Investment-Specific Technological Change and Asset Prices

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Abstract

I show that investment-specific technological change is a source of systematic risk. Positive shocks to investment technology lead to a reallocation of resources from consumption to investment. I use returns on stocks that produce investment goods minus returns on stocks that produce consumption goods as a proxy for the investment shock. The value of assets in place minus growth opportunities falls after positive shocks to investment technology, which suggests an explanation for the value puzzle. I formalize these insights in a dynamic general equilibrium model and find that the model's implications are supported by the data.

1 Introduction

The second half of the twentieth century saw remarkable technological innovations, the majority of which took place in equipment and software. However, technological innovations affect output only to the extent that they are implemented through the formation of new capital stock. Since firms need to invest if they are to benefit from advances in technology, these innovations do not necessarily benefit all firms equally. I refer to these types of innovations as an investment-specific technology shock, thus differentiating it from the technological shock that affects all capital, as in the standard neoclassical model. In this paper,

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I argue that investment-specific technological change is a source of systematic risk that is responsible for some of the cross-sectional variation in risk premia, both between different sectors in the economy and between value and growth firms.

I present a dynamic general equilibrium model that links investment-specific technological change to asset prices. In contrast to the standard one-sector model, shocks to investment technology (I-shocks) do not directly affect the production of the consumption good. Instead, they alter the real investment opportunity set in the economy by lowering the cost of new capital goods. Since the old capital stock is unaffected, the economy must invest to realize the benefits. As the economy trades off current against future consumption, there is a reallocation of resources from the production of consumption goods to investment goods. Under standard preference specifications (power utility), the marginal value of a dollar will be high in these states of the world, because the economy is willing to give up consumption today. Therefore, stocks are more expensive if they pay off in states when real investment opportunities are good. The types of firms that are likely to do well in these states are firms that produce capital goods and firms with a lot of growth opportunities. In the more general case in which the elasticity of intertemporal substitution (EIS) differs from the reciprocal of the coefficient of relative risk aversion, I find that the premium is negative as long at the EIS is sufficiently low.

My model provides new empirical implications about the cross-section of stock returns. Since investment-specific technological change is not directly observable, direct empirical tests are difficult to implement. However, one of the advantages of my model is that, by using the cross-section of stock prices, it provides restrictions that help identify investment-specific technological change. The model features two sectors of production, one that produces the consumption good and the other that produces the investment good. In this context, investment-specific technological change benefits firms that produce capital goods relative to firms that produce consumption goods. As a result, a portfolio of stocks producing investment goods minus stocks producing consumption goods (IMC) spans the investment shock. I construct an empirical equivalent of this portfolio and use it as a proxy for investment technology shocks.

The first implication of the model is that the investment-specific shock may carry a negative premium. If this is true, firms that are positively correlated with the investment-specific shock should have lower average returns. I show that sorting individual firms into portfolios on covariances with the IMC portfolio leads to portfolios that are mispriced by the CAPM or the Consumption CAPM (CCAPM). On the other hand, a two-factor model

that includes the IMC portfolio successfully prices the spread. I repeat the procedure using the entire cross-section of stock returns, following Fama and French (1992). Overall, I find that the estimates of the risk premium are negative, statistically significant and similar in magnitude, regardless of the test assets used.

The second implication is that the value of assets in place relative to the value of growth opportunities has a negative correlation with the I-shock. This negative correlation is important, because it offers a novel explanation for the value puzzle: a positive I-shock lowers the cost of new investment, which causes the value of future growth opportunities to increase and the value of assets in place to fall. As a result, growth stocks have lower expected returns because they do well in states in which real investment opportunities are good and the marginal value of wealth is high. I find that including the IMC portfolio in the (C)CAPM dramatically improves the ability of the model to price the cross-section of stocks sorted by book to market.

The model identifies the IMC portfolio as a proxy for investment-specific technological change. To examine this restriction more carefully, I first show that positive returns on the IMC portfolio are followed by an increase in the quantity of investment. Next, I consider the model's predictions about consumption and leisure. In the model, consumption falls in the short run but increases in the long run, and leisure temporarily falls. I find that both leisure and the discretionary component of consumption falls following positive returns on the IMC portfolio.

As an additional robustness check, I use the observed investment-output ratio to further examine the implications of my model. Using the calibrated solution of my model, I invert the investment-output ratio to back out the normalized investment shock implied by the data. I find that the stochastic discount factor (SDF) implied by the model that uses the extracted shock is consistent with my earlier conclusions. Growth stocks and investment firms have higher correlation with the implied SDF than value stocks and consumption firms, implying lower returns.

The paper is organized as follows. In Section 2 I discuss related studies. In Section 3 I present a general equilibrium model with investment-specific shocks, and in Section 4 I calibrate the model. In Section 5 I present the empirical results. Section 6 concludes. The Appendix contains all technical details.

2 Related studies

Jermann (1998) and Tallarini (2000) are early examples of work that explores the assetpricing implications of general equilibrium models. These studies build on the neoclassical RBC model and focus on aggregate quantities and prices. In this environment these authors find that the equity premium and risk-free rate puzzles are exacerbated, because high risk aversion implies endogenously smooth consumption. They extend the production economy model to allow for cross-sectional heterogeneity in firms, with the explicit purpose of linking firm characteristics, such as book to market and size, to expected returns.

