

The Coordination Role of Stress Test Disclosure in Bank Risk Taking

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Abstract

We examine whether stress-test disclosures distort banks' risk-taking decisions. We study a model in which a regulator may choose to rescue banks in the event of concurrent bank failures. Our analysis reveals a novel coordination role of stress-test disclosures. By disclosing stress tests, a regulator informs all banks of the failure likelihood of other banks, which facilitates bank's coordination in risk-taking. We find that disclosing stress tests always increases the rate of bank failure and, unless bank failure externalities are sufficiently severe, disclosure also increases banks' average risk and the bailout likelihood.

1 Introduction

One of the measures adopted by bank regulators in response to the financial crises of 2008 was to institute the public disclosure of stress tests to assess and publicly certify the stability and resilience of the largest banks. Stress tests are intended to expose the extent to which a bank is robust enough to endure a set of adverse macroeconomic scenarios and remain capable of performing

its lending operations. Advocates of public disclosure of stress tests argue that the disclosure of stress-test information enhances bank transparency and promotes financial stability, thereby avoiding costly bailouts. In the event of a systemic bank failure, regulators sometimes have to step in and bail out troubled financial institutions, and that entails substantial social costs. For instance, Bloomberg reported that by March 2009, adding up guarantees and lending limits, the Federal Reserve had committed \$7.77 trillion to rescue the financial system. It was claimed that the bailout of the financial system prevented a likely collapse of the economy into a major depression, and the magnitude of the externalities associated with a massive bank failure left little choice to regulators. However, such inexorability raises the issue of whether banks had anticipated the bailout and that anticipation created the problem in the first place. The prospect of a bailout upon the concurrent failure of multiple banks reduces the expected loss of failure for banks and, therefore, may induce banks to coordinate to make such concurrent failures more likely. However, such coordination may not be an easy feat unless there is an external mechanism that facilitates it. In this paper, we argue that the public disclosure of stress-test information can potentially constitute one such mechanism.

In this paper, we examine a setting with a continuum of banks that decide the risk of their loans. Banks face the surveillance of a bank regulator in the form of a stress test. The stress test reveals whether a bank is vulnerable to adverse macroeconomic shocks. We examine two scenarios: a no-disclosure scenario, in which the regulator receives the stress-test results from all banks but does not disclose them publicly; and a disclosure scenario, in which the regulator publicly discloses the stress-test information obtained from banks. Depending on the scenario, banks choose the risk of their loans. If a vulnerable bank obtains a bad loan outcome, it fails unless it receives an injection of capital from the regulator (a bailout). Finally, upon observing the loan outcomes of all banks, the regulator decides whether to inject capital to bail out failing banks to avoid the social costs of

concurrent bank failures.

Our analysis reveals a novel coordination role of stress-test disclosure and shows that disclosing stress tests may make risk decisions in the banking industry more extreme. Such a coordination role stems from the very nature of stress-test information and how this information may help banks assess the likelihood of a bailout more accurately. Specifically, the prospect of a bailout induces a strategic complementarity between banks' risk decisions. If a bank expects other banks to take more risk, it also anticipates an increase in the expected number of failing banks, and thus a higher likelihood of a bailout. This, in turn, makes taking risk much less costly for the bank, and therefore renders taking more risk the optimal choice. This strategic complementarity between risk decisions motivates banks to coordinate them. However, this coordination requires banks to conjecture each other's risk choices, and hence it is constrained by how much they know about each other. The disclosure of stress tests disseminates information among banks and reveals to each bank the extent to which other banks are likely to fail in the event of an adverse macroeconomic shock. Therefore, it helps banks assess the likelihood that the regulator may be compelled to bail out the banking system. The reduction in the uncertainty about the regulator's bailout decision makes it easier for banks to coordinate, and that may make their equilibrium risk choices more extreme than if stress tests had not been disclosed.

We take an ex-ante perspective and compare the expected equilibrium outcomes of a scenario in which stress tests are disclosed with those of a scenario in which stress tests are not disclosed. We focus on comparing three equilibrium variables: the average risk taken by banks, the likelihood of a bailout, and the rate of bank failure. In particular, we find that the disclosure of stress tests decreases the banks' average risk and reduces the likelihood of a bailout if the regulator is prone to bail out banks to begin with. This occurs in a scenario in which the bank regulator faces a steep social cost of bank failure, or the social cost of a bailout is low. However, if the regulator has weak

incentives to bail out banks (that is, if the social cost of bank failure is low while the social cost of a bailout is high), disclosing stress tests increases both the average risk and the probability of a bailout. The intuition behind this result can be grasped by gauging the relative impact of disclosing a bad stress-test outcome versus disclosing a good stress-test outcome on the coordination of banks' risk decisions. If the regulator is already prone to bailout banks, banks anticipate a high likelihood of a bailout, and therefore in the absence of any disclosure they coordinate into taking a high risk. Thus, disclosing stress tests can discipline banks from taking risk excessively in the event that the stress-test results reveal a low number of vulnerable banks. Consequently, stress-test disclosure reduces both banks' risk and the likelihood of a bailout on average. In the opposite situation, if the regulator is quite reticent to bail out banks, banks expect a low likelihood of a bailout and coordinate to take a low risk when there is no disclosure. However, disclosing stress-test results can produce an aggravating effect that facilitates banks' coordination into taking a high risk in the event that it indicates a high number of vulnerable banks. Therefore, stress-test disclosure induces banks to take a higher risk on average and increases the likelihood of a bailout.

We also compare the rate of bank failure in the disclosure and no-disclosure scenarios. While stress-test disclosure can either increase or decrease the average risk taken by banks, we find that it always increases the rate of bank failure. The key driving economic force is that the disclosure of stress-test information facilitates the coordination of banks into taking excessive risk precisely when the banking system is filled with vulnerable banks. This compounding effect largely contributes to the occurrence of bank failures, overwhelming other economic forces.

The organization of the paper is the following. In Section 2 we review the related literature. In Section 3 we explain the model setup. In Section 4 we analyze the model by first considering the no-disclosure scenario and then the disclosure scenario and we examine the equilibrium by applying the global games approach. In Section 5, we examine the economic consequences of stress-test

disclosure. Section 6 concludes the paper.

2 Literature Review

Our paper contributes to the stress-test disclosure literature. Goldstein and Sapra (2014) provide a thorough review of the benefits and costs of disclosing stress test in the extant literature. Goldstein and Sapra argue that stress-test disclosure may generate two benefits. First, more disclosure allows market participants to have better information about banks' operations, which improves the efficiency of their pricing decisions. The higher price efficiency in turn disciplines banks' behaviors. Second, disclosing stress-test results may hold bank regulators more accountable for their supervisory actions, thereby alleviating concerns that regulators may privately forbear banks that should not be allowed to continue. On the other hand, Goldstein and Sapra also highlight four potential costs. The first cost is due to the "Hirshleifer effect," that is, public disclosure may destroy risk-sharing opportunities among banks and impair the functioning of interbank markets (Hirshleifer, 1971; Goldstein and Leitner, 2013). Second, disclosure may lead to sub-optimal myopic decisions by banks through a "real-effect" channel of amplifying short-term price pressure (Gigler, Kanodia, Sapra and Venugopalan, 2014). Third, disclosure causes overweighting of the disclosed public information by banks' investors in economies with strategic complementarity, which in turn may trigger inefficient panic-based bank runs (Morris and Shin, 2002; Goldstein and Pauzner, 2005). Lastly, through a "feedback effect" channel, disclosure may interfere with regulators' learning from market prices for supervisory purposes, thus decreasing the efficiency of regulatory intervention (Bond and Goldstein, 2015). Our paper contributes to this literature by identifying a coordination role of stress-test disclosure that has not been previously studied.

More broadly, our paper is related to the literature on information disclosure in the banking

industry. Due to the size of this literature, as partly evidenced by the number of surveys, we refer readers to three recent surveys by Goldstein and Sapra (2014), Beatty and Liao (2014) and Acharya and Ryan (2015). This literature includes studies on potential consequences of many different disclosures in the banking industry. For instance, Plantin, Sapra and Shin (2008) argue that disclosure of assets' fair value leads to fire-sale, which causes downward spirals in assets prices and solvency problems for otherwise-sound banks. Prescott (2008) shows that disclosing bank supervisory information publicly may reduce banks' incentive to share the information with regulators in the first place. Corona, Nan and Zhang (2015) find that the disclosure of higher quality accounting information may encourage excessive risk-taking by banks through exacerbating competition in the deposit market.

