

The Dynamics of Mergers and Acquisitions in Oligopolistic Industries*

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Abstract

This paper develops a continuous time real options model to study the interaction between industry structure and takeover activity. In an asymmetric industry equilibrium, firms have an endogenous incentive to merge when restructuring decisions are motivated by operating and strategic benefits. The model predicts that (i) the likelihood of restructuring activities are greater in more concentrated industries or in industries more exposed to exogenous shocks; and (ii) the magnitude of returns arising from restructuring to both merger firms and rival firms are higher in more concentrated industries. While recent real options models contend that competition erodes the option value of waiting and hence accelerates the timing of mergers, increased competition in our model delays the timing of mergers.

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1 Introduction

Many aspects of mergers and acquisitions have been studied extensively in economics and finance.¹ Despite the substantial development of this literature, some important issues of the takeover process still lack deep understanding. For example, while existing models largely focus on firm characteristics to explain why firms should merge or restructure, they have not been entirely successful at uncovering the relation between industry structure and takeovers. A drawback shared by many dynamic models of mergers and acquisitions discussed later is that product market competition is abstracted away. The objective of our paper is to develop an industry equilibrium model that analyzes how product market competition affects the gains from a merger as well as the joint determination of the timing and terms of mergers. We also examine how industry characteristics affect merger returns to both merging and rival firms.

Recent empirical studies have documented ample evidence on the effect of industry characteristics on mergers and acquisitions. Analyzing a large number of industries during 1980s and 1990s, Andrade, Mitchell and Stafford (2001), Harford (2005), and Mitchell and Mulherin (1996) find that mergers occur in waves driven by industry-wide shocks, and that mergers strongly cluster by industry within a wave. For the airline industry, Borenstein (1990) and Kim and Singal (1993) document that airfares on routes affected by a merger increase significantly relative to routes not affected by a merger. Similarly, Kim and Singal (1993) and Singal (1996) identify a positive relation between airfares and industry concentration. Singal (1996) establishes that acquirers and targets earn abnormal returns and abnormal returns to rival firms are positively related to changes in industry concentration. In sum, these studies support the view that mergers not only affect industry equilibrium but also have substantial anticompetitive effects.²

While the preceding empirical evidence demonstrates that industry characteristics play an important role in the takeover process, the extant theoretical literature lacks a thorough analysis

¹See Weston et al. (1990) and Weston et al. (1998) for a comprehensive review of the literature.

²Based upon data from 1970s, empirical studies by Eckbo (1983) and Eckbo and Wier (1985) find little evidence that challenged horizontal mergers are anticompetitive when using an event study methodology. While Singal (1996) offers a lucid discussion of the early evidence, McAfee and Williams (1988) cast doubt on the ability of event studies to detect anticompetitive mergers. In addition, Holmstrom and Kaplan (2001), Shleifer and Vishny (2003), and Andrade and Stafford (2004) point out a number of differences between the takeover activity in the 1960s or 1970s and the recent merger waves of the 1980s or 1990s.

of this issue.³ Our model attempts to fill this gap. We build a dynamic industry equilibrium model of mergers and acquisitions in a real options framework in the spirit of Lambrecht (2004). Unlike Lambrecht (2004), we consider an asymmetric oligopolistic industry structure similar to that in Perry and Porter (1985). We do not assume exogenously specified operating synergies, such as economies of scale. Instead, we specify a tangible asset that helps increase output for a given average cost. The merged firm can acquire assets from its two partners. Thus, the merged firm is larger and its average and marginal costs are lower. In addition, after a merger, firms face a different industry structure. We derive a closed-form solution to an industry equilibrium in which the benefits from merging are determined endogenously as a result of product market competition and cost reduction.

Unlike Perry and Porter's (1985) static model which analyzes incentives to merge only, our dynamic model allows us to derive a number of predictions that are in line with existing empirical evidence. In addition, our dynamic model also provides some novel testable empirical predictions regarding merger returns as well as the timing and terms of mergers. First, we introduce exogenous shocks to industry demand and demonstrate that a merger occurs at the first time when the demand shock hits a trigger value from below. That is, mergers occur in a rising product market. In addition, we show that the likelihood of mergers is greater for industries that are more exposed to exogenous shocks. These results are consistent with empirical evidence on merger waves discussed earlier (see, e.g., Mitchell and Mulherin (1996)).

Second, we show that increased competition delays the timing of mergers. This result is in contrast to that derived in recent real options models, e.g., Grenadier (2002). Grenadier (2002) emphasizes that competition erodes the option value of waiting and thus accelerates option exercise. He presents a symmetric industry model in which anticompetitive profits result from exogenously reducing the number of identical firms that compete in the industry. Unlike his model, firms are asymmetric in our model and anticompetitive profits result from two small firms merging into a large firm. These anticompetitive profits are larger for more concentrated industries because the effect of price increase after a merger is larger in these industries. Thus, firms in less competitive industries optimally exercise their option to merge sooner.

³While the welfare implications of mergers in oligopolistic industries have been extensively analyzed in the industrial organization literature (e.g., Farrell and Shapiro (1990)), this literature does not study the issues of merger timing and merger returns, which are the focus of our paper.

Finally, our model’s closed-form solutions show that the likelihood of restructuring activities and the magnitude of cumulative returns to both the merging and rival firms are positively associated with industry concentration. This result holds true for both friendly mergers and hostile takeovers. As in Lambrecht (2004), we show that there is a Pareto optimal allocation of the ownership share in a friendly merger such that both the acquirer and the target exercises the merger option at the socially optimal threshold. By contrast, in a hostile takeover, the target precommits the terms of the takeover, and the acquirer subsequently decides on the timing given those terms. Compared to a friendly merger, the target asks for a larger ownership and the acquirer delays the takeover waiting for a larger restructuring surplus. For both friendly mergers and hostile takeovers, the likelihood of restructuring activities and the magnitude of cumulative returns from restructuring to both merging and rival firms are positively related to anticompetitive profit gains, which are positively related to industry concentration.

Our paper contributes to a growing body of research using the real options approach to analyze the dynamics of mergers and acquisitions. Morellec and Zhdanov (2005) generalize Shleifer and Vishny’s (2003) static model to incorporate imperfect information and uncertainty into a dynamic framework. Lambrecht and Myers (2005) study mergers and acquisitions in declining industries motivated by greater efficiency through layoffs, consolidation, and disinvestment. Margsiri, Mello and Ruckes (2005) analyze a firm’s decision to grow internally or externally by making an acquisition. Hackbarth and Morellec (2006) examine the risk dynamics throughout the merger episode. Morellec and Zhdanov (2006) analyze the interaction between financial leverage and takeover activity in a model with endogenous timing, while Leland (2006) considers the role of purely financial synergies in motivating mergers and acquisitions in a model with exogenous timing. All these papers do not consider the impact of product market competition on takeover activity.⁴ The papers closest to ours are Lambrecht (2004) and Bernile, Lyandres, and Zhdanov (2006). Bernile, Lyandres, and Zhdanov (2006) present a duopolistic industry model with two incumbents and a potential new entrant. Their model of Bertrand competition focuses on merger waves, which are under certain conditions deterred by the threat of entry. In contrast to their model, we study Cournot competition and do not consider entry.

⁴Using the real options approach, Fries, Miller, and Perraudin (1997), Lambrecht (2001) and, Miao (2005) develop industry equilibrium models to analyze the implications of industry structure for firm investment, financing, entry and exit decisions.