Zhang (2005) builds an industry equilibrium model and Gomes, Kogan, and Zhang (2003), Gourio (2005), and Gala (2006) build general equilibrium models in which investment frictions and idiosyncratic shocks result in ex-post heterogeneity in firms.

My work is most closely related to Gomes, Kogan, and Yogo (2007) who focus on ex-ante firm heterogeneity, i.e., heterogeneity that arises because of differences in the type of firm output rather than differences in productivity or accumulated capital. These authors build a general equilibrium model in which differences in the durability of a firm's output lead to differences in expected returns. They focus on differences between final goods producers; in contrast, I focus on differences between capital-good and final-good producers. My paper is also close to theirs in terms of the empirical methods used, since they also use the Input-Output tables from the Bureau of Economic Analysis (BEA) to classify firms based on the type of output they produce.

However, the general equilibrium models I mention above have a single aggregate shock and therefore imply a one-factor structure for the cross-section of stock returns. As a result, any difference in expected returns must be due to differences in market betas, since the conditional CAPM holds exactly. Even though true betas are unobservable, Lewellen and Nagel (2006) argue that they could not covary enough with the market premium to justify the observed premia. In addition, models with one shock cannot generate the pattern documented by Fama and French (1993), in which a portfolio of value minus growth firms (HML) is a factor in the time-series of returns in addition to the market portfolio. My model enriches the production technology of a standard general equilibrium asset pricing model by differentiating between types of technological shocks.

Although the production technology in my model is different from the models above, it has been used extensively in the macroeconomic literature. Investment-specific shocks were first considered by Solow (1960) in his growth model with vintage capital. Uzawa (1961) and Rebelo (1991) use a two-sector model to study endogenous growth. Greenwood, Hercowitz, and Krussell (1997, 2000) document a negative correlation between the price of new equipment and new investment. They interpret this finding as evidence for investmentspecific technological change and show that it can explain both the long-run behavior of output and its short-run fluctuations. They calibrate a Real Business Cycle (RBC) model with investment-specific technological change, using the price of new equipment as a proxy for the realizations of the investment-specific shock. They show that this shock can account for a large fraction of both short- and long-run output variability, in magnitudes of 30% to 60%. Fisher (2006) treats the investment shock as unobservable and uses a similar model to derive long-run identifying restrictions on a Vector Auto Regression (VAR). In Fisher's model, the identified investment technology shock explains up to 62% of output fluctuations over the business cycle. Justiniano, Primiceri, and Tambalotti (2008) estimate a medium scale Dynamic Stochastic General Equilibrium (DSGE) model and find that the investment shock accounts for a large fraction of the business-cycle fluctuations in output and hours worked.

One of the few papers that explores the asset pricing implications of investment shocks is Boldrin, Christiano, and Fisher (2001). They consider a model similar to mine with two production sectors, habit preferences and investment-specific shocks. They calibrate their model to match the equity premium, but their main focus is on improving on the quantity dynamics. Jovanovic (2007) has no shocks to investment technology, but features "seeds" that can subsequently be converted into trees, and thus delivers similar implications about aggregate quantities. In contrast to these papers, my interest is on deriving implications about the cross-section of stock returns. The closest example is Panageas and Yu (2006) who build a general equilibrium model with different vintages of capital, but they focus on the comovement between asset returns and consumption over the long-run.

A recent body of work explores the effects of technological innovation on on asset prices. Most of the literature, for instance, Greenwood and Jovanocic (1999), Hobijn and Jovanovic (2001), DeMarzo, Kaniel and Kremer (2006), Pastor and Veronesi (2006), has not focused on the implications of technological innovation for the cross-section of asset prices.

My paper is also related to recent work that links the properties of firm cash flows to expected returns, for example Campbell and Vuolteenaho (2004), Lettau and Wachter (2006), Santos and Veronesi (2006b), and Lustig and Van Nieuwerburgh (2006). These models are based on the observation that growth firms' cash flows of have higher duration than do the cash flows produced by value firms, and thus may be more sensitive to changes in the financial investment opportunity set, in the spirit of Merton's ICAPM (1973). On the other hand,

Bansal, Dittmar, and Lundblad (2006), Bansal, Dittmar, and Kiku (2007), Bansal, Kiku, and Yaron (2007) explore the fact that growth and value firms' cash flows have differential properties to explain the value premium. Because all these models are based on an exchange economy, changes in the financial investment opportunity set are either exogenously specified or arise through preferences. Finally, Gourio (2006) and Novy-Marx (2008) argue that the value firms face higher operating leverage and are thus riskier in bad times when the price of risk is high. My work complements the papers above by considering time-varying *real* investment opportunities in a model with production.

My paper is also related to the large number of studies that explore the ability of the consumption-based model to explain the cross-section of expected returns. These studies focus on measurement issues (Ait-Sahalia, Parker, and Yogo, 2004; and Jagannathan and Wang, 2005); long horizons (Bansal, Dittmar, and Lundblatt, 2005; Parker and Julliard, 2005; and Malloy, Moskowitz, and Vissing-Jørgensen, 2006); conditional versions of the CCAPM (Lettau and Ludvigson, 2001; and Santos and Veronesi, 2006a); multiple good economies (Lustig and Van Nieuwerburgh, 2005; Pakos, 2004; Piazzesi, Schneider, and Tuzel, 2006; Yogo, 2006; Uhlig, 2007; Van Binsbergen, 2007) and pure production-based models (Cochrane, 1991, 1996; Li, Vassalou, and Xing, 2006; Belo, 2006; and Liu, Whited and Zhang, 2007).

