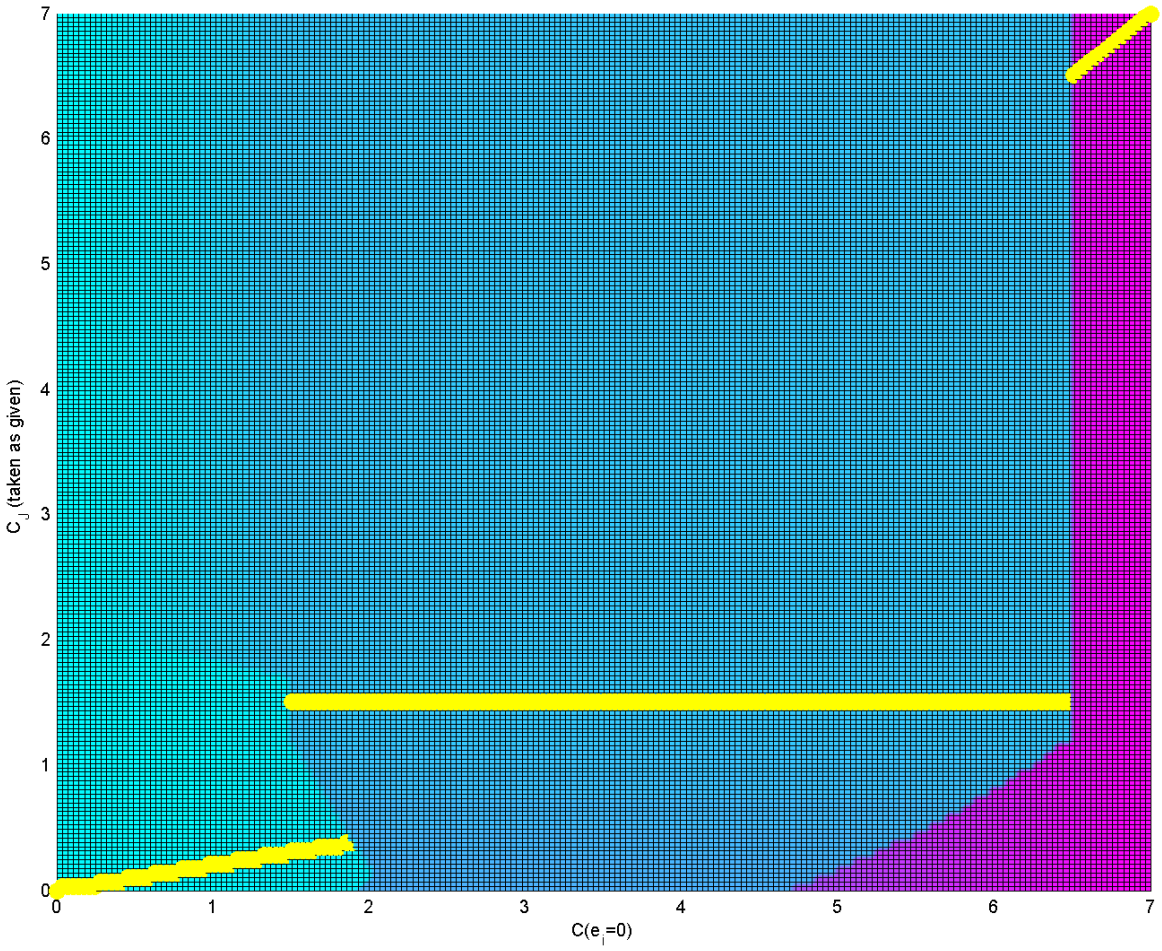
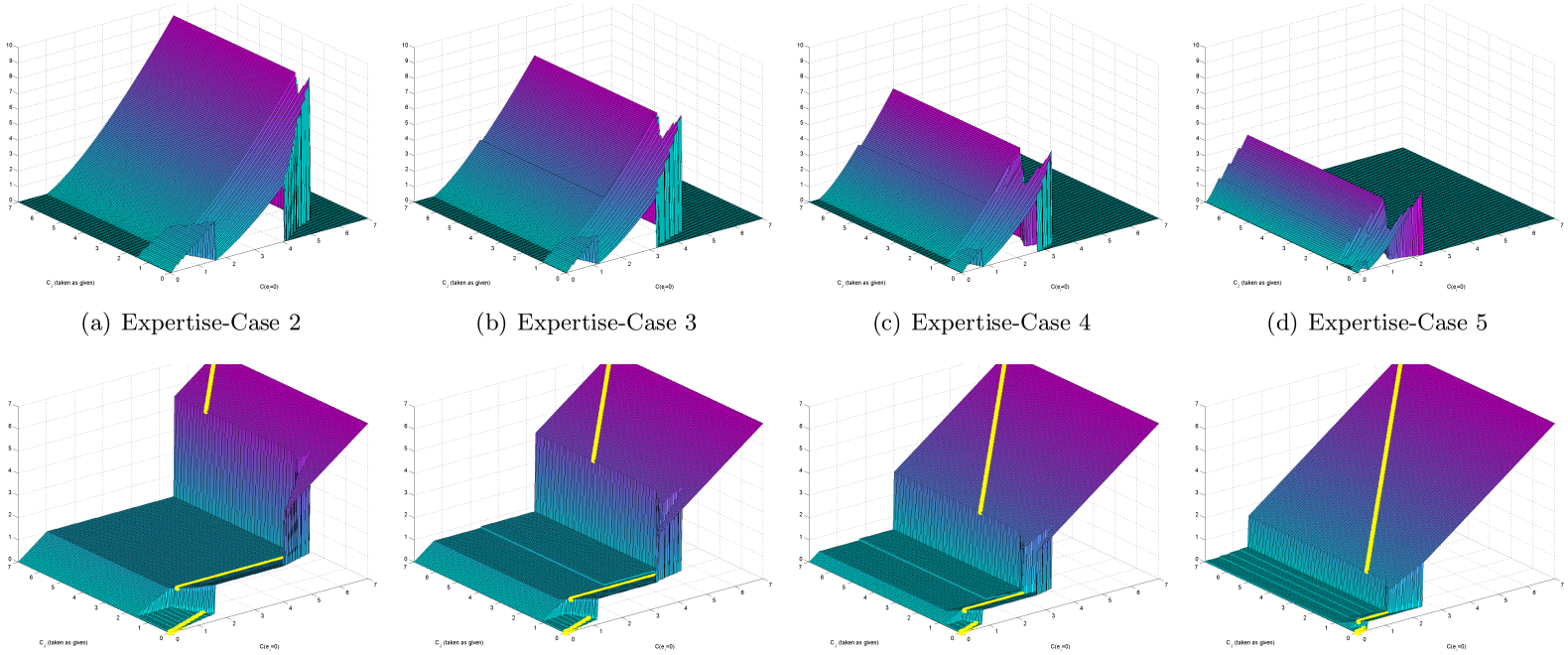


Figure 4: Symmetric Equilibria when Volatility is Constant. The highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that lead to symmetric pure-strategy Nash equilibria in financial expertise. Figure is generated by setting: $\delta = 0.9$, $\lambda = 1$, $\Delta = 1$, $\theta = 1.1$, $\pi = 0.05$, and $\sigma = 10$.



(a) Expertise-Case 2 (b) Expertise-Case 3 (c) Expertise-Case 4 (d) Expertise-Case 5

(e) Cost-Case 2 (f) Cost-Case 3 (g) Cost-Case 4 (h) Cost-Case 5

Figure 5: Response Surfaces when Volatility is Stochastic. Panels (a)-(d) show the best responses in the level of expertise given the initial cost of investigation and the cost level of potential counterparties. Panels (e)-(h) show the costs that result from optimal investment in expertise, given the same variables. The highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that lead to symmetric pure-strategy Nash equilibria in financial expertise. Figures are generated by setting: $\delta = 0.9$, $\lambda = 1$, $\Delta = 1$, $\theta = 1.1$, $\pi = 0.05$, and $\sigma = 8$ for Case 2, $\sigma = 6$ for Case 3, $\sigma = 4$ for Case 4, and $\sigma = 2$ for Case 5.