Lambrecht (2004) studies mergers motivated by economies of scale. Although he does not focus on industry structure, he does analyze the case of a duopoly industry and shows that market power strengthens symmetric firms' incentives to merge. Unlike his model, we consider an oligopolistic Cournot–Nash model with asymmetric firms. In addition, in order to focus on the role of industry structure, we abstract from economies of scale.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 analyzes the timing of a socially optimal merger. Sections 4 and 5 examine, respectively, the implications of industry structure for the timing and terms of friendly and hostile control transactions. Section 6 concludes. Proofs are relegated to an appendix.

2 The Model

We incorporate an asymmetric oligopolistic industry structure into a real options framework. After outlining the framework's assumptions, we characterize industry equilibrium when asymmetric firms play Cournot–Nash strategies. We then examine the incentive to merge when restructuring decisions are motivated by operating and strategic benefits.

2.1 Assumptions

Time is continuous and uncertainty is modeled by a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. To construct a dynamic equilibrium model of firms' restructuring decisions, we consider an industry populated by infinitely-lived firms whose assets generate a continuous stream of cash flows. The industry consists of n large firms and m small firms producing a single homogeneous output, where $n \geq 0$ and $m \geq 2$ are integers. To have sufficient symmetry for analytical convenience, we follow Perry and Porter (1985) and assume that each large firm is identical and owns capital k , and that each small firm is also identical, but owns capital $k/2$. The industry's total capital stock is in fixed supply and normalized to unity. In addition, capital does not depreciate over time. Thus, the industry's capital stock at each point in time satisfies

$$nk + mk/2 = 1. \tag{1}$$

The cost structure is important in the model. We denote by $C(q, K)$ the cost function of a firm that owns a fraction K of the capital stock and produces output q . The output q is

produced with a combination of the fixed capital input, K , and a vector of variable inputs, Z , according to a smooth concave production function, $q = F(Z, K)$. Then the cost function $C(q, K)$ is obtained from the cost minimization problem. Unlike Lambrecht (2004), we assume that the production function F has constant returns to scale.⁵ This implies that $C(q, K)$ is linearly homogenous in (q, K) . For analytical tractability, we adopt the quadratic specification of the cost function, $C(q, K) = q^2 / (2K)$. This cost function may result from the Cobb-Douglas production function $q = \sqrt{KZ}$, where Z may represent labor input. It is important to note that both the average and marginal costs decrease with the capital asset K . Salant et al. (1983) show that if average cost is constant independent of firm size, merger is unprofitable in a Cournot oligopoly with linear demand. It is profitable if and only if duopolists merge into monopoly. As pointed out by Perry and Porter (1985), the constant average cost assumption does not provide a sensible description of mergers.

The capital asset plays an important role in the model. It allows us to address the industry asymmetries caused by mergers of a subset of firms. A merged firm combines the capital assets of the two entities to produce output. It faces a different optimization problem immediately after restructuring because of its altered cost function and because of new strategic considerations that arise from the change in industry structure.

We suppose the industry's inverse demand at time t is given by the following linear function

$$P(t) = aY(t) - bQ(t), \quad (2)$$

where $Q(t)$ is the industry's output at time t , $Y(t)$ denotes the industry's demand shock at time t observed by all firms, and a and b are positive constants. Here, a represents exposure of demand to industry-wide shocks and b represents the price sensitivity of demand. We assume that the demand shock is governed by the following geometric Brownian motion process:

$$dY(t) = \mu Y(t)dt + \sigma Y(t) dW(t), \quad Y(0) = y_0, \quad (3)$$

where μ and σ are positive constants and $(W_t)_{t \geq 0}$ is a standard Brownian motion defined on $(\Omega, \mathcal{F}, \mathcal{P})$.

⁵To complement existing models, we do not allow for synergies due to economies of scale (Lambrecht (2004)), or synergies due to an efficiency-enhancing capital reallocation (Morellec and Zhdanov (2005)).

We assume that firms are risk neutral and discount future cash flows by $r > 0$. We also assume that all firms are Cournot–Nash players and management acts in the best interests of shareholders. Therefore, all corporate decisions are rational and value-maximizing choices. To ensure that the present value of profits is finite, we make the following assumption:

Assumption 1 *The parameters μ , σ , and r satisfy the condition $2(\mu + \sigma^2/2) < r$.*

Firms may decide to restructure if it is in their best interest. To analyze the effect of industry structure on mergers in the simplest possible way, we follow Perry and Porter (1985) and assume that only two small firms can merge to form a large firm. We do not consider cases in which a large firm or more than two small firms enter into a takeover. This assumption preserves the simple asymmetric industry structure through the merger episode. Thus, after a merger only the number of small and large firms changes, but all small firms or all large firms remain to be identical. Takeovers are costly in reality. When two small firms i and j merge, they incur sunk costs denoted by $X_i > 0$ and $X_j > 0$, which capture fees to investment banks and lawyers as well as costs of restructuring and integration of the two small firms into a large organization.

2.2 Industry Equilibrium

Let $q_f(t)$ denotes the quantity selected by a small ($f = s$) or a large ($f = l$) firm. Then each small firm's instantaneous profit is given by

$$\pi_s(t; n) = [aY(t) - bQ(t)]q_s(t) - q_s(t)^2/k, \quad (4)$$

and each large firm's instantaneous profit is given by

$$\pi_l(t; n) = [aY(t) - bQ(t)]q_l(t) - q_l(t)^2/(2k), \quad (5)$$

where

$$Q(t) = \sum_i q_i(t) = nq_l(t) + mq_s(t) \quad (6)$$

is the industry output. Note that the argument n in the profit function indicates that there are n large firms in the industry prior to a merger. This notation is useful for our merger analysis below since the number of large firms changes after a restructuring option is exercised. We will

use similar notation below. Given the preceding instantaneous profits, we can compute firm value, or the present value of profits,

$$V_f(y; n) = \mathbb{E}^y \left[\int_0^\infty e^{-rt} \pi_f(t; n) dt \right], \quad (7)$$

for $f = s, l$, where $\mathbb{E}^y[\cdot]$ denotes the conditional expectation operator given that the current shock takes the value $Y(t) = y$.

As in Grenadier (2002), we define strategies and industry equilibrium as follows. The strategy $\{(q_s^*(t), q_l^*(t)) : t \geq 0\}$ constitutes an industry (Markov perfect Nash) equilibrium if, given information available at date t , (1) $q_s^*(t)$ is optimal for a small firm, when it takes other firms' strategies $nq_l^*(t) + (m-1)q_s^*(t)$ as given, and (2) $q_l^*(t)$ is optimal for a large firm, when it takes other firms' strategies $(n-1)q_l^*(t) + mq_s^*(t)$ as given. Because firms play a dynamic game, there could be many Markov perfect Nash equilibria as is well known in game theory. Instead of finding all equilibria, we will focus on the equilibrium where small and large firms play static Cournot strategies. As is well known, the static Cournot strategies that firms play at each date constitute a Markov perfect Nash equilibrium. The following proposition characterizes this industry equilibrium.

Proposition 1 (i) *In the Cournot–Nash industry equilibrium, the optimal output of any small firm and any large firm is respectively given by*

$$q_s^*(t) = \frac{a(b + k^{-1})}{\Delta(n)} Y(t), \quad (8)$$

$$q_l^*(t) = \frac{a(b + 2k^{-1})}{\Delta(n)} Y(t), \quad (9)$$

where⁶

$$\Delta(n) \equiv (b + k^{-1})(b + 2(b + 1)k^{-1}) - b^2n > 0. \quad (10)$$

(ii) *Moreover, the equilibrium industry output and price are respectively given by*

$$Q^*(t) = \frac{a[b(2k^{-1} - n) + 2k^{-2}]}{\Delta(n)} Y(t), \quad (11)$$

$$P(t) = \frac{a(b + k^{-1})(b + 2k^{-1})}{\Delta(n)} Y(t). \quad (12)$$

(iii) *Industry output (price) decreases (increases) with the number of large firms n .*

⁶It follows from (1) that $k^{-1} > n$. Thus, we can show that both $\Delta(n)$ and $\Delta(n + 1)$ are positive.