Finally, my paper is related to Makarov and Papanikolaou (2007), who identify the latent factors that affect stock returns based on heteroscedasticity. One of the factors they recover is highly correlated with the investment minus consumption portfolio and the value minus growth (HML) factor of Fama and French (1993). I provide a general equilibrium model that links these two facts and that suggests an additional proxy for investment-specific shocks.

3 General equilibrium model

I build a general equilibrium model to formalize the idea that investment-specific shocks create a reallocation of resources between the consumption and the investment sector. The two-sector specification I consider is adapted from the model of Rebelo (1991) who studies endogenous growth.

3.1 Information

The information structure obeys standard technical assumptions. Specifically, there exists a complete $(\Omega, \mathcal{F}, \mathcal{P})$ probability space supporting the Brownian motion $Z_t = (Z_t^X, Z_t^Y)$. \mathcal{P} is the corresponding Wiener measure, and \mathcal{F} is a right-continuous increasing filtration generated by Z.

3.2 Firms and technology

Production in the economy takes place in two separate sectors, one producing the consumption good (numeraire) and one producing the investment good.

3.2.1 Consumption sector

The consumption goods sector (C-sector) produces the consumption good, C, with two factors of production, sector specific capital K_C and labor L_C

$$C_t \le X_t K_{C,t}^{\beta_C} L_{C,t}^{1-\beta_C},\tag{1}$$

where β_C is the elasticity of output with respect to capital. Output in the C-sector is subject to a disembodied productivity shock X that evolves according to

$$dX_t = \mu_X X_t dt + \sigma_X X_t dZ_t^X.$$
⁽²⁾

The X shock (C-shock) increases the productivity of *all* capital in the consumption sector. This is the standard shock in existing one-sector general equilibrium models. The capital allocated to the C-sector depreciates at a rate δ , while the investment in consumption-specific capital is denoted by I_C . Thus, capital in the final goods sector evolves according to

$$dK_{C,t} = I_{C,t} dt - \delta K_{C,t} dt.$$
(3)

Investment in the C-sector is subject to adjustment costs. If the firm has capital K_C and wants to increase its capital by I_C , it consumes $c(\frac{I_C}{K_C})K_C$ total units of the investment good, where $c(\cdot)$ is an increasing and convex function. The value of a representative firm in the consumption sector equals

$$S_{C,t} = E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(X_s K_{C,s}^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} - \xi_s c \left(\frac{I_{C,s}}{K_{C,s}} \right) K_{C,s} \right) \, ds, \tag{4}$$

where w is the relative price of labor and ξ is the relative price of the investment good, or equivalently the cost of new capital.

3.2.2 Investment sector

The investment goods sector (I-sector) produces the investment good using sector specific capital K_I and labor L_I . I simplify the model by assuming that the capital stock in the investment sector is fixed.¹ One can therefore think of the investment sector as using land and labor to produce the investment good. The output of the I-sector can be used to increase the capital stock in the C-sector

$$c\left(\frac{I_{C,t}}{K_{C,t}}\right)K_{C,t} \le Y_t K_I^{\beta_I} L_{I,t}^{1-\beta_I}.$$
(5)

The shock Y, which represents the investment shock, affects the productivity of the investment sector. A positive shock to Y increases the productivity of the investment sector, which implies that the economy can produce the same amount of new investment using fewer resources (L_I) . Therefore, a positive investment shock will imply a fall in the cost of producing new capital, whereas the old capital stock will be unaffected. The elasticity of output with respect to labor in the investment sector equals $1 - \beta_I$. The investment shock follows

$$dY_t = \mu_Y Y_t \, dt + \sigma_Y Y_t dZ_t^Y. \tag{6}$$

Firms in the investment sector represent claims on the land (K_I) used to produce investment goods. The value of a representative firm in the investment sector equals

$$S_{I,t} = E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(\xi_s Y_s K_I^{\beta_I} L_{I,s}^{1-\beta_I} - w_s L_{I,s} \right) \, ds, \tag{7}$$

where w is the wage and ξ is the relative price of the investment good in terms of the consumption good.

Finally, if one defines the investment rate, $i_C \equiv \frac{I_C}{K_C}$, the adjustment cost function takes the form

$$c(i_C) = (c_1 + i_C)^{\lambda} - c_1^{\lambda}$$
 (8)

and c_1 is chosen so that c'(0) = 1.

¹Endogenizing capital accumulation adds an additional state variable and complicates the numerical solution considerably, but does not add any new insights. The reason is that allowing K_I to vary over time does not directly affect risk premia just the risk-free rate.