In addition, our paper is related to a stream of papers that have also studied how the prospect of a bailout affects economic decisions. Arya and Glover (2006) examine a principal-multiagent moral hazard setting in which the principal will bail out the agents only when the outcome is very bad. This, however, encourages the agents to collude on the low-productivity choices ex ante. Acharya and Yorulmazer (2007) study a setting in which two banks may have incentive to increase the correlation between their investment returns, anticipating a potential bailout by a regulator. This is because, when the two banks' returns are highly correlated, they tend to fail concurrently, forcing the regulator to bailout the banks. On the contrary, when the correlation is low, the two banks are more likely to fail at different times, leaving the regulator with a better option of having the surviving bank to acquire the failed one. Therefore, in order to maximize the ex ante probability of a bailout, the two banks choose to maximize the correlation between their returns. Lastly, Farhi and Tirole (2011) examine banks' liquidity decisions and the regulator's interest rate policy. More specifically, when all banks choose to be more illiquid, they become more likely to be financially distressed, which forces the regulator to lower the interest rate with a higher probability. This

in return encourages banks to further reduce their liquidity. In contrast, our paper focuses on examining the coordination among banks' risk-taking decisions and the role played by stress-test disclosure in facilitating this coordination.

3 The Model Setup

We consider a banking industry with a continuum of banks, indexed by $i \in [0, 1]$. Each bank i is endowed with an investment project. The outcome of the project depends on the bank's risk choice $S_i \in [0, 1]$, the macroeconomic state $\omega \in \{G, B\}$, and the bank's type, $\theta_i \in \{H, L\}$. One can think of the macroeconomic state as the macroeconomic conditions in which banks operate, potentially described with a set of measures such as an unemployment rate, a GDP growth, etc. We assume that the macroeconomic state is uncertain, and that all parties share the common belief that the probability of a good macroeconomic state is q , $\Pr(\omega = G) = q \in [0, 1]$. The bank's type θ_i represents a unique characteristic of each bank i that reflects how well the bank can endure bad macroeconomic conditions. For instance, some banks may have large liquidity buffers that allow them to withstand a crisis, while other banks with smaller liquidity buffers may not survive. We also refer to θ_i as the bank i 's stress-test information, as each bank's stress-test result reflects its ability to survive a crisis. We call a bank with $\theta_i = H$ a "high-type" bank and one with $\theta_i = L$ a "low-type" bank. Each bank knows its own type, θ_i , but not the types of other banks. We further assume that the proportion of low-type banks is \tilde{p} , with $\tilde{p} \sim U[0, 1]$. That is, we assume that the proportion of low-type banks is unknown to all parties ex ante, but they all share a common prior about such proportion that is uniformly distributed.

In our model, each bank is required to communicate its stress-test information, θ_i , to the regulator. In reality, the Federal Reserve works together with each bank to obtain and verify the bank's stress-test information. In that light, we assume that banks report their stress-test information to

the regulator truthfully. In our analysis, we focus on characterizing the equilibria in two scenarios. In the first scenario, the regulator does not disclose stress-test information to the public. We call this scenario the “no-disclosure scenario.” In the second scenario, the regulator publicly discloses the stress-test information of all banks. We call this second scenario the “disclosure scenario.” We further assume that, if the regulator discloses the stress-test results, it does so truthfully.

We now explain our modelling of banks’ risk choices S_i . For simplicity we assume that, if the macroeconomic state is good (i.e., $\omega = G$), any bank i ’s project succeeds and generates a cash flow of S_i , regardless of the bank’s type. If the macroeconomic state is bad (i.e., $\omega = B$), the outcome of the project depends on the bank’s risk choice, S_i , and the bank’s type, θ_i . In particular, with probability $1 - S_i$, the project succeeds and generates a cash flow S_i . With probability S_i , the project fails and its outcome is contingent on the bank’s type: if $\theta_i = H$, the project generates a cash flow $K > 0$, which is the minimum capital that a bank needs to survive; if $\theta_i = L$, the project generates a zero cash flow, and that leads to a bank failure unless the bank obtains an external capital injection of K .¹ There are several ways to justify the assumption that a bank cannot survive without an external capital injection. For example, as discussed in Goldstein and Leitner (2015), a bank may have a debt liability, and if the bank is unable to repay its obligations, the bank fails. Alternatively, we could interpret K as the level of cash holdings in a bank below which the bank suffers a run.

Bank failures are costly to the economy. We assume that bank failures have negative externalities on the economy, and we capture the aggregation of such externalities by a function $C(n)$, where $n \in [0, 1]$ is the proportion of failing banks. We often refer to $C(\cdot)$ as the *social cost of bank failures*, and we assume $C(0) = 0$, $C(1) = \infty$, $C'(0) = 0$, $C'(n) > 0$, and $C''(n) > 0$ for all $n \in [0, 1]$. Notice that $C''(n) > 0$ implies an increasing marginal cost of bank failures. This

¹It is not necessary to make any assumption about the size of K because, as it will be clear later on, in equilibrium the expected return upon success is always larger than that upon a failure.

captures the general idea that bank failures are more costly the more banks fail concurrently. To avoid the social cost of bank failures, the regulator can make an injection K into each failing bank (henceforth, a “bailout”), but bailing out a failing bank entails another social cost of λK , with $\lambda > 0$, which we often refer to as the *social cost of a bailout*. The social cost of a bailout can arise, for instance, as a consequence of the distortions produced by the increase in taxation required to collect the bailout funds. Alternatively, a bailout can increase government debt to the extent of potentially placing the government itself under financial distress. Notice that the regulator does not consider the direct payment of K as a part of the social cost of bailouts because this injection is a pure transfer to the banks, which does not change the total welfare in the economy. For simplicity, we assume that the bailout policy cannot be targeted, in the sense that if the regulator decides to bail out banks, it bails out every failing bank. This assumption is descriptive of the set of bailout actions taken in the 2008-2009 crisis, such as the Term Auction Facility (TAF) which provides a liquidity backstop for all major banks, the Capital Purchase Program (CPP) with which the government injected capital into banks without solicitation, and the guarantee of short-term debt and transaction deposits of all insured banks by the Federal Deposit Insurance Corporation (FDIC). In our model, since banks have chosen their risk level before the regulator’s decision and banks’ cash flows are sunk, the regulator decides whether to bail out only to minimize the sum of the social cost of bank failures and the social cost of a bailout.

The time-line of the model is illustrated in Figure 1. At date 0, each bank communicates its stress-test information (that is, its type θ_i) to the regulator. The banks’ stress-test results, denoted by $\{\theta_i\}_{i \in [0,1]}$, are disclosed publicly only in the disclosure scenario. At date 1, each bank chooses its risk level S_i . At date 2, the macroeconomic state as well as the banks’ project outcomes are realized and publicly observed, and the regulator decides whether to bail out failing banks.

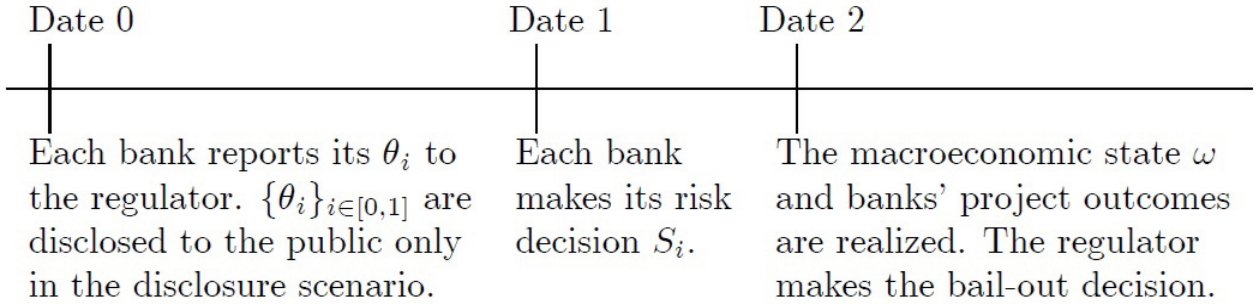


Figure 1: Time line.