Part (i) of Proposition 1 characterizes firms' individually optimal output choices in our Cournot–Nash framework. It then aggregates individually optimal output choices into industry output, $Q^*(t)$, and industry equilibrium price, $P(t)$, in part (ii). As stated in part (iii), this industry equilibrium implies that industry output is a decreasing function of the number of large firms n . Since mergers will increase the number of large firms, they will raise industry output and lower price. This result is consistent with the economic intuition that mergers may reduce industry competition by raising firms' market power. In this model, it is possible that the price effect is dominated by the quantity effect and hence mergers may not lead to an increase in profits. The next section therefore derives conditions under which two small firms will have an incentive to merge.

2.3 Incentive to Merge

To analyze the incentive for two small firms to form a large firm, we first derive expressions for firm values of small and large firms before a takeover taking the industry equilibrium characterized in Proposition 1 as given. We then determine firm values after a merger based on the change in industry structure and the industry equilibrium stipulated in Proposition 1. The incentive to merge then follows from comparing the different firm values.

Proposition 2 *Suppose Assumption 1 holds. For $f = s, l$, the equilibrium firm value is given by*

$$V_f(y; n) = \frac{\Pi_f(n) y^2}{r - 2(\mu + \sigma^2/2)}, \quad (13)$$

where the profit multipliers of small and large firms are given by

$$\Pi_s(n) = \frac{a^2 (b + k^{-1})^3}{\Delta(n)^2}, \quad (14)$$

$$\Pi_l(n) = \frac{a^2 (b + 2k^{-1})^2 (b + k^{-1}/2)}{\Delta(n)^2}. \quad (15)$$

Note that after a merger there are $n + 1$ identical large firms and $m - 2$ identical small firms in the industry. Thus, the value of the large firm after a merger is given by $V_l(y; n + 1)$. It follows from Proposition 2 that the benefit from merging is given by

$$V_l(y; n + 1) - 2V_s(y; n) = \frac{[\Pi_l(n + 1) - 2\Pi_s(n)] y^2}{r - 2(\mu + \sigma^2/2)}. \quad (16)$$

To have takeover incentives, the term $\Pi_l(n+1) - 2\Pi_s(n)$ must thus be positive. This term represents the profitability of an anticompetitive merger. After a merger, the number of small firms in the industry is reduced and hence the industry's market structure is also changed. It is straightforward to show that the output of two small firms prior to a merger exceeds the output of one merged firm using Proposition 1. Thus, an incentive to merge requires that the increase in industry price be sufficient to offset the reduction in output of the merged firm. Based on these arguments, we can summarize the following conditions for takeover incentives.

Proposition 3 *Let $\Delta(n)$ be given in (10) and define the critical value*

$$\Delta^* \equiv \frac{b^2}{1-A}, \quad (17)$$

where A is given by

$$A \equiv (b + 2k^{-1}) \sqrt{\frac{b + k^{-1}/2}{2(b + k^{-1})^3}}. \quad (18)$$

(i) If

$$\max_n \Delta(n) = \Delta(0) < \Delta^*, \quad (19)$$

then there will always be an incentive to merge. (ii) If

$$\min_n \Delta(n) = \Delta(1/k - 1) > \Delta^*, \quad (20)$$

then there will never have an incentive to merge. (iii) If

$$\Delta(0) > \Delta^* > \Delta(1/k - 1), \quad (21)$$

then when n is large enough there will be an incentive to merge.

In this proposition, (19) and (21) provide two conditions for takeover incentives in our Cournot–Nash framework.⁷ That is, when the increase in price outweighs the decrease in output such that net effect leads to an increase in instantaneous profits, the two small firms have an incentive to form a large organization. Specifically, these two conditions depend on the industry demand function through b , the industry capital stock, k , and the industry structure through the number n of existing large firms prior to the restructuring. Moreover, the proposition shows

⁷Similar conditions have been derived by Perry and Porter (1985) in their static model.

that there could be no incentives to merge if the condition in (20) holds. Finally, observe that this oligopolistic industry also encompasses the situation in which mergers would not occur unless the industry was sufficiently concentrated to start with.

It is straightforward to show that there is always an incentive to merge when the industry consists of a small-firm duopoly.⁸ A similar result is obtained by Salant et al. (1983) and Perry and Porter (1985) in a static industry model. More recently, Lambrecht (2004) analyzes a dynamic model in which duopolists merge to form a monopolist motivated by economies of scale, while Morellec and Zhdanov (2005) assume an exogenously specified synergy gain for a merger of two firms in an industry.

In the analysis to follow, we make the following assumption:

Assumption 2 *Suppose $\Pi_l(n+1) - 2\Pi_s(n) > 0$ so that there is an incentive to merge.*

A sufficient condition for this assumption is given by case (i) or (iii) in Proposition 3. Given this assumption, there exists an incentive to merge. In this case, equation (16) implies that the benefit from mergers increases with the demand shock so that it is procyclical. When considering a merger, firms tradeoff the stochastic benefit from merging against the fixed cost of merging. Since firms have the option but not the obligation to merge, the surplus from merging has a call option feature. This merger surplus is therefore given by the positive part of the (net) payoff function from merging; that is,

$$S(y; n) = [V_l(y; n+1) - 2V_s(y; n) - X_i - X_j]^+, \quad (22)$$

for the case when two small firms i and j merge to form the $(n+1)$ th large firm.

The following lemma is useful for our merger analysis below.

Lemma 1 *Under Assumption 2, the profit differential $\Pi_l(n+1) - 2\Pi_s(n)$ increases with a and n , and the profit ratio $\Pi_l(n+1) / \Pi_s(n)$ increases with n .*

To interpret this lemma, recall that the parameter n represents the number of large firms in the industry prior to a merger. This parameter proxies for industry concentration. To see this

⁸One can immediately verify that condition (19) is satisfied for $k = 1$, $n = 0$, and $m = 2$, which is a limiting case of our model and the one considered e.g. by Lambrecht (2004). Thus, there are benefits for small firms to restructure even if there are no economies of scales nor efficiency-enhancing synergies.

point, we consider the industry's Herfindahl index, which is defined as the sum of the squares of the market shares of each firm in the industry. That is, the Herfindahl index $H(t)$ at date t is given by

$$H(t) = n \left(\frac{q_l(t)}{Q(t)} \right)^2 + m \left(\frac{q_s(t)}{Q(t)} \right)^2. \quad (23)$$

Using (1) and Proposition 1, we can derive that

$$H(t) = \frac{n(2k^{-2} - b^2) + 2(b + k^{-1})^2 k^{-1}}{[b(2k^{-1} - n) + 2k^{-2}]^2}, \quad (24)$$

which is monotonically increasing in n .⁹

We also know that the parameter a represents the exposure of industry demand to the industry-wide shock. A larger value of a implies that the increase in industry price and the market size is higher in response to an increase in the exogenous demand shock. Lemma 1 then demonstrates that the anticompetitive gains from a merger measured in terms of either profit differential or profit ratio are larger in more concentrated industries. In addition, when measured in terms of profit differential, these gains are also larger in industries that are more exposed to the industry-wide shock.

The conditions in Proposition 3 and Assumption 2 are not easy to check analytically. We thus use Figure 1 to illustrate them numerically. This figure also shows that both the profit differential and the profit ratio are not monotonic with the price sensitivity parameter b . This non-monotonicity results from two opposing effects of an increase in b . Specifically, it raises price, but lowers output. Thus, the economic impact of changes in the price sensitivity parameter on both the profit differential and the profit ratio are ambiguous because it depends on whether the price or the quantity effect dominates.