3.3 Households

There exists a continuum of identical households with recursive utility preferences. Households maximize a utility index J, that is defined recursively by;

$$J_0 = E_0 \int_0^\infty h(C_t, N_t, J_t) dt.$$
 (9)

where C_t is consumption and N_t is leisure that the household enjoys in period t. Following Duffie and Epstein (1992), the aggregator is defined as:

$$h(C, N, J) = \frac{\rho}{1 - \theta^{-1}} \left(\frac{(CN^{\psi})^{1 - \theta^{-1}}}{((1 - \gamma)J)^{\frac{\gamma - \theta^{-1}}{1 - \gamma}}} - (1 - \gamma)J \right)$$
(10)

Here ρ will play the role of the time-preference parameter, γ controls risk aversion, and θ the elasticity of intertemporal substitution (EIS). Utility is defined over the composite good $C N^{\psi}$, and ψ controls the relative shares of consumption and leisure. These preferences are exactly as in Angeletos and Panousi (2007), while in the time-additive case belong to the class studied by King, Plosser and Rebelo (1988).

Households supply $1 - N_t$ units of labor that can be freely allocated between the two sectors,

$$L_{C,t} + L_{I,t} = 1 - N_t. (11)$$

Shifts in the allocation of labor between the two sectors allow the economy to intratemporally trade off consumption versus investment. An alternative interpretation of L is as a perishable good which can be used as input in either of the two sectors or consumed directly, for example oil.

Households trade a complete set of state contingent securities in the financial markets. Finally, the parameters in the model are assumed to satisfy

$$u \equiv \rho \frac{1 - \gamma}{1 - \theta^{-1}} - (1 - \gamma)(\mu_X - \delta \beta_C) + \frac{1}{2}\sigma_X^2 \gamma(1 - \gamma) > 0$$
(12)

and

$$\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 > 0 \tag{13}$$

The first restriction ensures that the value function for the social planner is bounded, whereas the second ensures that the state variables have a stationary distribution.

3.4 Competitive equilibrium

Definition 1. A competitive equilibrium is defined as a collection of stochastic processes $C^*, N^*, K_C^*, L_I^*, L_C^*, I_C^*, \pi^*, \xi^*, w^*$ such that (i) households chose C^* to maximize (9) given w^* and π^* (ii) firms choose I_C^*, L_I^* and L_C^* , given π^*, w^* and ξ^* , to maximize equations (4) and (7) (iii) K_C^* satisfies equation (3) given I_C^* (iv) all markets clear according to (1),(5) and (11)

Here, I focus on the social planner's problem, i.e., the problem of optimal allocation of labor. I demonstrate in the Appendix that, as in other models with dynamically complete financial markets and no externalities, a competitive equilibrium can be constructed from the solution to the social planner's problem.

Proposition 1. The social planner's value function is

$$J(X, Y, K_C) = \frac{(X K_C^{\beta_C})^{1-\gamma}}{1-\gamma} f(\omega), \qquad (14)$$

where $\omega \equiv \ln\left(\frac{Y}{K_C}\right)$ and $f(\omega)$ satisfies the ODE

$$0 = \left\{ \rho \frac{1-\gamma}{1-\theta^{-1}} f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} (L_C^* {}^{1-\beta_C} (1-L_C^* - L_I^*)^{\psi})^{1-\theta^{-1}} + c^{-1} \left(e^{\omega} K_I^{\beta_I} L_I^* {}^{1-\beta_I} \right) (\beta_C (1-\gamma) f(\omega) - f'(\omega)) + -u f(\omega) + (\mu_Y + \delta\beta_C) f'(\omega) + \frac{1}{2} \sigma_Y^2 (f''(\omega) - f'(\omega)) \right\}.$$
(15)

The allocation of labor between the two sectors is given by $L_C^* = \frac{(1-\beta_C)(1-l(\omega))}{1+\psi-\beta_C}$ and $L_{I,t}^* = l(\omega)$ where

$$l(\omega) = \arg\min_{l} \left(\rho \frac{1-\gamma}{1-\theta^{-1}} f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \left(\left(\frac{(1-\beta_{C})(1-l)}{1+\psi-\beta_{C}} \right)^{1-\beta_{C}} \left(1 - \frac{(1-\beta_{C})(1-l)}{1+\psi-\beta_{C}} - l \right)^{\psi} \right)^{1-\theta^{-1}} + c^{-1} \left(e^{\omega} K_{I}^{\beta_{I}} l^{1-\beta_{I}} \right) \left(\beta_{C} (1-\gamma) f(\omega) - f'(\omega) \right) \right)$$
(16)

The state variable, ω , has dynamics

$$d\omega_t = \left(\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - i_C(\omega)\right) dt + \sigma_Y dZ_t^Y.$$
(17)

where

$$i_C(\omega) = c^{-1} \left(e^{\omega} K_I^{\beta_I} l(\omega)^{1-\beta_I} \right)$$
(18)

Proof See Appendix.

I solve for equilibrium policies numerically, and show the details of the solution in the Appendix.

In equilibrium, there is only one state variable that determines optimal policy, ω , and it can be interpreted as the ratio of "effective" capital stocks in the two sectors. Most importantly, innovations to ω come only from the I-shock (Y). Finally, as long as $\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 > 0$, the state variable has a unique stationary distribution. This guarantees that in equilibrium one sector does not dominate the economy.