4 The Equilibrium

We analyze the model by backward induction, starting from date 2. At date 2, upon the realization of the macroeconomic state as well as the project outcomes, the regulator decides whether to bail out failing banks by examining the trade off between the social cost of bank failures, $C(n)$, and the total social cost of bailouts, $n\lambda K$. The regulator only bails out banks if the social cost of bank failures is larger than the social bailout costs. Since the social cost of bank failures is convex in n , the regulator bails out the failing banks if and only if n is sufficiently large. We summarize this result in the lemma below.

Lemma 1 *Conditional on the proportion of failing banks n , the regulator bails out banks if and only if $n \geq \hat{n}$, where \hat{n} is given by*

$$C(\hat{n}) = \hat{n}\lambda K.$$

Lemma 1 suggests that the regulator's bail-out decision is determined by the trade off between the social cost of bailouts and the social cost of bank failures. It indicates that when the number of failing banks go beyond a threshold, the regulator must bail out as the social cost of bank failures outweighs the social cost of a bailout. In addition, the lower the bail-out cost parameter λ and/or

the more convex the bank-failure cost function $C(\cdot)$, the more likely that the regulator will bail out failing banks.

To avoid corner solutions in risk taking behavior we assume $\hat{n} < \frac{1}{2(1-q)} < 1 - \frac{K}{2}$. The first inequality ensures that the regulator bails banks out with a positive probability, and the second inequality ensures that banks' risk decisions are interior solutions. If these inequalities are not satisfied, the equilibrium may degenerate into either a scenario in which the regulator never bails banks out, or a scenario in which all banks take the maximum risk. In either case, the equilibrium is extreme and not interesting. Therefore, we exclude them from our analysis.

4.1 The Socially Optimal Risk Benchmark

Before characterizing the equilibrium risk decisions in the two scenarios, we first study the socially optimal risk decision that maximizes social welfare in the absence of private information. In this benchmark, the types of all banks are publicly observable, and thus the realized portion of low-type banks, denoted by p , is also observable. The social welfare is defined to be the expected aggregate cash flows generated by all banks net of both the social cost of bank failures and the social cost of bailouts. The social welfare can be formally expressed as follows:

$$\begin{aligned}
& (1-p) \{qS_H + (1-q) [(1-S_H)S_H + S_HK]\} \\
& +p [qS_L + (1-q)(1-S_L)S_L] \\
& - (1-q) [(1-I_{pS_L \geq \hat{n}}) C(pS_L) + I_{pS_L \geq \hat{n}} pS_L \lambda K],
\end{aligned} \tag{1}$$

where, S_H and S_L represent the high-type and low-type banks' risk choices respectively, and the indicator function in these terms, $I_{pS_L \geq \hat{n}}$, takes a value of 1 if $pS_L \geq \hat{n}$ and a value of 0 otherwise. The first line in expression (1) represents the expected cash flow generated by high-type banks

which constitute a $(1 - p)$ percentage of the banks. Since by assumption high-type banks never fail, there are neither bank failure costs nor bailout costs associated with these banks. The second and third lines in (1) represent the expected welfare impact of low-type banks which amount to a p percentage of banks. The second line represents the expected cash flow generated by low-type banks, and the third line represents the expected social cost of bank failures and the expected social cost of bailouts caused by low-type banks.

Denote the socially optimal risk choices for high-type and low-type banks as S_H^{FB} and S_L^{FB} respectively, by solving the first-order conditions from the social welfare expression in (1) we obtain,

$$S_H^{FB} = \frac{1}{2(1-q)} + \frac{K}{2}.$$

and

$$S_L^{FB} < \frac{1}{2(1-q)}.$$

We find that the low-type bank's socially optimal risk choice is always lower than $\frac{1}{2(1-q)}$. To understand this, notice that if there were no bank-failure cost nor bail-out cost, then the socially optimal risk level for the low-type bank would be $\frac{1}{2(1-q)}$. However, once we consider the social costs, because a higher risk level taken by low-type banks leads to more bank failures and thus greater social costs, the socially optimal risk S_L^{FB} that maximizes welfare must be lower than $\frac{1}{2(1-q)}$.

We formally state the socially optimal risk choices in the lemma below.

Lemma 2 $S_H^{FB} = \frac{1}{2(1-q)} + \frac{K}{2}$, $S_L^{FB} < \frac{1}{2(1-q)} < S_H^{FB}$.

Next, we examine banks' equilibrium risk decisions in the no-disclosure and disclosure scenarios separately. We analyze each scenario and then compare banks' risk decisions.

4.2 No-disclosure Scenario

We now consider the scenario in which the regulator does not disclose to the public the banks' stress-test information. Given the no-disclosure policy, at date 1, a bank i makes its risk decision, S_i , to maximize its expected payoff. If the bank is a high-type bank, its expected project payoff is

$$qS_i + (1 - q) [(1 - S_i) S_i + S_i K].$$

By solving the first-order condition, we obtain the high-type bank's equilibrium risk level, denoted by S_H^N , where N stands for no-disclosure. The expression for S_H^N is

$$S_H^N = \frac{1}{2(1 - q)} + \frac{K}{2}.$$

One can see that the high-type bank's equilibrium risk level is socially efficient as it equals to the socially optimal risk level (i.e., $S_H^N = S_H^{FB}$).² Moreover, since the high-type bank's risk decision is not affected by the possibility of a bailout, its risk decision is independent of other banks' decisions.

Next, we derive the equilibrium risk decision by a low-type bank, denoted by S_L^N . Given other banks' equilibrium risk choices $\{S_H^N, S_L^N\}$, the low-type bank chooses S_i to maximize its expected project payoff:

$$qS_i + (1 - q) [(1 - S_i) S_i + S_i \Pr(n \geq \hat{n} | S_H^N, S_L^N) K].$$

The payoff for the low-type bank is the same as that for the high-type bank except that, in a bad macroeconomic state, if the low-type bank's project is unsuccessful, the bank obtains a zero cash flow unless the regulator decides to bail out the bank by injecting a capital of K . We show that

²We verify that $S_H^N > K$ (see detailed analysis in the appendix). That is, a high-type bank earns a higher payoff when the project succeeds than when the project fails.

the probability of a bail out is

$$\Pr(n \geq \hat{n} | S_H^N, S_L^N) = 1 - \frac{\hat{n}}{S_L^N}.$$

With first-order condition derived from the above expression for the low-type bank's payoff, we obtain that the equilibrium risk level for the low-type bank satisfies,

$$S_L^N = \frac{1}{2(1-q)} + \frac{K \left(1 - \frac{\hat{n}}{S_L^N}\right)}{2} > \frac{1}{2(1-q)} > S_L^{FB}. \quad (2)$$

The implicit equation in (2) suggests that, because of the possibility of a bailout, low-type banks take a higher risk than the socially optimal level. Equation (2) also shows that the prospect of a bailout makes low-type banks' risk choices strategic complements to each other. If other low-type banks choose a higher risk S_L^N , the probability that these banks fail increases. A higher probability of concurrent bank failures leads to a higher bailout probability, and the higher bailout probability in turn encourages more risk-taking by every single low-type bank.

Solving equation (2) gives a unique solution for S_L^N , which we summarize in the proposition below.

Proposition 1 *In the no-disclosure scenario, there exists a unique equilibrium in which high-type banks choose*

$$S_H^N = \frac{1}{2(1-q)} + \frac{K}{2} = S_H^{FB},$$

low-type banks choose

$$S_L^N = \frac{K}{4} + \frac{1}{4(1-q)} + \frac{\sqrt{\left(\frac{1}{1-q} + K\right)^2 - 8K\hat{n}}}{4} > S_L^{FB},$$

and the regulator chooses to bail out banks if $n = pS_L^N > \hat{n}$, which happens with a probability $1 - \frac{\hat{n}}{S_L^N}$.

It is apparent that the low-type banks' equilibrium risk choice is strictly decreasing in \hat{n} . This is because for a lower \hat{n} , the expected bailout injection is larger, and that encourages low-type banks to take more risk. In other words, if it is likely that the regulator will bail out failing banks, low-type banks are induced to take higher risk.

4.3 Disclosure Scenario

We now consider the scenario in which the regulator discloses banks' stress-test information, $\{\theta_i\}_{i \in [0,1]}$, to the public. As a result, the realized proportion of low-type banks p becomes public knowledge, which helps banks to forecast the regulator's bailout decision.