[Insert Figure 1 Here]

⁹To show this, we observe that

$$\frac{\partial H(t)}{\partial n} = \frac{(b^3 k^4 n - 2bk^2 n - 6b^2 k^2 - 2b^3 k^3 - 8bk - 4)(bk^2 n - 2bk - 2)}{k^6 [b(2k^{-1} - n) + 2k^{-2}]^4},$$

which is positive because equation (1) implies that $nk < 1$ and hence

$$(b^3 k^4 n - 2bk^2 n - 6b^2 k^2 - 2b^3 k^3 - 8bk - 4) < -2bk^2 n - 6b^2 k^2 - b^3 k^3 - 8bk - 4 < 0$$

and $(bk^2 n - 2bk - 2) < -bk - 2 < 0$.

3 The timing of a socially optimal merger

To serve as a benchmark for the later analysis where both the timing and the terms of the merger will be jointly determined, we start by examining a socially optimal merger. We suppose that there is a social planner that determines the timing of the merger. The social planner selects the timing of the merger to maximize the total surplus of the merger accruing to the two small firms i and j . However, the social planner does not need to determine the negotiated terms of the merger as to how the merger surplus is divided between the two firms because the terms are irrelevant for social optimality.

Formally, the social planner's objective is to select a merger time τ by solving the following optimization problem

$$OM(y; n) = \sup_{\tau} \mathbb{E}^y [e^{-r\tau} S(Y_{\tau}; n)], \quad (25)$$

where $OM(y; n)$ denotes the value of the merger option when the current shock takes the value y and there are n firms prior to a merger. Using Proposition 2 and the merger surplus in (22), we can derive

$$S(y; n) = \left[\frac{(\Pi_l(n+1) - 2\Pi_s(n)) y^2}{r - 2(\mu + \sigma^2/2)} - X_i - X_j \right]^+. \quad (26)$$

This equation shows that the merger surplus $S(y; n)$ increases with the industry's demand shock y . Thus, as is familiar from the real option theory, there exists a value-maximizing threshold y^* such that restructuring is optimal as soon as the demand shock $Y(t)$ touches y^* the first time from below.

To solve for the optimal threshold y^* , we denote by $OM(y, y^*; n)$ the value of the merger option when the trigger is y^* and the current shock takes value $y \leq y^*$. We then have

$$OM(y, y^*; n) = \mathbb{E}^y [e^{-r\tau_{y^*}} S(Y(\tau_{y^*}); n)], \quad (27)$$

where τ_{y^*} denotes the first passage time of the process $Y(t)$ to the socially optimal merger threshold y^* . It is easy to show that

$$OM(y, y^*; n) = S(y^*; n) \left(\frac{y}{y^*} \right)^{\beta}, \quad (28)$$

for all $y \leq y^*$ (see e.g. Karatzas and Shreve (1999)), where β denotes the positive root to the characteristic equation

$$0.5 \sigma^2 \beta (\beta - 1) + \mu \beta - r = 0. \quad (29)$$

Note that it is straightforward to prove that $\beta > 2$ given Assumption 1. For y^* to be optimal, the option to merge must satisfy the following smooth-pasting condition:

$$\left. \frac{\partial OM(y, y^*; n)}{\partial y} \right|_{y=y^*} = 0. \quad (30)$$

This condition is also equivalent to the first-order condition (see e.g. Dumas (1991))

$$\frac{\partial OM(y, y^*; n)}{\partial y^*} = 0. \quad (31)$$

We now can derive the following result.

Proposition 4 *Suppose that Assumptions 1 and 2 hold. (i) The socially optimal restructuring takes place when the demand shock $Y(t)$ reaches the merger threshold y^* the first time from below, where y^* is given by*

$$y^* = \sqrt{\frac{\beta(X_i + X_j)}{\beta - 2} \frac{r - 2(\mu + \sigma^2/2)}{\Pi_l(n+1) - 2\Pi_s(n)}}. \quad (32)$$

(ii) *Prior to the merger $y \leq y^*$, the value of the joint merger option is given by*

$$OM(y, y^*; n) = \left[\frac{(\Pi_l(n+1) - 2\Pi_s(n))y^{*2}}{r - 2(\mu + \sigma^2/2)} - X_i - X_j \right] \left(\frac{y}{y^*} \right)^\beta. \quad (33)$$

(iii) *The merger threshold y^* decreases with a and n .*

Proposition 4 highlights several interesting features of mergers and acquisitions. First, part (i) of Proposition 4 shows that in an oligopolistic industry, a socially optimal merger occurs in a rising product market. This result also holds true for a friendly merger or a hostile merger as will be shown later. Thus, consistent with empirical evidence documented by Maksimovic and Phillips (2001), cyclical product markets generate a procyclical merger wave. This result is also consistent with Mitchell and Mulherin's (1996) empirical finding that industry shocks contribute to the merger and restructuring activities observed during 1980s.

To interpret equation (32), we use (26) to rewrite it as

$$S(y^*; n) = \frac{2}{\beta - 2} (X_i + X_j) > 0. \quad (34)$$

This equation implies that at the time of merger the surplus of the merger exceeds a positive value. This reflects the option value of waiting because mergers are irreversible.

Second, we turn to part (ii) of Proposition 4, which characterizes the value of the option to merge and the optimal option exercise strategy. As shown in Lambrecht (2004), the socially optimal timing of mergers depends on the growth rate and volatility of the state variable that affects the firms' core businesses valuations as well as the implementation and restructuring costs, but in this framework it also depends on the industry structure as captured by the term $\Pi_l(n+1) - 2\Pi_s(n)$. Notably, equation (33) admits an intuitive interpretation. The value of the option to merge is equal to the surplus, $S(y^*; n)$, generated at the time of the merger multiplied by a discount factor $(y/y^*)^\beta$. This discount factor can be interpreted as the Arrow-Debreu price of a primary claim that delivers \$1 at the time and in the state when the merger occurs. It can also be regarded as the probability of the demand shock $Y(t)$ reaching the merger threshold y^* the first time from below given that the current level of the demand shock is y .

Finally, part (iii) of Proposition 4 reports interesting and new comparative statics results. The standard comparative statics results regarding the timing of irreversible investments, such as mergers and acquisitions, are well known in the real options literature (see, e.g., Dixit and Pindyck (1994)). For example, an increase in the industry's demand uncertainty delays the timing of mergers, and an increase in the drift of the industry's demand shock speeds up the timing of mergers. So we suppress this discussion.

The novel comparative statics results in our paper are related to industry characteristics. First, part (iii) of Proposition 4 implies that the socially optimal merger threshold declines with the parameter a . Since the parameter a represents the exposure of the industry demand to the exogenous shock, one should expect to observe more mergers and acquisitions in industries whose demand is more exposed or more sensitive to the exogenous shock.¹⁰ This result is consistent with the empirical evidence documented by Mitchell and Mulherin (1996). They find that industries that experience the greatest amount of merger and restructuring activity in the 1980s are those that are exposed the most to industry shocks. The intuition behind the preceding result is that an increase in a raises industry demand for a given positive shock.

¹⁰The probability of a merger taking place over a time interval $[0, T]$ is given by

$$\Pr \left(\sup_{0 \leq \tau \leq T} Y(\tau) \geq y^* \right) = \mathcal{N} \left[\frac{\ln(y_0/y^*) + (\mu - \sigma^2/2) T}{\sigma\sqrt{T}} \right] + \left(\frac{y_0}{y^*} \right)^{-(2\mu - \sigma^2)/\sigma^2} \mathcal{N} \left[\frac{\ln(y_0/y^*) - (\mu - \sigma^2/2) T}{\sigma\sqrt{T}} \right],$$

where \mathcal{N} is the standard normal cumulative distribution function. This probability decreases with the merger threshold y^* .