The main mechanism that determines the price of risk for the I-shock is the allocation of labor between the sectors, $l(\omega)$. The behavior of $l(\omega)$ can be understood from the first order conditions of the planner's problem. To obtain some intuition, I consider the case in which investors have time-separable preferences ($\gamma = \theta^{-1}$), and the labor supply is fixed ($\psi = 1$).

$$\pi_t = e^{-\rho t} U_C, \tag{19}$$

$$\xi_t = \frac{J_{K_C}}{U_C} \frac{1}{c'(i_{C,t}^*)} = X_t K_{C,t}^{\beta_C - 1} \frac{\beta_C (1 - \gamma)f - f'}{(1 - \gamma)(1 - l(\omega))^{\gamma(\beta_C - 1)}} \frac{1}{c'(ae^{\omega_t} l(\omega_t)^{1 - \beta_I})}, \quad (20)$$

$$\xi_t Y_t \frac{\alpha K_I^{\beta_I}}{X_t K_{C,t}^{\beta_C}} = \frac{l(\omega_t)^{1-\beta_I}}{(1-l(\omega_t))^{1-\beta_C}} \frac{1-\beta_C}{1-\beta_I}.$$
(21)

Here, π is the shadow cost of the resource constraint in the C-sector, (1), and $\xi\pi$ is the shadow cost of the resource constraint in the I-sector, (5). This implies that π is the state price density and that ξ is the relative price of the investment good in terms of the consumption good.

Equation (19) is standard and states that in equilibrium, the marginal valuations in each state equal the shadow cost of the resource constraint in the C-sector, which is the state price density.

Equation (20) states that the relative price of output in the I-sector (ξ) equals the marginal value of capital in the C-sector divided by the marginal installation cost and marginal utility. In the one-sector model without adjustment costs, the relative price of the investment good is always one and the marginal utility equals the marginal value of capital. In my model, ξ is a function of the I-shock, because a positive investment shock increases the supply of the investment good and therefore lowers its relative price. In addition, ξ depends on the C-shock (X), because a positive shock to productivity in the consumption sector increases the demand for the investment good and therefore its relative price. This is

why, in equilibrium, the C-shock affects both sectors symmetrically.

Equation (21) states that in equilibrium, the marginal product of labor in both sectors must be equal. This condition determines $l(\omega)$. The effect of the investment-specific shock on the allocation of labor depends on how Y affects $\xi(Y, \cdot)Y$. As long as $\xi(Y, \cdot)Y$ is increasing in Y, a positive shock to Y will increase the profits of firms in the investment sector as well as the marginal product of labor in the I-sector. Therefore, the allocation of labor to the I-sector, $l(\omega)$, must temporarily increase, inducing a fall in consumption. In the future, the capital stock in the consumption sector increases, reversing the short-run fall in consumption. The end result is that consumption displays a U-shaped response to a positive investment shock. Consumption initially falls because the economy allocates more resources to the Isector in order to take advantage of the improvement in technology. Eventually, the new technology starts bearing fruit and the growth rate of consumption increases.

In general, investors evaluate states based on three factors: their consumption in that state (C_t) , their leisure (N_t) and their continuation utility (J_t) . The decision how much to work and how to allocate between the two sectors affects consumption and leisure contemporaneously and their continuation utility through investment. the pricing kernel or stochastic discount factor (SDF) takes the form:

$$\frac{d\pi_t}{\pi_t} = -r_{f,t} dt - b_Y(\omega_t) dZ_t^Y - \gamma \,\sigma_X \, dZ_t^X$$
(22)

where

$$b_Y(\omega_t) = -\left((\theta^{-1}(1 - \beta_C) + \psi(\theta^{-1} - 1)) \frac{l'(\omega_t)}{1 - l(\omega_t)} + \frac{f'(\omega_t)}{f(\omega_t)} \frac{\gamma - \theta^{-1}}{\gamma - 1} \right) \sigma_Y$$
(23)

The function $b_Y(\omega)$ reflects the price of risk for the investment-specific shock. When solving the model, I find that $l(\omega)$ is an increasing function of ω . This is important because it means that both consumption and leisure temporarily *fall* after a positive I-shock, which tends to increase investors' valuation of that state. However, the labor allocation decision also affects investor's continuation utility through investment, so continuation utility will be higher after a positive I-shock. Given that $f'(\omega)/f(\omega) < 0$, if investors have preference for later resolution of uncertainty, i.e. $(\gamma \theta < 1)$, this will increase their valuation of that state, whereas if they have preferences for early resolution of uncertainty $(\gamma \theta > 1)$, this will tend to lower state prices.

4 Computation and Calibration

In this section I present the numerical solution of the model. I provide the solution details in the Appendix.

4.1 Parameters

I calibrate the model using the parameters in Table 1. The value of $\theta = 0.35$ consistent with Vissing-Jørgensen (2002) who estimates an EIS of 0.3 to 0.4 for stockholders using micro-level data. The value for the EIS is substantially higher than the one reported by Van Binsbergen, Fernandez-Villaverde, Koijen and Rubio-Ramirez (2008), who estimate a DSGE model via Maximum Likelihood and find an EIS of 0.06. Finally, my choice for the EIS falls within the estimates reported by Hall (1988), and lies between the values used in Campanale, Castro and Clementi (2007) and Bansal, Kiku and Yaron (2007).