The equilibrium risk choice of high-type banks in this disclosure scenario, denoted by S_H^D , remains the same as in the no-disclosure scenario. This is because high-type banks never receive capital injections from the regulator, and thus observing p or not does not affect their risk decisions. That is, high-type banks choose $S_H^D = S_H^N = \frac{1}{2(1-q)} + \frac{K}{2} = S_H^{FB}$.

However, the risk-taking incentive for low-type banks in the disclosure scenario, denoted by S_L^D , changes because of the public release of stress-test information. In particular, conditional on $\tilde{p} = p$ and other banks' choices $\{S_H^D, S_L^D\}$, a low-type bank's risk choice satisfies

$$S_L^D(p) = \frac{1}{2(1-q)} + \frac{K \Pr(n \geq \hat{n}; p, S_H^D, S_L^D)}{2} > S_L^{FB}.$$

Given $\tilde{p} = p$, the proportion of failing banks is $pS_L^D(p)$. For a specific low-type bank, if all other low-type banks choose a low risk level such that $pS_L^D(p) < \hat{n}$, then the regulator never bails out

(i.e., $\Pr(n \geq \hat{n} | p, S_H^D, S_L^D) = 0$), and thus this specific low-type bank chooses

$$S_L^{D1}(p) = \frac{1}{2(1-q)}.$$

However, if other low-type banks choose a high risk level such that $pS_L^D(p) \geq \hat{n}$, the regulator bails out banks with certainty (i.e., $\Pr(n \geq \hat{n} | p, S_H^D, S_L^D) = 1$). In this case, the specific low-type bank chooses a high risk level,

$$S_L^{D2}(p) = \frac{1}{2(1-q)} + \frac{K}{2}.$$

Similar to the no-disclosure scenario, the possibility of a bailout renders the low-type banks' risk choices strategic complements to each other. However, in contrast with the no-disclosure scenario, the stress-test disclosure conveys information about the proportion of low-type banks that may fail, and that helps banks to better forecast the regulator's bailout decision, facilitating the coordination among banks in taking risk. This improvement in coordination facilitated by the stress-test disclosure can in turn lead to multiple equilibria. Specifically, we may have (1) a *low-risk equilibrium* in which all low-type banks choose $S_L^{D1} = \frac{1}{2(1-q)} > S_L^{FB}$ and the regulator never bails out, and (2) a *high-risk equilibrium* in which all low-type banks choose $S_L^{D2} = \frac{1}{2(1-q)} + \frac{K}{2} > S_L^{D1} > S_L^{FB}$ and the regulator always bails out failing banks. In the following lemma, we characterize the parameter regions in which these equilibria exist.

Lemma 3 *In the disclosure scenario,*

if $p < \frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{2}}$, the low-risk equilibrium is the unique equilibrium;

if $p > \frac{\hat{n}}{\frac{1}{2(1-q)}}$, the high-risk equilibrium is the unique equilibrium;

if $p \in \left[\frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{2}}, \frac{\hat{n}}{\frac{1}{2(1-q)}} \right]$, the two equilibria coexist.

Lemma 3 highlights the coordination role of disclosing stress-test results. Recall that in the no-

disclosure scenario when banks' stress-test information is not disclosed, p is unknown. Therefore, it is difficult for banks to forecast the regulator's bailout decision, which is driven by the proportion of banks that fail and ultimately depends on the realized proportion of low-type banks p . The lack of information regarding \tilde{p} hinders effective coordination among banks in their risk choices, leading to a unique equilibrium in the no-disclosure scenario. In contrast, when the stress-test information is publicly disclosed, the uncertainty regarding \tilde{p} is fully resolved by effectively revealing its realization p . This information helps banks to better assess the regulator's bailout decision and, in turn, facilitates the coordination among banks' risk-taking decisions. As it becomes easier for banks to coordinate, multiple equilibria may arise. In sum, the disclosure of stress tests informs banks about each other's types, and thereby facilitates the coordination among banks' risk decisions, which ultimately fuels a self-fulfilling prophecy. To obtain comparative statics results and generate better regulatory insights, we resort to a global game approach to obtain unique equilibrium outcome.

The global game technique has been widely used in coordination games with multiple equilibria to obtain uniqueness. The equilibrium selection obtained by the global games approach has been supported by evidence in numerous experimental studies (Cabrales et al, 2004; Heinemann et al, 2004; Anctil et al, 2004; Anctil et al, 2010).³ To make use of this technique, we assume that upon the stress-test disclosure by the regulator, each bank observes a private noisy signal of the realized portion of low-type banks p , $\tilde{x}_i = p + \tilde{\varepsilon}_i$. The noise $\tilde{\varepsilon}_i$ is distributed in the interval $[-\eta, \eta]$, with a cumulative distribution function $F(\cdot)$ and a density function $f(\cdot)$. The noise terms, $\tilde{\varepsilon}_i$, are independent across banks. This heterogeneity in the information observed by different banks can be understood as, for instance, a difference in the interpretation of the regulator's disclosures

³The global games technique was first introduced by Carlsson and van Damme (1993) and later on popularized by Morris and Shin (1998). A majority of the extant global games literature focuses on models with binary actions. A notable exception is Guimaraes and Morris (2007) who examine a currency attack model with continuous actions. We apply the technique developed in Guimaraes and Morris (2007) to a banking setting in which banks' risk decisions are continuous, and obtain a unique equilibrium.

and/or a different random sampling of the stress tests to obtain an estimate of p . In applying the global games technique, we consider the limiting case in which η goes to zero, such that the noise $\tilde{\varepsilon}_i$ becomes negligible and p is (almost) perfectly observable by all banks. Concurring with the results in the global games literature, the introduction of conditionally independent private signals in the disclosure scenario breaks common knowledge and restores the uniqueness of the equilibrium. We summarize the result in the proposition below.

Proposition 2 *In the limit of $\eta \rightarrow 0$, there exists a unique equilibrium characterized by a threshold*

$$\hat{p} = \frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{4}}, \text{ such that,}$$

if $p < \hat{p}$, the low-risk equilibrium is the unique equilibrium;

if $p \geq \hat{p}$, the high-risk equilibrium is the unique equilibrium.

The information revealed by the stress-test disclosure has important effects on the low-type banks' risk choices and the regulator's bailout decision. If the stress-test result shows a small proportion of low-type banks ($p < \hat{p}$), disclosing the stress-test result produces a *disciplining effect*. Knowing that most banks are of high-type and will not fail, a low-type bank anticipates a small likelihood of a bailout, and thus prefers to take a lower risk. This, in turn, induces other low-type banks to take a lower risk since low-type banks' risk choices are strategic complements. As low-type banks are coordinated in taking low risk, the regulator is less likely to bail out, which further discourages low-type banks from taking risk. Through this downward spiral, the equilibrium converges to a stable point in which low-type banks choose low risk and the regulator never bails out, which is in turn justified by low-type banks' choices of low risk and the resulting low frequency of bank failure.

On the contrary, if the stress-test result indicates a large proportion of low-type banks ($p \geq \hat{p}$), disclosing the stress-test result has an *aggravating effect*. That is, given the looming risk of bank failure, low-type banks anticipate a high bailout likelihood and thus coordinate into taking high

risk, which then leads to an increase in bank failures and forces the regulator to bail out more often. Eventually, through such an upward spiral, the equilibrium converges to a stable point in which low-type banks take maximal risk and the regulator bails out failing banks with certainty.

5 Equilibrium Analysis

With the unique equilibrium in both the no-disclosure and the disclosure scenarios fully characterized, we now compare the two scenarios to analyze the economic consequences of stress-test disclosure. We will focus on three equilibrium variables most commonly mentioned in policy debates: banks' risk-taking decision, the probability of a bailout, and the frequency of bank failures. Since in reality the stress-test disclosure policy is set before the realization of the stress-test results, we look at all three variables from an ex-ante perspective.⁴

5.1 The Effect of Stress-Test Disclosure on Risk Taking

In this section, we compare banks' risk-taking decisions between the no-disclosure and the disclosure scenarios. Since high-type banks always choose the socially optimal risk level in both scenarios, we only need to compare the expected risk levels of low-type banks, $E[S_L^N]$ and $E[S_L^D]$. The results are summarized in the proposition below.