Thus, it raises anticompetitive gains $\Pi_l(n+1) - 2\Pi_s(n)$ as shown in Lemma 1, thereby raising the surplus $S(y; n)$ from a merger.

We next turn to the analysis of industry concentration. Part (iii) of Proposition 4 implies that one should expect to see more mergers and acquisitions taking place in more concentrated (i.e. less competitive) industries. The economic intuition behind this result is simple. A relatively higher level of industry concentration is associated with relatively larger anticompetitive profits by Lemma 1, which raise the incentive to merge, *ceteris paribus*. In particular, a larger magnitude of anticompetitive profits leads to an increase in restructuring benefits and thus firms in *less* competitive industries will optimally exercise their option to merge, in expectation, sooner.

This implication about the optimal timing of mergers in our Cournot–Nash framework is in sharp contrast to most of the earlier findings in the real options literature. Notably, Grenadier (2002) demonstrates in his symmetric industry equilibrium model with irreversible investment that firms in *more* competitive industries will optimally exercise their investment options, in expectation, sooner. We attribute the startling difference in results to differences in the economic modeling of industry structure. In our asymmetric industry equilibrium model, anticompetitive profits result from mergers of two small firms which form a large firm. In Grenadier’s (2002) model, anticompetitive profits result from exogenously reducing the number of identical firms that compete in the industry. In addition, Grenadier (2002) studies an incremental investment problem, while we analyze a single discrete option exercise decision.

In the next two sections, we consider the more realistic situation where a social planner does not exist and hence individual optimizing behavior will determine restructuring decisions. That is, when considering a merger between two firms, each firm maximizes the present value of the merger surplus accruing to itself. We will endogenously determine both the timing and the terms of mergers as a result of an option exercise game. In analyzing option exercise games, we consider two restructuring mechanisms. In Section 4, we assume that the two firms decide on the (socially optimal) timing of the merger first. They then negotiate how to share the maximized surplus from merging. This restructuring mechanism is similar to a friendly merger. In Section 5, we analyze the alternative restructuring mechanism in which the target firm first decides on the division of the surplus and subsequently the acquiring firm decides on

the timing of the merger. This mechanism is similar to a hostile takeover.

4 The timing and terms of a friendly merger

In this section, we consider a friendly merger. We will show that in a friendly merger, firms can exercise their merger options at the socially optimal threshold given in Proposition 4 if the ownership share is appropriately assigned to the merging firms. We will analyze the impact of competition and industry structure on the division of surplus from merging.

Let ξ_f denotes the ownership share of firm $f = i, j$ in the merged entity (with $\xi_i + \xi_j = 1$). Then the merger surplus accruing to firm f , denoted by $S_f(y, \xi_f; n)$, is again given by the positive part of the (net) payoff function from merging; that is,

$$S_f(y, \xi_f; n) = [\xi_f V_l(y; n+1) - V_s(y; n) - X_f]^+. \quad (35)$$

Let y_f^* denote the merger threshold selected by firm f . The value of firm f 's option to merge, denoted by $OM_f(y, y_f^*, \xi_f; n)$, is given by

$$OM_f(y, y_f^*, \xi_f; n) = \mathbb{E}^y \left[e^{-r\tau_{y_f^*}} S_f \left(Y(\tau_{y_f^*}), \xi_f; n \right) \right], \quad (36)$$

where $\tau_{y_f^*}$ denotes the first passage time of the process $Y(t)$ to the merger threshold y_f^* selected by firm f starting from the value y .

As in Coase (1960), the socially optimal merger policy derived in the previous section can be achieved. In particular, a Pareto optimal sharing rule, ξ_f , which provides incentives for both firms to agree to a merger at the socially optimal threshold y^* must satisfy the following two conditions:

$$\left. \frac{\partial OM_f(y, y_f^*, \xi_f; n)}{\partial y_f^*} \right|_{y_f^*=y^*} = 0, \quad \text{and} \quad \xi_i + \xi_j = 1. \quad (37)$$

Put differently, we want to select a sharing rule such that both firms will prefer to enter into a merger agreement at the socially optimal threshold y^* . That is, we impose the requirement that $y_i^* = y_j^* = y^*$ because it ensures maximal surplus available to the two firms.

Proposition 5 *Suppose that Assumptions 1 and 2 hold. (i) In the absence of any transaction costs or other frictions, the value-maximizing merger threshold y_f^* for each firm $f = i, j$ is equal*

to the socially optimal threshold y^* , given in Proposition 4. (ii) Prior to the merger $y \leq y_f^*$, the value of firm f 's option to merge is given by

$$OM_f(y, y_f^*, \xi_f^*; n) = \left[\frac{(\xi_f^* \Pi_l(n+1) - \Pi_s(n)) y_f^{*2}}{r - 2(\mu + \sigma^2/2)} - X_f \right] \left(\frac{y}{y_f^*} \right)^\beta, \quad (38)$$

for $f = i, j$. (iii) The unique sharing rule that induces both firms to exercise their merger options at the threshold y^* is given by

$$\xi_i^* = \frac{X_i}{X_i + X_j} + \frac{\Pi_s(n)}{\Pi_l(n+1)} \left[1 - \frac{2X_i}{X_i + X_j} \right]. \quad (39)$$

(iv) In addition, if $X_i > X_j$, then the ownership share ξ_i^* increases with n .

One main difference between Propositions 4 and 5 is that Proposition 5 provides closed-form expressions for each firm's individually optimal post-merger ownership of the merged entity and for each firm's value of the merger option. Equation (38) admits a similar interpretation to (33). That is, the value of firm f 's merger option is equal to the merger surplus $S_f(y_f^*, \xi_f^*; n)$ accrued to firm f at the merger time, multiplied by a discount factor $(y/y_f^*)^\beta$, representing either Arrow-Debreu price or merger probability.

From the expression in equation (39), one can see that a firm's post-merger ownership share depends on the restructuring costs X_i and X_j as well as the ratio of the post-merger profits to the pre-merger profits. In the symmetric case where $X_i = X_j$, the two merging partners have equal ownership shares, $\xi_i^* = \xi_j^* = 1/2$. We thus consider the more interesting asymmetric case and assume $X_i > X_j$ without loss of generality.

Since industry concentration increases with n , part (iv) of Proposition 5 shows that if a firm incurs a larger merging cost than its merging partner, then this firm's ownership share increases with industry concentration and its partner's ownership share decreases with industry concentration. The intuition behind this result is the following. When the industry is more concentrated, the increase in price after the merger has a stronger effect so that the ratio of the post-merger profits to the pre-merger profits is higher by Lemma 1. Since the firm with the higher restructuring cost has a higher exercise price of its merger option, for its merger option value to be maximized, it needs to demand a higher benefit from merging. As a result, the firm that incurs higher restructuring costs will in equilibrium require a correspondingly higher ownership share in the merged entity.

We now turn to cumulative returns resulting from a friendly merger. The stock value of a small merging firm $f = i, j$ before the merger, denoted by $E_f(Y(t); n)$, is equal to the value of its assets in place plus the value of the merger option:

$$E_f(Y(t); n) = V_s(Y(t); n) + OM_f(Y(t), y^*, \xi_i; n). \quad (40)$$

We express the cumulative merger return as a fraction of the stand-alone equity value, $V_s(Y(t); n)$. Thus, the cumulative return to the merging firm $f = i, j$ at time $t \leq \tau_{y^*}$ is given by

$$R_{f,M}(Y(t), n) = \frac{E_f(Y(t); n) - V_s(Y(t); n)}{V_s(Y(t); n)} = \frac{OM_f(Y(t), y^*, \xi_i; n)}{V_s(Y(t); n)}. \quad (41)$$

The cumulative return to the merging firm f at the time of merger announcement is equal to the preceding expression evaluated at $t = \tau_{y^*}$ and $Y(t) = y^*$.