The value $\psi = 3$ is in line with King, Plosser, and Rebelo (1988), Christiano and Eichenbaum (1992) and Angeletos and Panousi (2007) and ensures that the steady-state fraction of available time worked approximately matches the US data. I pick a conservative value for $\gamma = 2$, and the values for β_I and β_C imply that the labor share of output is 70%.

4.2 Consumption, Output, Investment and Hours

I report the unconditional moments generated by the model versus their empirical counterparts in Table 2. I use annual data for consumption, output investment and hours and compare them with data generated by my model. I simulate the model at a monthly frequency and aggregate the data to form annual observations. I simulate 10,000 paths, each with a length of 50 years. In the model, consumption expenditures is defined as the output of the consumption sector, since consumption is the numeraire good. Investment expenditures equals the relative price of the investment good times the output of the investment sector. Output is the sum of consumption and investment expenditures and hours is the fraction of time worked.

My model does overall a good job matching the mean and standard deviation of consumption, investment hours and output growth, with the empirical estimates falling inside the 90% confidence intervals generated by the model. The model also matches the autocorrelation of hours and investment growth, but fails to match the autocorrelation of consumption and output. Consumption and output in the model are highly autocorrelated, substantially more so than actual data, mainly due to the presence of adjustment costs to capital. I also examine whether the model matches the correlation of innovations in consumption, hours, investment and output. I compute innovations by first estimating a VAR(1) and then report the correlation of the residuals, both for the data and the model.

Overall, I find that the model does a reasonable job matching the patterns of comovement in the data, especially the correlation between innovations to consumption and investment growth (0.47 vs 0.52). The model misses a bit on the correlation between innovations in labor supply and output and investment growth (0.63 and 0.97 vs 0.85 and 0.86 respectively).

In addition, I decompose the variance of innovations in consumption, hours, output and investment growth in Table 3. Consumption is mostly driven by the C-shock (X), whereas investment expenditures, output and hours are driven mostly by the I-shock (Y). These results are in line with Justiniano, Primiceri and Tambalotti (2008).

In order to gain some intuition about the results, I plot certain key variables of the model as a function of the state variable ω in Figures 1(a)-(f). On the one hand, both consumption and leisure are declining functions of ω . States where the productivity of the investment sector is high relative to the capital stock (high ω states), are high marginal valuation states, as shown in Figure 1(f). The price of new capital goods is a declining function of ω , which increases Tobin's Q.

In Figure 2, I plot the dynamic responses to an investment-specific shock, with the timeperiod being 1 year. All quantities are computed relative to the steady state benchmark $(\omega = \overline{\omega})$ as responses to a one-standard deviation shock in Y. In Figure 2(a), consumption falls in the short run, as more resources are allocated to the investment sectors, but increases permanently in the long run relative to the old steady-state. Output, as shown in Figure 2(b) displays a similar response. Figures 2(c)-(d) show that labor supply and investment initially increase and then fall back to their initial levels. Figure 2(e) shows that the price of capital goods falls after the shock and then slowly increases. Tobin's Q in Figure 2(f) increases following the investment shock and then falls back to its equilibrium level. The I-shock is therefore responsible for the positive comovement of investment expenditures, output and hours, whereas it tends to create negative comovement between investment and consumption expenditures. Figure 2(g) shows that the investors' marginal valuation (state price density) increases following the shock and then falls to a level below the old steady state, because the economy features more capital relative to the old steady state and therefore more consumption.

In Figure 3, I plot the dynamic responses of the same variables to a shock in the consumption sector (X). The shock in the consumption sector is a random walk so it leads to permanent increases in consumption, output and the relative price of the investment good, but does not affect labor supply or the allocation of labor across sectors. Therefore, the C-shock (X) is responsible for the positive correlation in consumption and investment expenditures and output. Finally, note that the increase in consumption leads to lower state prices, so the shock in the consumption sector has a positive premium.

4.3 Asset prices

4.3.1 Investment and consumption firms

There are two representative firms in my model, one that produces the consumption good and one that produces the investment good. The market value of each firm is

$$S_t^C = E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(X_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - w_s L_{C,s}^* - \xi_s c \left(\frac{I_{C,s}^*}{K_{C,s}^*} \right) K_{C,s}^* \right) \, ds, \qquad (24)$$

$$S_t^I = E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(\xi_s \alpha Y_s K_I^{\beta_I} (L_{I,s}^*)^{1-\beta_I} \right) \, ds.$$

$$\tag{25}$$

The value of each sector includes all cash flows that accrue to the owners of the capital stock.

Proposition 2. The ratio of the value of the investment goods sector, S_t^I , over the consumption goods sector, S_t^C , equals

$$\frac{S_t^I}{S_t^C} = \frac{\beta_I f'(\omega)}{\beta_C (1-\gamma) f(\omega) - f'(\omega)},\tag{26}$$

and is an increasing function of ω . **Proof** See Appendix.

A positive I-shock increases the productivity of the investment sector and there increases its value relative to the consumption sector. In Figure 1(h) I plot the relative value of the investment sector as a function of ω , and in Figures 2(k) and 3(k), I plot the response of the relative valuation of the two sectors following a one-standard-deviation shock to the investment (Y) and consumption (X) shocks respectively. Consistent with the proposition above, a positive shock to investment technology increases the value of investment firms relative to consumption firms, whereas the shock to the consumption sector has no effect.