Proposition 3 *There exists a cutoff $\hat{n}^T = \frac{1}{4(1-q)} + \frac{K}{8}$, such that the expected risk in the disclosure scenario is lower than the risk in the no-disclosure scenario (i.e., $E[S_L^D] < E[S_L^N]$) if and only if $\hat{n} < \hat{n}^T$.*

Proposition 3 highlights a key result of this paper. It suggests that the disclosure of stress tests reduces the expected risk level if and only if the bailout threshold is sufficiently low ($\hat{n} < \hat{n}^T$),

⁴For instance, the Dodd-Frank act mandates the disclosure of banks' stress-test results, regardless of whether the results are good or not.

which happens when the social cost of bank failures is high relative to the social cost of a bailout. The intuition behind Proposition 3 can be illustrated with the trade-off between the disciplining effect of disclosing good stress-test results and the aggravating effect of disclosing bad stress-test results. To see how \hat{n} affects this trade-off, consider two extreme examples. First, consider the case of a very small threshold \hat{n} . That is, when the social cost of bank failures is tremendous compared with the social cost of a bailout. In this case the regulator bails out almost with certainty even without the disclosure of \tilde{p} . Anticipating the almost-certain likelihood of bailout, low-type banks coordinate to take the maximal risk (i.e., S_L^N approaches $\frac{1}{2(1-q)} + \frac{K}{2}$). In this case, disclosing the stress-test results helps to restrain risk taking. This is because if the stress-test results show that the proportion of low-type banks p is large, the aggravating effect of the disclosure is negligible as low-type banks choose the maximal amount of risk no matter whether the stress-test results are disclosed. However, if the disclosure shows that the proportion of low-type banks is small, low-type banks are greatly disciplined into taking low risk. Second, consider now the other extreme case in which \hat{n} is very large (i.e., \hat{n} is close to the upper bound $\frac{1}{2(1-q)}$). In this case the regulator bails out failing banks with a close-to-zero probability. With little hope for a bailout, low-type banks coordinate into taking the minimal risk with or without the disclosure of \tilde{p} . In this scenario, disclosing the stress-test results actually encourages low-type banks to take more risk. The reason is that if the stress-test results show a small proportion of low-type banks, the disciplining effect of disclosure does not make much of a difference because low-type banks are already choosing the minimal risk regardless of the disclosure. However, if the result shows a large proportion of low-type banks, the aggravating effect of disclosure is very strong as it coordinates low-type banks into taking the maximal risk.

To the extent that in reality the regulator cannot afford not to bail banks out (i.e., when the social cost of bank failures is high), our finding suggests that one way to mitigate banks' excessive

risk taking is to disclose the stress-test information to the public. This seems to be consistent with the observation in the recent crisis that the Dodd-Frank act mandates the disclosure of stress-test results of “systematically important” financial institutions. These institutions, once failed, would result in large social costs and force regulators to bail them out.

5.2 The Effect of Stress-Test Disclosure on Bail-out Probability

It may also be interesting to examine how stress-test disclosure affects the expected bail-out probability, i.e., $E[\Pr(n \geq \hat{n})]$. The comparison of the bailout probabilities in the two scenarios, in fact, is equivalent to the comparison of the expected risk levels taken by the low-type banks.⁵ Intuitively, when the low-type banks take more risk, the proportion of failing banks is higher (n becomes larger), and therefore the probability of a bailout is also higher. Therefore, based on our previous result in Proposition 3, we can conclude that the disclosure of stress tests reduces the expected bailout probability if and only if $\hat{n} < n^T$.

Proposition 4 *The expected bail-out probability in the disclosure scenario is lower than that in the no-disclosure scenario if and only if $\hat{n} < \hat{n}^T$.*

5.3 The Effect of Stress-Test Disclosure on Bank Failures

Since the disclosure of stress-tests may play a coordination role in banks’ risk taking and increase the likelihood of concurrent bank failures, we are also interested in how this disclosure affects the frequency of bank failures. The frequency of bank failures is characterized by the proportion of failing banks n , which is equal to pS_L . We summarize the ex-ante effect of disclosure on the expected proportion of failing banks, $E[\tilde{p}S_L(\tilde{p})]$, in the following proposition.

⁵Detailed analysis is in the appendix.

Proposition 5 *The expected proportion of failing banks in the disclosure scenario is always higher than that in the no-disclosure scenario, i.e., $E[\tilde{p}S_L^D(p)] > E[\tilde{p}S_L^N]$.*

Proposition 5 indicates that, although stress-test disclosure may help to restrain risk taking, it always leads to more bank failures. In the light of Proposition 3, this result may seem surprising. Formally, one can write the expected proportion of failing banks in the disclosure scenario as,

$$E[\tilde{p}S_L^D(\tilde{p})] = E[\tilde{p}]E[S_L^D(\tilde{p})] + Cov[\tilde{p}, S_L^D(\tilde{p})],$$

while in the no-disclosure scenario, $E[\tilde{p}S_L^N] = E[\tilde{p}]E[S_L^N]$. In these expressions, as stated in Proposition 3, the expected risk in the disclosure scenario $E[S_L^D(\tilde{p})]$ is smaller than the expected risk in the no-disclosure scenario $E[S_L^N]$ for $\hat{n} < \hat{n}^T$. However, the covariance term in the disclosure scenario $Cov[\tilde{p}, S_L^D(\tilde{p})]$ is always positive and large enough to dominate. To understand this, notice that low-type banks coordinate to take high risk if the stress-test disclosure shows a large proportion of low-type banks. In other words, low-type banks coordinate into taking excessive risk precisely when the stress-test result reveal that the banking system is filled with vulnerable banks (i.e., p is high). Thus, the covariance term reflects a compounding effect that contributes to increasing the occurrence of bank failure.

6 Conclusions

The policy of disclosing stress tests publicly was initially adopted by banking regulators as a reaction to the financial crisis of 2008, with the expectation that a transparent regulatory oversight to control banks' specific and systemic risk would discipline banks and reassure investors of the stability of the financial system. However, there is a consequence of stress-test disclosure that was usually neglected. That is, the receivers of the public disclosure of stress-test information

not only include investors in banking markets but also banks themselves, and the improvement of information sharing among banks facilitates banks' coordination on risk taking. We find that in a strong banking system, the disclosure of stress tests disciplines banks from taking excessive risk, while in a vulnerable banking system the disclosure aggravates banks' excessive risk-taking. We also examine the economic consequences of disclosing stress-test information from an ex ante perspective. We find that the stress-test disclosure may encourage banks to take higher risk and increase the likelihood of a regulatory bailout, especially when the social cost of bank failures is low while the social cost of a bailout is high. In addition, we also find that the disclosure of stress tests induces more bank failures. Our study may provide regulatory implications regarding the policies of stress tests as well as bailouts, and may help us better understand the interactions between disclosure and risk taking in banking industries.

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Appendix I: Proofs

Proof of Lemma 1

Proof. First, at $n = 1$, the difference between the social cost of bank failure and the social cost of a bailout, i.e., $C(n) - \lambda nK$, is

$$C(1) - \lambda K = \infty > 0,$$

since $C(1) = \infty$. Second, at $n = 0$, $C(n) - \lambda nK = C(0) = 0$. In addition,

$$\frac{\partial(C(n) - \lambda nK)}{\partial n} \Big|_{n=0} = C'(0) - \lambda K = -\lambda K < 0,$$

therefore, for $n = \varepsilon > 0$, where ε is a small positive number, $C(\varepsilon) - \lambda \varepsilon K < 0$. Therefore, by the intermediate value theorem, there exists a \hat{n} such that $C(\hat{n}) = \lambda \hat{n}K$.

We next prove that \hat{n} is unique. With some abuse of notation, denote the smallest root that solves $C(n) = \lambda nK$ as \hat{n} . Therefore, for $n < \hat{n}$, $C(n) - \lambda nK < 0$. It must be the case that at $n = \hat{n}$,

$$\frac{\partial(C(n) - \lambda nK)}{\partial n} \Big|_{n=\hat{n}} > 0,$$

since $C(1) - \lambda K > 0$ and $C(\varepsilon) - \lambda \varepsilon K < 0$. In addition, $C(n) - \lambda nK$ is strictly convex in n , because

$$\frac{\partial^2(C(n) - \lambda nK)}{\partial n^2} = C''(n) > 0.$$

Combined with $\frac{\partial(C(n) - \lambda nK)}{\partial n} \Big|_{n=0} < 0$ and by the intermediate value theorem, there exists a unique n' such that $C'(n') = \lambda K$. For $n < (>)n'$, $C'(n) < (>)\lambda K$. Recall that $\frac{\partial(C(n) - \lambda nK)}{\partial n} \Big|_{n=\hat{n}} > 0$ and thus $\hat{n} > n'$. Therefore, $C(n') - \lambda n'K < 0$. In the region $n \in [0, n']$, $C(n) - \lambda nK$ is strictly decreasing in n and $C(n) < \lambda nK$. That is, there is no root in $[0, n']$ that solves $C(n) = \lambda nK$.