Similarly, we can compute the cumulative merger return to a rival firm at the time of the merger announcement. The stock value of a rival firm $f = s, l$ prior to the announcement of a merger at date $t \leq \tau_{y^*}$ is given by

$$V_f(Y(t); n) + \mathbb{E} \left[e^{-r(\tau_{y^*}-t)} (V_f(Y(\tau_{y^*}); n+1) - V_f(Y(\tau_{y^*}); n)) | Y(t) \right]. \quad (42)$$

That is, it is equal to the firm value before merger plus an option value from the merger. This option value results from the fact that the value of the rival firm becomes $V_f(Y(\tau_{y^*}); n+1)$ after the merger since there are $n+1$ large firms in the industry. The cumulative return to a small or large rival firm before the merger is given by

$$R_{f,R}(Y(t); n) = \frac{\mathbb{E} \left[e^{-r(\tau_{y^*}-t)} (V_f(Y(\tau_{y^*}); n+1) - V_f(Y(\tau_{y^*}); n)) | Y(t) \right]}{V_f(Y(t); n)} \quad (43)$$

for $f = s, l$. We will focus on the cumulative return at the time of the merger announcement, when $t = \tau_{y^*}$ and $Y(\tau_{y^*}) = y^*$. The following proposition characterizes the cumulative returns at the time of the merger announcement.

Proposition 6 *Suppose that Assumptions 1 and 2 hold. (i) The cumulative merger return to the merging firms at the time of restructuring is given by*

$$R_{f,M}(y^*; n) = \frac{2X_f}{\beta(X_i + X_j)} \left[\frac{\Pi_l(n+1)}{\Pi_s(n)} - 2 \right], \quad (44)$$

for $f = i, j$. (ii) The cumulative merger return to a small or large rival firm at the time of restructuring is given by

$$R_{f,R}(y^*; n) = \left[1 + \frac{b^2}{\Delta(n+1)} \right]^2 - 1, \quad (45)$$

for $f = s, l$. (iii) All the above returns are positive and increase with n .

Proposition 6 highlights several interesting aspects of cumulative merger returns in our Cournot–Nash framework. First, equation (44) reveals that the cumulative return has three determinants including an anticompetitive effect, a hysteresis effect, and a size effect. The last two effects represented by β and X_f are also derived by Lambrecht (2004) in the presence of economies of scale. By contrast, our model with constant returns to scale does not have a synergy effect discussed by Lambrecht (2004). Instead, we have the anticompetitive effect represented by the ratio $[\Pi_l(n+1) - 2\Pi_s(n)]/\Pi_s(n)$. This effect reflects the fact that after a merger, the number of small (large) firms in the industry decreases (increases) and hence the market structure and competitive landscape of the industry change. Consistent with economic intuition, the term $[\Pi_l(n+1) - 2\Pi_s(n)]/\Pi_s(n)$ therefore indicates the percentage gain in profits resulting from the change in the industrial organization of the market. Second, merging firms have higher cumulative returns in more concentrated industries. The intuition is that the anticompetitive effect on the industry’s equilibrium price after a merger is stronger for those industries. As a consequence, merging firms derive higher restructuring benefits. Third, equation (45) reveals that the cumulative return to a small or a large rival firm at the time of merger announcement is positive, too. Like the cumulative returns to merging firms, the cumulative returns to rival firms increase with industry concentration. The intuition is that the industry’s equilibrium price rises after the merger and rival firms also benefit from this price increase. This benefit increases with industry concentration.

5 The timing and terms of a hostile takeover

After analyzing friendly mergers, we now turn to the analysis of hostile takeovers.¹¹ As suggested by Lambrecht (2004), a hostile takeover corresponds to a Stackelberg leader–follower game in which the target first precommits to the terms it requires and the acquirer sub-

¹¹See, e.g. Schwert (2000), for the characteristics of friendly and hostile transactions.

sequently decides on the timing of the restructuring taking the (unnegotiable) terms of the target as given. While hostile takeovers result in an inefficient timing of mergers, we will study the role of industry characteristics for the timing and the terms of takeovers as well as the cumulative returns to the acquirer and the target.

We use subscripts i and j to denote the acquirer and the target, respectively. Let \bar{y}^* denote the takeover threshold selected by the acquirer. Let $OT_f(y, \bar{y}^*, \xi_f; n)$ represent the value of the takeover option for firm $f = i, j$ when the current shock takes the value y , where ξ_j is the ownership share demanded by the target with $\xi_i + \xi_j = 1$. Let $\tau_{\bar{y}^*}$ denote the first passage time of the process $Y(t)$ to the takeover threshold \bar{y}^* starting from the initial value y . For $y \leq \bar{y}^*$, we then have

$$OT_f(y, \bar{y}^*, \xi_f; n) = \mathbb{E}^y [e^{-r\tau_{\bar{y}^*}} S_f(Y(\tau_{\bar{y}^*}), \xi_f; n)], \quad (46)$$

where the surplus to firm f , $S_f(\cdot)$, is given in (35).

The target and the acquirer's optimization problem can be formulated in the following two steps. (i) Given the threshold strategy $\bar{y}^*(\xi_i)$ selected by the acquirer firm i , the target firm j chooses ownership share ξ_j so as to solve

$$\max_{\xi_j} OT_j(y, \bar{y}^*, \xi_j; n), \quad (47)$$

subject to $\xi_i + \xi_j = 1$. (ii) The acquirer firm i chooses $\bar{y}^*(\xi_i)$ to solve the maximization problem,

$$\max_{\bar{y}^*} OT_i(y, \bar{y}^*, \xi_i; n). \quad (48)$$

We now can derive the following result.

Proposition 7 *Suppose that Assumptions 1 and 2 hold. (i) The value-maximizing takeover threshold of the Stackelberg game is given by*

$$\bar{y}^* = \sqrt{\frac{\beta}{\beta - 2} \left(\frac{\beta}{\beta - 2} X_i + X_j \right) \frac{r - 2(\mu + \sigma^2/2)}{\Pi_l(n + 1) - 2\Pi_s(n)}}. \quad (49)$$

(ii) *Prior to the takeover $y \leq \bar{y}^*$, the value of firm f 's option to merge, for $f = i, j$, is given by*

$$OT_f(y, \bar{y}^*, \bar{\xi}_f; n) = \left[\frac{(\bar{\xi}_f \Pi_l(n + 1) - \Pi_s(n)) \bar{y}^{*2}}{r - 2(\mu + \sigma^2/2)} - X_f \right] \left(\frac{y}{\bar{y}^*} \right)^\beta. \quad (50)$$

(iii) The ownership share of the acquiring firm ($f = i$) is given by

$$\bar{\xi}_i = \frac{X_i}{\beta/(\beta-2)X_i + X_j} + \frac{\Pi_s(n)}{\Pi_l(n+1)} \left[1 - \frac{2X_i}{\beta/(\beta-2)X_i + X_j} \right]. \quad (51)$$

(iv) The takeover threshold \bar{y}^* decreases with a and n , and $\bar{y}^* > y^*$. (v) In addition, we have that $\bar{\xi}_i < \xi_i^*$ and that if $(\beta-4)X_i > (\beta-2)X_j$, then the ownership share $\bar{\xi}_i$ increases with n .

We now compare the timing and terms of the hostile takeover obtained in this proposition with those of the friendly merger derived in Proposition 5. First, as in a friendly merger, the takeover threshold depends on the anticompetitive profits and industry characteristics represented by the parameters a, b and n . In particular, this threshold decreases with a and n . Thus, a hostile takeover is more likely in a more concentrated industry, or in an industry with demand being more sensitive to exogenous shocks.