Proposition 2 is important because it establishes that the relative value of the two sectors is a state variable in the economy, since it is a monotone transformation of ω . The same is not

true for the relative price of the investment good, ξ , which also depends on the consumption shock (X), as shown in equation (20) and Figure 3(e). Therefore, the cross-section of stock prices may contain additional information about real investment opportunities relative to the prices of investment goods or Tobin's Q. I can use this information to identify investmentspecific technological change in the data, since a portfolio of investment minus consumption stocks (IMC) correlates positively with the I-shock. Hence, I can use returns on the IMC portfolio to test the asset pricing implications of the model, namely that the pricing kernel loads on the I-shock with a negative premium. A corollary is that the IMC portfolio should have negative expected returns after adjusting for market risk. The model implies that investment firms have returns that are on average 1.9% lower than consumption firms, which is close to the number in the data (1.6%), as shown in Table 4.

Proposition 2 will be the main motivation for the majority of the empirical analysis in this paper. Proposition 2 relies on two assumptions: first the fact that technology shocks are permanent, and second that utility is homothetic and there are no demand or preference shocks. To illustrate the first point, consider what happens when X is a pure i.i.d. process. In this case, a positive C-shock will not translate in an increase in investment demand since it is a purely transitory shock to wealth and might lead to higher valuations of C-firms relative to I-firms. However, many authors, including Fisher (2006) and Gali (1999) point out that permanent shocks are the natural way to model purely technological disturbances. In addition, Alvarez and Jermann (2005) find that a permanent component to consumption is necessary to justify the spread between equity and long-horizon bonds. Regarding the second assumption, if households have preferences that display decreasing relative risk aversion, a positive C-shock will act as a demand shock, by altering households' willingness to substitute over time and might therefore affect the relative valuations of the I and C-sector. If that is the case, the IMC portfolio should predict and increase in both the price and quantity of investment. I will examine this implication in Section 5.3.

4.3.2 Market portfolio and risk-free asset

The sum of the market values of the two sectors equals the market portfolio or the value of the entire dividend stream,

$$S_{t}^{M} = S_{t}^{C} + S_{t}^{I} = E_{t} \int_{t}^{\infty} \frac{\pi_{s}}{\pi_{t}} (C_{s} - w_{s}) ds$$

= $\frac{(X_{t} K_{C,t}^{\beta_{C}})^{1-\gamma}}{1-\gamma} \left(\beta_{C} (1-\gamma) f(\omega) + (\beta_{I} - 1) f'(\omega)\right).$ (27)

Innovations in the C-shock, (dZ^X) , and the I-shock, (dZ^Y) drive the returns on the market portfolio. Table 3 shows that most of the variability of returns on the market portfolio is due to the I-shock.

When solving the model, I find that the value of the market portfolio is positively correlated with the C-shock and negatively correlated with the I-shock. The latter can be seen in Figure 1(g). When computing the response of the value of the market portfolio to the investment-specific shock, I find that it falls in the short run but it increases in the long run. A positive I-shock increases future dividends but increases discount rates. If agents have preferences such that $\theta < 1$, the discount rate effect dominates and leads to a fall in the price of the dividend stream. Given that the I-shock has a negative price of risk, this helps increase the equity premium. As shown in Figure 2(j), in the long-run, the investment shock leads to higher levels of capital accumulation and therefore to a higher value of the market portfolio. The shock in the consumption sector, permanently raises the value of the market portfolio, as shown in Figure 3(j).

The investment shock's dynamic effects on the value of the stock market can be also be understood as follows. Suppose that I interpret a positive investment shock as the availability of a new type of capital that has the same productivity as the old capital but is cheaper. This will lower the valuation of existing or old capital, which becomes obsolete, and will therefore lower the value of the stock market. However, in the long run, the economy will accumulate more of the new type of capital so the valuation of the stock market will rise.

The model generates a fairly respectable equity premium of 4.1% while matching the volatility of the market portfolio. Table 3 shows that most of the variability of the market portfolio comes from the I-shock. Part of the reason why this is the case is that both expected dividend growth and the risk-free rate are driven by the I-shock.

The risk-free rate is equal to minus the drift of the pricing kernel implied by the model, as shown in Equation 22. Table 4 also shows the moments of the risk-free rate implied by the model. It matches the level of the risk-free rate, however, similar to models where the EIS is low, the risk-free rate is very volatile, roughly twice as much as in the data.

4.3.3 Value and Growth

Because markets are complete, claims to cashflows can be decomposed in such a way as to create the analog of value and growth firms. The value of a firm can be separated into the value of assets in place and the value of future growth opportunities, as in Berk, Green and Naik (1999). When investment is frictionless, existing assets represent the entire value of the firm, since a perfectly elastic supply of capital means that firms earn zero rents in equilibrium. In contrast, adjustment costs prevent the supply of capital from being perfectly elastic, hence future growth opportunities have additional value.