In the region $n \in [n', 1]$, $C(n) - \lambda nK$ is strictly increasing in n . That is, there is a unique root in $[n', 1]$ that solves $C(n) = \lambda nK$, which is $n = \hat{n}$. Overall, the root that solves $C(n) = \lambda nK$ is unique. For $n < (>) \hat{n}$, $C(n) < (>) \lambda nK$. That is, the regulator bails out banks if and only if $n > \hat{n}$. ■

Proof of Lemma 2

Proof. The first-order condition of S_H^{FB} is

$$q + (1 - q)(1 - 2S_H^{FB} + K) = 0,$$

which gives

$$S_H^{FB} = \frac{1}{2(1-q)} + \frac{K}{2}.$$

To show that $S_L^{FB} < \frac{1}{2(1-q)}$, it suffices to verify that for $S_L > \frac{1}{2(1-q)}$, the social welfare is strictly decreasing in S_L . To see this, notice that in the welfare function, the low-type banks' expected cash flow is maximized at $S_L = \frac{1}{2(1-q)}$. Therefore, for $S_L > \frac{1}{2(1-q)}$, the low-type banks' expected cash flow is decreasing in S_L . In addition, for the term of social costs, when $\frac{1}{2(1-q)} \geq \frac{\hat{n}}{p}$, $S_L > \frac{1}{2(1-q)} \geq \frac{\hat{n}}{p}$, which gives $pS_L > \hat{n}$ and the social costs term becomes $-(1-q)pS_L\lambda K$ which is decreasing in S_L . When $\frac{1}{2(1-q)} < \frac{\hat{n}}{p}$, for $S_L \in (\frac{1}{2(1-q)}, \frac{\hat{n}}{p}]$, $pS_L \leq \hat{n}$ and the social costs term becomes $-(1-q)C(pS_L)$ which is decreasing in S_L . For $S_L > \frac{\hat{n}}{p}$, the social costs term becomes $-(1-q)pS_L\lambda K$ which is decreasing in S_L . Overall, the welfare is decreasing in S_L for $S_L > \frac{1}{2(1-q)}$. As a result, the socially optimal risk $S_L^{FB} < \frac{1}{2(1-q)}$. ■

Proof of Proposition 1

Proof. Given other banks' equilibrium risk choices $\{S_H^N, S_L^N\}$, a low-type bank chooses S_i to maximize its expected project payoff:

$$qS_i + (1 - q) [(1 - S_i) S_i + S_i \Pr(n \geq \hat{n} | S_H^N, S_L^N) K].$$

The equilibrium risk level for a low-type bank satisfies

$$S_L^N = \frac{1}{2(1-q)} + \frac{K \Pr(n \geq \hat{n} | S_H^N, S_L^N)}{2}. \quad (3)$$

Notice that $S_L^N < 1$ because $S_L^N < \frac{1}{2(1-q)} + \frac{K}{2} < 1$.

We now derive the probability of a bailout given other banks' equilibrium risk choices $\Pr(n \geq \hat{n} | S_H^N, S_L^N)$.

Since the portion of low-type banks is \tilde{p} and each low-type bank fails with a probability S_L^N , the proportion of banks that fail is given by $n = \tilde{p}S_L^N$. The bailout probability is then equal to

$$\Pr(n \geq \hat{n}) = \Pr(\tilde{p}S_L^N \geq \hat{n}) = \Pr\left(\tilde{p} \geq \frac{\hat{n}}{S_L^N}\right).$$

Notice that, since $S_L^N > \frac{1}{2(1-q)}$, the assumption that $\hat{n} < \frac{1}{2(1-q)}$ ensures that $S_L^N > \hat{n}$ and $1 > \frac{\hat{n}}{S_L^N}$, which, thereby, avoids the discussion of corner solutions. That is, the regulator always chooses to bail out with some strictly positive probability (bailouts always occur if \tilde{p} is close to 1). Given that $\tilde{p} \sim U[0, 1]$,

$$\Pr\left(\tilde{p} \geq \frac{\hat{n}}{S_L^N}\right) = 1 - \frac{\hat{n}}{S_L^N},$$

which leads to

$$S_L^N = \frac{1}{2(1-q)} + \frac{K \left(1 - \frac{\hat{n}}{S_L^N}\right)}{2}. \quad (4)$$

This first-order condition of S_L^N reduces into a quadratic equation of S_L^N and thus can have at most 2 roots. Recall that since we assume $\frac{\hat{n}}{2(1-q)} < 1$, $S_L^N > \hat{n}$. We now show that there exists a unique $S_L^N \in (\hat{n}, 1)$. First, at $S_L^N = \hat{n}$, the RHS of the equation is $\frac{1}{2(1-q)}$, which is larger than the LHS \hat{n} given our assumption $\frac{\hat{n}}{2(1-q)} < 1$. At $S_L^N = 1$, the RHS is $\frac{1}{2(1-q)} + \frac{K(1-\hat{n})}{2}$ and smaller than the LHS 1 given our assumption $\frac{1}{2(1-q)} + \frac{K(1-\hat{n})}{2} < \frac{1}{2(1-q)} + \frac{K}{2} < 1$. Therefore, by the intermediate value theorem, there exists a root in $(\hat{n}, 1)$ that solves the first-order condition. Moreover, there can only exist an odd number of solutions in $(\hat{n}, 1)$. Since the first-order condition is quadratic, there exists a single root, i.e., the equilibrium is unique and equal to

$$S_L^N = \frac{K}{4} + \frac{1 + \sqrt{[1 + K(1-q)]^2 - 8K\hat{n}(1-q)^2}}{4(1-q)}.$$

We now verify that $S_H^N > K$ and $S_L^N > K \Pr(n \geq \hat{n}; S_H^N, S_L^N)$. From the first-order condition of S_H^N , we have

$$q + (1-q)(1 - 2S_H^N + K) = q + (1-q)(1 - S_H^N + K - S_H^N).$$

In order to make the first-order condition zero, it must be the case that $S_H^N > K$. Otherwise, the first-order condition is always positive because $S_H^N < 1$. Similarly, we can verify that $S_L^N > K \Pr(n \geq \hat{n}; S_H^N, S_L^N)$. ■

Proof of Lemma 3

Proof. Notice that given the banks' risk choices $S_L^D(p)$, there can only be two equilibria, either $pS_L^{D2} \geq \hat{n}$ or $pS_L^{D1} < \hat{n}$. We first consider the equilibrium in which $pS_L^{D2} \geq \hat{n}$. In this equilibrium, the regulator bails out banks with certainty and thus $S_L^{D2} = \frac{1}{2(1-q)} + \frac{K}{2}$. To have $p \left(\frac{1}{2(1-q)} + \frac{K}{2} \right) \geq \hat{n}$, it must be the case that $p \geq \frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{2}}$. Second, we consider the other equilibrium $pS_L^{D1} < \hat{n}$. In

this equilibrium, the regulator never bails out banks and thus $S_L^{D1} = \frac{1}{2(1-q)}$. To have $p \frac{1}{2(1-q)} < \hat{n}$, it must be the case that $p \leq \frac{\hat{n}}{2(1-q)}$. To summarize, for $p < \frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{2}}$, the unique equilibrium is that $S_L^{D1} = \frac{1}{2(1-q)}$ and for $p > \frac{\hat{n}}{2(1-q)}$, the unique equilibrium is $S_L^{D2} = \frac{1}{2(1-q)} + \frac{K}{2}$. For $p \in \left[\frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{2}}, \frac{\hat{n}}{2(1-q)} \right]$, there are two equilibria, $S_L^{D1} = \frac{1}{2(1-q)}$ and $S_L^{D2} = \frac{1}{2(1-q)} + \frac{K}{2}$. ■