Second, compared to a friendly merger, a takeover takes place, in expectation, inefficiently late. The intuition is similar to that in Lambrecht (2004). Compared to a friendly merger, the target j asks for a larger ownership share. This reduces the acquirer i 's ownership share in that $\bar{\xi}_i < \xi_i^*$. The acquirer i then requires the total surplus to be larger and consequently delays the timing of the takeover. The extra delay is reflected by the squared option multiplier $(\beta/(\beta-2))^2$ in (49). This multiplier results from the standard real options theory. With takeovers the acquirer needs to pay an additional bid premium. This bid premium acts as an additional sunk cost to the acquirer.

We next turn to cumulative takeover returns. As in Section 4, we define the cumulative return to the takeover firm $f = i, j$ at the time $t \leq \tau_{\bar{y}^*}$ as

$$R_{f,T}(Y(t), n) = \frac{E_f(Y(t); n) - V_s(Y(t); n)}{V_s(Y(t); n)} = \frac{OT_f(Y(t), \bar{y}^*, \xi_f; n)}{V_s(Y(t); n)}, \quad (52)$$

and to the small or large rival firms by equation (43). The following proposition characterizes the cumulative returns at the time of the takeover announcement.

Proposition 8 *Suppose that Assumptions 1 and 2 hold. (i) At the time of the takeover announcement, the cumulative takeover return to the acquirer is given by*

$$R_{i,T}(\bar{y}^*; n) = \frac{2X_i}{\beta(\beta/(\beta-2)X_i + X_j)} \left[\frac{\Pi_l(n+1)}{\Pi_s(n)} - 2 \right], \quad (53)$$

and the cumulative takeover return to the target is given by

$$R_{j,T}(\bar{y}^*; n) = \frac{2}{\beta} \left[\frac{\Pi_l(n+1)}{\Pi_s(n)} - 2 \right]. \quad (54)$$

(ii) The cumulative return to a small or large rival firm at the time of the takeover announcement is given by

$$R_{f,R}(\bar{y}^*; n) = \left[1 + \frac{b^2}{\Delta(n+1)} \right]^2 - 1, \quad (55)$$

for $f = s, l$. (iii) All the above returns are positive and increase with n . (iv) In addition, $R_{i,T} < R_{i,M}$ and $R_{j,T} > R_{j,M}$.

The analytic expressions for cumulative takeover returns allow us to make a transparent comparison between the two restructuring mechanisms. As in the case of a friendly merger characterized in Proposition 6, the cumulative returns are determined by an anticompetitive effect, a hysteresis effect, and a size effect. In the hostile takeover, there is also a bidding effect as in Lambrecht (2004). This effect is represented by the second term in the expression for the ratio of the takeover returns:

$$\frac{R_{j,T}(\bar{y}^*; n)}{R_{i,T}(\bar{y}^*; n)} = \frac{\beta}{\beta - 2} + \frac{X_j}{X_i}. \quad (56)$$

Unlike Lambrecht (2004), there is no synergy effect. Instead, the anticompetitive effect in our model implies that the takeover returns to the acquirer and the target as well as to the rivals are higher in more concentrated industries. This result is stated in part (iii) of the proposition and its intuition are similar to those for the case of a friendly merger.

Next, it is interesting to compare the cumulative returns to rival firms across the restructuring mechanisms. Parts (ii) of Propositions 6 and 8 imply that the cumulative returns to rivals are unaffected by the restructuring mechanism. This implies that these cumulative returns cannot be used empirically to differentiate a friendly merger from a hostile takeover.

Finally, part (iv) implies that the acquirer fares better under a friendly merger, while the target fares better under a hostile takeover. As in Lambrecht (2004), the target's increased bargaining power in a hostile takeover raises its cumulative return by raising the factor from $X_i/(X_i + X_j)$ in (44) to unity in (54). In addition, this increased bargaining power is reflected in equation (53) by an increase in the acquirer's merger cost by the hysteresis factor $\beta/(\beta - 2) > 1$, which decreases the acquirer's cumulative return.

6 Conclusion

This paper develops a real options model of mergers and acquisitions that jointly determines the industry's product market equilibrium, and the timing and terms of takeovers. The analysis in the paper explicitly recognizes the role of the strategic product market interaction resulting from the deal and derives equilibrium restructuring strategies by solving option exercise games between bidding and target shareholders.

The model's predictions are generally consistent with the available empirical evidence. Importantly, the model also generates some new predictions. First, while increased product market competition among heterogeneous firms lowers cumulative returns in takeover deals, it does not speed up the acquisition process. Unlike other real options models, anticompetitive profits from a merger in our model, are larger for less competitive industries due to a higher price adjustment at the time of the restructuring. As a result, we arrive at the surprising conclusion that firms in less competitive industries optimally exercise their real options earlier. Second, we show that the likelihood of restructuring activities and the magnitude of cumulative returns to both the merging and rival firms are positively associated with industry concentration. In addition, the merger likelihood is greater in industries that are more exposed to demand shocks. These predictions are empirically testable and such an empirical study is left for future research.

One limitation of our analysis is that we assume exogenous mergers in the sense that the merger structure (who merges with whom and who remains independent) is exogenously imposed. It would be interesting to consider endogenous mergers when each firm makes individual merger decision and responds to mergers by other firms. In this case, multiple mergers may arise and the order of mergers are endogenous. This extension is nontrivial even in a static model (see, e.g., Qiu and Zhou (2007)). Incorporating endogenous mergers into a dynamic model would be a fruitful topic for future research.

Appendix

Proof of Proposition 1: In the Cournot–Nash industry equilibrium, each small firm’s objective is to

$$\max_{q_s(t)} \pi_s(t), \quad (\text{A.1})$$

while taking other small and large firms’ output strategies as given, where $\pi_s(t)$ is reported in equation (4). This maximization problem has the following first-order condition for each of the small firms:

$$aY(t) - bQ(t) = (b + 2k^{-1})q_s(t). \quad (\text{A.2})$$

Similarly, each large firms’ objective is to

$$\max_{q_l(t)} \pi_l(t), \quad (\text{A.3})$$

while taking other small and large firms’ output strategies as given, where $\pi_l(t)$ is reported in equation (5). This maximization problem has the following first-order condition for each of the large firms:

$$aY(t) - bQ(t) = (b + k^{-1})q_l(t). \quad (\text{A.4})$$

Using (1) and (6), we can solve the system of first-order conditions (A.2) and (A.4) to obtain the equilibrium expressions for $q_s^*(t)$ and $q_l^*(t)$ in equations (8) and (9) along with the formula for $\Delta(n)$ in equation (10). Aggregating individual firms’ output choices then yields the industry’s optimal output level $Q^*(t)$ in equation (11). Based on this result, the industry’s equilibrium price process $P(t)$ in equation (12) immediately follows from substituting $Q^*(t)$ into equation (2) and simplifying. ■

Proof of Proposition 2: Substituting the equilibrium output choice $q_s^*(t)$ and $q_l^*(t)$ in equations (8) and (9) back into the expression of instantaneous operating profits in equations (4) and (5) produces the closed-form solutions in equations (14) and (15). Using these equations to evaluate equation (7), we can derive the expression for equilibrium firm value $V_i(y; n)$ for $i = s, l$ that is reported in equation (13). ■

Proof of Proposition 3: We first find the critical value Δ^* in equation (17) by solving the equation

$$\Pi_l(n+1) - 2\Pi_s(n) = 0. \quad (\text{A.5})$$

Economically, equation (A.5) represents a break-even condition for the incentive to merge. To begin, notice that the functional form of $\Delta(n)$ in equation (10) has the following useful property:

$$\Delta(n) = \Delta(n+1) + b^2. \quad (\text{A.6})$$

Thus, using (14), and (15), we can write

$$\Pi_l(n+1) - 2\Pi_s(n) = \frac{a^2(b+2k^{-1})^2(b+k^{-1}/2)}{\Delta(n+1)^2} - \frac{2a^2(b+k^{-1})^3}{\Delta(n)^2} \quad (\text{A.7})$$

By inserting equations (A.6) and (A.7) into the break-even condition in equation (A.5), rearranging, and simplifying, we obtain:

$$\frac{2a^2(b+k^{-1})^3}{\Delta(n)^2} \left[\frac{A^2}{(1-b^2/\Delta(n))^2} - 1 \right] = 0, \quad (\text{A.8})$$

where the positive constant A is given in equation (18). By solving equation (A.8) for $\Delta(n)$, we can determine the critical value Δ^* in equation (17). Since one can easily verify that the term $\Pi_l(n+1) - 2\Pi_s(n)$ is increasing in the number of large firms, we can therefore derive the three conditions for the incentive to merge in equations (19), (20), and (21). ■

Proof of Lemma 1: It follows from equation (A.7) that $\Pi_l(n+1) - 2\Pi_s(n)$ increases with a . In addition, using (A.6), we know that it also decreases with $\Delta(n)$. Since $\Delta(n)$ decreases with n , $\Pi_l(n+1) - 2\Pi_s(n)$ increases with n . Using equations (14)-(15), we can show that

$$\begin{aligned} \frac{\Pi_l(n+1)}{\Pi_s(n)} &= \frac{(b+2k^{-1})^2(b+k^{-1}/2)}{(b+k^{-1})^3} \frac{\Delta(n)^2}{\Delta(n+1)^2} \\ &= \frac{(b+2k^{-1})^2(b+k^{-1}/2)}{(b+k^{-1})^3} \left(1 + \frac{b^2}{\Delta(n+1)} \right)^2. \end{aligned}$$

From this equation, we can see that $\Pi_l(n+1)/\Pi_s(n)$ increases with n . ■

Proof of Proposition 4: Using equation (27), we can derive

$$OM(y, y^*; n) = S(y^*; n) \mathbb{E}^y [e^{-r\tau_{y^*}}], \quad (\text{A.9})$$

for some undetermined threshold $y^* \geq y$. By Karatzas and Shreve (1999), we know that

$$\mathbb{E}^y [e^{-r\tau_{y^*}}] = \left(\frac{y}{y^*}\right)^\beta, \quad (\text{A.10})$$

where β is the positive root of the characteristic equation (29). Thus, we obtain equation (28) and a fortiori (33). We can use either equation (30) or (31) to determine the option value-maximizing merger threshold reported in equation (32). We can then rewrite the value of the merger option in equation (33) in terms of the optimal threshold y^* . Finally, the comparative statics results in part (iii) of the proposition follow from Propositions 2-3. ■

Proof of Proposition 5: We use similar arguments as in the proof of Proposition 4 to derive the merger option value given in equation (38). The requirement that friendly mergers take place at the socially optimal threshold from Proposition 4 allows us to use the merger threshold y^* in equation (32). The negotiated terms of the merger ξ_i^* in equation (39) then follow from solving the equilibrium condition in equation (37) where y^* is given in equation (32). ■

Proof of Proposition 6: Evaluating equations (41) and (43) at the merger threshold $Y(t) = y^*$ and using the results from Propositions 2 and 5, we find the cumulative merger returns in equations (44) and (45). Finally, part (iii) follows immediately from Lemma 1. ■

Proof of Proposition 7: Similar arguments as in the proof of Proposition 4 deliver the takeover option value given in (50). The acquiring firm i selects the takeover threshold \bar{y} such that

$$\frac{\partial OT_i(y, \bar{y}, \xi_i; n)}{\partial \bar{y}} = 0, \quad (\text{A.11})$$

which yields

$$\left\{ \left(\frac{2-\beta}{\beta \bar{y}} \right) [\xi_i \Pi_l(n+1) - \Pi_s(n)] \bar{y}^2 + [r - 2(\mu + \sigma^2/2)] X_i \right\} \left(\frac{y}{\bar{y}} \right)^\beta = 0. \quad (\text{A.12})$$

Clearly, the solution to equation (A.12) is a monotone and decreasing function of ξ_i , which we denote by $\bar{y}(\xi_i)$. By inverting this solution, we obtain the acquiring firm's ownership share $\xi_i(\bar{y})$ as a function of its value-maximizing takeover threshold \bar{y} , so that the target's maximization problem in equation (47) can be reformulated as follows:

$$\max_{\bar{y}} OT_j(y, \bar{y}, \xi_i(\bar{y}); n). \quad (\text{A.13})$$

After substituting $\xi_i(\bar{y})$ into the target's option value OT_j and using again the identify $\xi_j = 1 - \xi_i$, we need to solve the following first-order condition for the target

$$\frac{\partial OT_j(y, \bar{y}, \xi_i(\bar{y}); n)}{\partial \bar{y}} = 0, \tag{A.14}$$

which yields the solution for the value-maximizing takeover threshold in equation (49). Substituting the resulting expression for the takeover threshold $\bar{y}^*(n)$ into the solution for $\xi_i(\bar{y})$ from equation (A.12) generates the ownership share for the acquirer reported in equation (51). Finally, parts (iv) and (v) follow immediately from inspecting the analytic solutions and Lemma 1. ■

Proof of Proposition 8: Evaluating the expressions for cumulative merger returns to the acquirer, target, and rivals corresponding to equations (41) and (43) at the takeover threshold $Y(t) = \bar{y}^*$ and using the results from Propositions 2 and 7, we obtain the cumulative merger returns in equations (53), (54), and (55). Finally, parts (iii) and (iv) follow immediately from inspecting the analytic solutions and Lemma 1. ■

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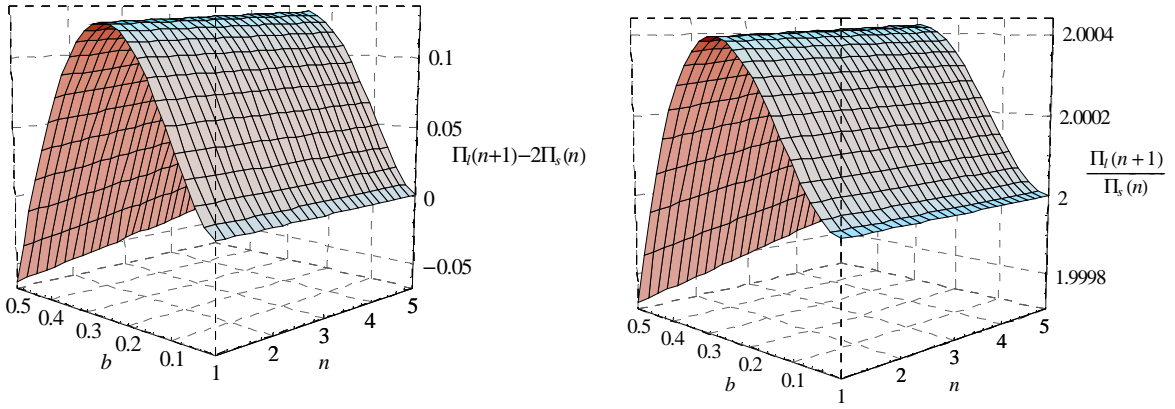


Figure 1. Profit Differential and Profit Ratio.

This figure plots the profit differential $\Pi_l(n+1) - 2\Pi_s(n)$ and the profit ratio $\Pi_l(n+1) / \Pi_s(n)$ as functions of the price sensitivity b and the number of large firms n when $k=0.2$ and $a=100$.