Proof of Proposition 2

Proof. The proof is similar to the proof in Guimaraes and Morris (2007). First, we denote by $H(p|x_i)$ the cumulative distribution over \tilde{p} for a bank that observes x_i and $H(p|x_i)$ is given by

$$H(p|x_i) = \frac{\int_0^p u(\tilde{p}) f(x_i - \tilde{p}) d\tilde{p}}{\int_0^1 u(\tilde{p}) f(x_i - \tilde{p}) d\tilde{p}} = \frac{\int_0^p f(x_i - \tilde{p}) d\tilde{p}}{\int_0^1 f(x_i - \tilde{p}) d\tilde{p}} = \frac{F(x_i) - F(x_i - p)}{F(x_i) - F(x_i - 1)},$$

where $u(\tilde{p}) = 1$ is the prior density function of \tilde{p} . In the limit of $\eta \rightarrow 0$, $H(p|x_i)$ becomes

$$\lim_{\eta \rightarrow 0} H(p|x_i) = \lim_{\eta \rightarrow 0} \frac{F(x_i) - F(x_i - p)}{F(x_i) - F(x_i - 1)} = 1 - F(x_i - p),$$

because as $\eta \rightarrow 0$, $x_i = p + \varepsilon > \eta$ which implies that $F(x_i) = 1$. Similarly, $x_i = p + \varepsilon < 1 - \eta$, which implies that $x_i - 1 < -\eta$ and thus $F(x_i - 1) = 0$.

Second, we show that there exists a threshold equilibrium in which the regulator bails out failed banks if and only if $p \geq \hat{p}$. To see this, notice that given the regulator bails out if $p \geq \hat{p}$, a bank believes that the regulator bails out with a probability $1 - H(\hat{p}|x_i) = F(x_i - \hat{p})$. As a result, the bank chooses a risk that is equal to

$$\begin{aligned} S^*(\varepsilon_i; p) &= \frac{1}{2(1-q)} + \frac{KF(x_i - \hat{p})}{2} \\ &= \frac{1}{2(1-q)} + \frac{KF(p + \varepsilon_i - \hat{p})}{2}. \end{aligned}$$

Given each bank chooses $S^*(\varepsilon_i; p)$, the portion of banks that fail is

$$n(S^*(\varepsilon_i; p); p) = \int_{-\eta}^{\eta} p S^*(\varepsilon_i; p) d\varepsilon_i = \int_{-\eta}^{\eta} \left[\frac{1}{2(1-q)} + \frac{KF(p + \varepsilon_i - \hat{p})}{2} \right] d\varepsilon_i,$$

where since $F(\cdot)$ is strictly increasing in p , $n(S^*(\varepsilon_i; p); p)$ is strictly increasing in p . Therefore, there exists a unique threshold \hat{p} that makes $n(S^*(\varepsilon_i; \hat{p}); \hat{p}) = \hat{n}$. For $p > (<) \hat{p}$, $n(S^*(\varepsilon_i; p); p) > (<) \hat{n}$ and the regulator bails out (does not bail out). Therefore, the threshold equilibrium is indeed an equilibrium.

Lastly, we derive the threshold \hat{p} . Recall that the probability of $p \geq \hat{p}$ is $1 - H(\hat{p}|x_i) = F(x_i - \hat{p})$.

Thus any bank observing a signal

$$\xi(\hat{p}, l) = \hat{p} + F^{-1}(l),$$

attaches probability l to $p \geq \hat{p}$. Moreover, since $F(x_i - \hat{p})$ increases in x_i , any bank observing a signal less than $\xi(\hat{p}, l)$ attaches a probability less than l to $p \geq \hat{p}$. Thus if the true state is \hat{p} , the proportion of banks assigning probability l or less to $p \geq \hat{p}$ is

$$\begin{aligned} \Gamma(l|\hat{p}) &= \Pr(x_i \leq \xi(\hat{p}, l)) \\ &= \Pr(x_i \leq \hat{p} + F^{-1}(l)) \\ &= \Pr(\hat{p} + \varepsilon_i \leq \hat{p} + F^{-1}(l)) \\ &= \Pr(\varepsilon_i \leq F^{-1}(l)) \\ &= F(F^{-1}(l)) \\ &= l. \end{aligned}$$

If a bank believes that $p \geq \hat{p}$ with a probability l , the bank chooses a risk level:

$$S^*(l) = \frac{1}{2(1-q)} + \frac{Kl}{2}.$$

At $p = \hat{p}$, the proportion of banks that failed is

$$\begin{aligned} n &= \int_0^1 S^*(l) \hat{p} d\Gamma(l|\hat{p}) \\ &= \int_0^1 S^*(l) \hat{p} dl. \end{aligned}$$

Since at $p = \hat{p}$, the regulator is indifferent between bail out and not to bail out, we must have

$$\int_0^1 S^*(l) \hat{p} dl = \hat{n},$$

which gives

$$\hat{p} = \frac{\hat{n}}{\int_0^1 S^*(l) dl} = \frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{4}}.$$

Therefore, there exists a threshold equilibrium such that the regulator bails out failed banks if and only if $p \geq \frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{4}}$. Moreover, this equilibrium is also the unique equilibrium, following a general result from Frankel, Morris and Pauzner (2003) that show in games with strategic complementarity, arbitrary numbers of players and actions, and slightly noisy signals, the equilibrium is unique as the noise goes to zero. ■

Proof of Proposition 3

Proof. Recall that in the no-disclosure scenario, S_L^N is given by

$$S_L^N = \frac{K}{4} + \frac{1}{4(1-q)} + \frac{\sqrt{\left(\frac{1}{1-q} + K\right)^2 - 8K\hat{n}}}{4}.$$

Since S_L^N is independent of p , the expected risk $E[S_L^N] = S_L^N$.

The expected risk in the disclosure scenario is,

$$E[S_L^D] = \frac{1}{2(1-q)}\hat{p} + \left(\frac{1}{2(1-q)} + \frac{K}{2}\right)(1-\hat{p}),$$

where the ex ante probability that the stress-test result is good is $\Pr(p < \hat{p}) = \hat{p} = \frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{4}}$ and the probability that the stress-test result is bad is $1 - \hat{p}$.

Therefore, plugging in the expressions for $E[S_L^D]$ and $E[S_L^N]$, one can reduce $E[S_L^D] > S_L^N$ into

$$\frac{K}{4} + \frac{1}{4(1-q)} + \frac{\sqrt{\left(\frac{1}{1-q} + K\right)^2 - 8K\hat{n}}}{4} < \frac{1}{2(1-q)} + \frac{K}{4}.$$

Notice that the LHS is strictly decreasing in \hat{n} . Moreover, recall that we assume $\hat{n} \in \left[0, \frac{1}{2(1-q)}\right]$.

At $\hat{n} = 0$, the LHS becomes $\frac{K}{4} + \frac{1}{2(1-q)} > \frac{1}{2(1-q)} + \frac{K}{4}$, the RHS. In addition, at $\hat{n} = \frac{1}{2(1-q)}$, the LHS becomes $\frac{1}{2(1-q)} < \frac{1}{2(1-q)} + \frac{K}{4}$, the RHS. Therefore, there exists a unique cutoff \hat{n}^T such that $E[S_L^D] > S_L^N$ if and only if $\hat{n} > \hat{n}^T$. Solving $\frac{K}{4} + \frac{1}{4(1-q)} + \frac{\sqrt{\left(\frac{1}{1-q} + K\right)^2 - 8K\hat{n}}}{4} = \frac{1}{2(1-q)} + \frac{K}{4}$ gives $\hat{n}^T = \frac{1}{4(1-q)} + \frac{K}{8}$. ■

Proof of Proposition 4

Proof. From the first-order condition (2), the low-type banks' equilibrium risk-taking decisions S_L are linearly increasing in the probability of bailout $\Pr(n \geq \hat{n})$. Therefore, a comparison of

$E[\Pr(n \geq \hat{n})]$ is equivalent to the comparison of the expected risk level taken by the low-type banks, $E[S_L]$. From Proposition 3, we thereby conclude that the stress-test disclosure reduces the expected bail-out probability if and only if the regulator has a strong incentive to bail out failing banks (i.e., the bailout threshold $\hat{n} < n^T$). ■

Proof of Proposition 5

Proof. From an ex ante perspective, in the disclosure scenario, the expected portion of bank failure is $E[\tilde{p}S_L^D(\tilde{p})]$. From Proposition 2, $S_L^D = S_L^{D1} = \frac{1}{2(1-q)}$ if $\tilde{p} < \hat{p}$ and $S_L^D = S_L^{D2} = \frac{1}{2(1-q)} + \frac{K}{2}$ if $\tilde{p} \geq \hat{p}$. Thus

$$E[\tilde{p}S_L^D(\tilde{p})] = \int_0^{\hat{p}} \tilde{p}S_L^{D1} d\tilde{p} + \int_{\hat{p}}^1 \tilde{p}S_L^{D2} d\tilde{p} = \frac{\hat{p}^2}{2}S_L^{D1} + \frac{1-\hat{p}^2}{2}S_L^{D2}.$$

In the no-disclosure scenario, the expected portion of bank failure is

$$E[\tilde{p}S_L^N] = E[\tilde{p}]E[S_L^N] = \frac{S_L^N}{2}.$$

The first equality is because S_L^N is independent of \tilde{p} .

The difference $E[\tilde{p}S_L^D(\tilde{p})] - E[\tilde{p}S_L^N]$ is given by

$$\begin{aligned} \frac{\hat{p}^2}{2}S_L^{D1} + \frac{1-\hat{p}^2}{2}S_L^{D2} - \frac{S_L^N}{2} &= \frac{\hat{p}^2}{2}(S_L^{D1} - S_L^{D2}) + \frac{S_L^{D2} - S_L^N}{2} \\ &= -\frac{\hat{p}^2}{2}\frac{K}{2} + \frac{1}{2}\frac{K}{2}\frac{\hat{n}}{S_L^N} \\ &= \frac{K}{4}\left(\frac{\hat{n}}{S_L^N} - \hat{p}^2\right). \end{aligned}$$

The second equality is by plugging in the expressions of $S_L^{D1} = \frac{1}{2(1-q)}$, $S_L^{D2} = \frac{1}{2(1-q)} + \frac{K}{2}$ and $S_L^N = \frac{1}{2(1-q)} + \frac{K\left(1-\frac{\hat{n}}{S_L^N}\right)}{2}$. Thus $E[\tilde{p}S_L^D(\tilde{p})] - E[\tilde{p}S_L^N] > 0$ if and only if $S_L^N < \frac{\hat{n}}{\hat{p}^2}$. From the proof of Proposition 1, S_L^N is the unique fixed point that solves $x = \frac{1}{2(1-q)} + \frac{K\left(1-\frac{\hat{n}}{x}\right)}{2}$. In addition, for

any $x > S_L^N$, $\frac{1}{2(1-q)} + \frac{K(1-\frac{\hat{n}}{x})}{2} > x$. Thus $S_L^N < \frac{\hat{n}}{\hat{p}^2}$ if and only if at $x = \frac{\hat{n}}{\hat{p}^2}$,

$$\frac{1}{2(1-q)} + \frac{K\left(1 - \frac{\hat{n}}{\hat{p}^2}\right)}{2} > \frac{\hat{n}}{\hat{p}^2}. \quad (5)$$

From $\hat{p} = \frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{4}}$, $\hat{n} = \left(\frac{1}{2(1-q)} + \frac{K}{4}\right)\hat{p}$. Plugging this expression of \hat{n} into (5) gives

$$-\frac{K}{2}\hat{p}^3 + \left(\frac{1}{2(1-q)} + \frac{K}{2}\right)\hat{p} - \frac{1}{2(1-q)} - \frac{K}{4} < 0.$$

In addition, since $\hat{n} < \frac{1}{2(1-q)}$, $\hat{p} = \frac{\hat{n}}{\frac{1}{2(1-q)} + \frac{K}{4}} < \frac{\frac{1}{2(1-q)}}{\frac{1}{2(1-q)} + \frac{K}{4}}$.

Denote $f(\hat{p}) = -\frac{K}{2}\hat{p}^3 + \left(\frac{1}{2(1-q)} + \frac{K}{2}\right)\hat{p} - \frac{1}{2(1-q)} - \frac{K}{4}$. To show that $f(\hat{p}) < 0$ for all $\hat{p} \in \left(0, \frac{\frac{1}{2(1-q)}}{\frac{1}{2(1-q)} + \frac{K}{4}}\right)$, we first show that $f(\hat{p})$ is strictly increasing in $\left(0, \frac{\frac{1}{2(1-q)}}{\frac{1}{2(1-q)} + \frac{K}{4}}\right)$, i.e.,

$$\begin{aligned} f'(\hat{p}) &= -\frac{3K}{2}\hat{p}^2 + \frac{1}{2(1-q)} + \frac{K}{2} \\ &> \frac{1}{2(1-q)} + \frac{K}{2} - \frac{3K}{2} \left(\frac{\frac{1}{2(1-q)}}{\frac{1}{2(1-q)} + \frac{K}{4}}\right)^2 \\ &= \frac{\left(\frac{1}{2(1-q)} + \frac{K}{2}\right) \left(\frac{1}{2(1-q)} + \frac{K}{4}\right)^2 - \frac{3K}{2} \left(\frac{1}{2(1-q)}\right)^2}{\left(\frac{1}{2(1-q)} + \frac{K}{4}\right)^2} \\ &= \frac{\frac{1}{2(1-q)} \left[\left(\frac{1}{2(1-q)} + \frac{K}{4}\right)^2 - \frac{K}{2(1-q)}\right] + \frac{K}{2} \left[\left(\frac{1}{2(1-q)} + \frac{K}{4}\right)^2 - \left(\frac{1}{2(1-q)}\right)^2\right]}{\left(\frac{1}{2(1-q)} + \frac{K}{4}\right)^2} \\ &= \frac{\frac{1}{2(1-q)} \left[\frac{1}{2(1-q)} - \frac{K}{4}\right]^2 + \frac{K^2}{8} \left[\frac{K}{4} + \frac{1}{(1-q)}\right]}{\left(\frac{1}{2(1-q)} + \frac{K}{4}\right)^2} \\ &> 0. \end{aligned}$$

Thus $f(\hat{p}) < 0$ if and only if at $\hat{p} = \frac{\frac{1}{2(1-q)}}{\frac{1}{2(1-q)} + \frac{K}{4}}$, $f(\hat{p}) < 0$, i.e.,

$$\begin{aligned}
& -\frac{K}{2} \left(\frac{\frac{1}{2(1-q)}}{\frac{1}{2(1-q)} + \frac{K}{4}} \right)^3 + \left(\frac{1}{2(1-q)} + \frac{K}{2} \right) \frac{\frac{1}{2(1-q)}}{\frac{1}{2(1-q)} + \frac{K}{4}} - \frac{1}{2(1-q)} - \frac{K}{4} \\
= & \frac{\left(\frac{1}{2(1-q)} + \frac{K}{2} \right) \left(\frac{1}{2(1-q)} + \frac{K}{4} \right)^2 \frac{1}{2(1-q)} - \frac{K}{2} \left(\frac{1}{2(1-q)} \right)^3 - \left(\frac{1}{2(1-q)} + \frac{K}{4} \right)^4}{\left[\frac{1}{2(1-q)} + \frac{K}{4} \right]^3} \\
= & \frac{\left(\frac{1}{2(1-q)} + \frac{K}{4} \right)^2 \left[\left(\frac{1}{2(1-q)} + \frac{K}{4} \right)^2 - \left(\frac{K}{4} \right)^2 \right] - \frac{K}{2} \left(\frac{1}{2(1-q)} \right)^3 - \left(\frac{1}{2(1-q)} + \frac{K}{4} \right)^4}{\left[\frac{1}{2(1-q)} + \frac{K}{4} \right]^3} \\
= & -\frac{\left(\frac{1}{2(1-q)} + \frac{K}{4} \right)^2 \left(\frac{K}{4} \right)^2 + \frac{K}{2} \left(\frac{1}{2(1-q)} \right)^3}{\left[\frac{1}{2(1-q)} + \frac{K}{4} \right]^3} \\
< & 0.
\end{aligned}$$

Therefore, we have verified that $f(\hat{p}) < 0$ for all $\hat{p} \in \left(0, \frac{\frac{1}{2(1-q)}}{\frac{1}{2(1-q)} + \frac{K}{4}} \right)$. As a result, $E[\tilde{p}S_L^D(\tilde{p})] > E[\tilde{p}S_L^N]$. ■