I focus on the value of assets in place and growth opportunities in the C-sector, since in the baseline model the capital stock in the I-sector is fixed. The value of assets in place in the consumption sector equals the value of all dividends accruing from existing assets

$$S_t^V = \max_{\hat{L}_{C,s}} E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(X_s (K_{C_t} e^{-\delta(s-t)})^{\beta_C} \hat{L}_{C,s}^{1-\beta_C} - w_s \hat{L}_{C,s} \right) \, ds.$$
(28)

In the absence of arbitrage, the value of growth opportunities must equal the residual value

$$S_t^G = S_t^C - S_t^V. (29)$$

This decomposition creates fictitious value and growth firms in the economy without the cost of modeling individual firms explicitly.

Lemma 1. The relative value of assets in place over growth opportunities in the consumption sector equals

$$\frac{S_t^V}{S_t^G} = \frac{g(\omega)}{\beta_C (1-\gamma) f(\omega) - f'(\omega) - g(\omega)},\tag{30}$$

where the function $g(\omega)$ satisfies the ODE in the Appendix. **Proof** See Appendix.

Lemma 1 implies that innovations to S_t^V/S_t^G are independent of the C-shock, dZ^X , and are spanned by the investment-specific shock, dZ^Y .

I find that the relative value of assets in place minus the value of growth opportunities in the consumption sector is decreasing in ω , as shown in figures 1(i) and 2(l). This is important because it offers a novel explanation for the value effect. A positive I-shock lowers the cost of new capital in the C-sector, increasing the value of future growth opportunities relative to the value of installed assets. Therefore, since the I-shock carries a negative premium, this implies that growth stocks have lower average returns than value stocks. On the other hand, the relative value of assets in place relative to growth opportunities is unaffected by the X shock, as shown in Figure 3(l).

In addition to explaining the value premium, my model can clarify the findings of Fama and French (1993), that the (HML) portfolio can explain the cross-section of realized as well as expected stock returns, or that it represents a source of systematic risk not spanned by the market portfolio. Lemma 1 implies that a portfolio of value minus growth stocks also spans the investment-specific shock, thus justifying the existence of a value factor. Conversely, models that explain the value premium with only one aggregate shock cannot generate comovement between value or growth firms independent of the market portfolio. The presence of a second aggregate shock highlights one crucial difference between my paper and those of Gomes, Kogan, and Zhang (2003) and Gala (2006), who argue that value firms are riskier than growth firms in bad times, and that therefore, a conditional CAPM should price the value spread. In my paper, the value premium arises due to exposure to a second aggregate shock, the I-shock, and therefore the conditional CAPM does not hold.

Although stylized, the decomposition of aggregate value into value of assets in place and future growth opportunities helps focus on the direct effects of investment shocks on aggregate dynamics, instead of on indirect effects through the aggregation heterogenous firms.

The model predicts that a portfolio that is long the value of growth opportunities will underperform a portfolio that is long the value of assets in place by 4.4%, as shown in Table 4. In the absence of within-sector firm heterogeneity, I refer to this as the fictitious value premium in the model. If one views growth firms as deriving a higher fraction of their value from growth opportunities than value firms, this number provides an upper bound on the actual value premium in the model.

4.3.4 Comparative statics

Here I perform a limited set of comparative statics with respect to the main parameters (θ , γ and λ). I focus on three moments of the data: the volatility of investment, (σ_I), the Sharpe Ratio of the IMC portfolio (SR_{IMC}) and the volatility of the risk-free rate, (σ_{rf}). I examine the volatility of investment to measure the impact of the I-shock on quantities. I examine the Sharpe Ratio of the IMC portfolio because it is monotonically linked to the price of risk of the I-shock, and the volatility of the risk-free rate because it is one of the moments that the model has difficulty matching. The results are shown in Figure 4.

I find that the parameter that governs the magnitude of the risk premium for the I-shock to be the EIS (θ) and not risk aversion (γ). The magnitude of risk premia is higher when the EIS is lower. In contrast, I find that increasing γ while holding θ fixed actually lowers risk premia. In addition, premia are higher when adjustment costs are steeply increasing (low λ), as this leads to increased volatility of investment and consumption growth.

The volatility of the risk-free rate depends on the steepness of the adjustment costs, and

the EIS (θ). A low EIS and steeply increasing adjustment costs (low λ) generate a volatile risk-free rate.

In order to provide some intuition as to the relative contribution of the EIS and risk aversion in how investors value states (the price of Arrow-Debreu securities), consider a state where a positive investment shock has occurred. Welfare will increase because future consumption will be higher and because households are risk averse, this will tend to lower the price of the Arrow-Debreu security that pays off in the state where the I-shock has occurred. On the other hand, the reason why future consumption growth is higher is because households have chosen to sacrifice consumption (and leisure) today. Because households prefer smooth consumption over time, this tends to increase the price of the corresponding Arrow-Debreu security. In fact, as can be seen in Figure 4(l), increasing the EIS above 1 may result in a positive price of risk for the I-shock.

The results suggest that a low EIS may be important in general equilibrium models with production, if they are also to fit asset prices. A low EIS helps along two dimensions. First, it increases the volatility of aggregate stock returns. This mechanism is known and is consistent with the findings of Campanale, Castro and Clementi (2007) and Lochstoer and Kaltenbrunner (2007) who solve models where the EIS is less than 1. Their models perform essentially very similar to Jermann (1998), whose model features habit formation. The key insight in Campanale, Castro and Clementi (2007) is that the reason why habit formation models had been successful in the past was mainly because they implied a low EIS rather than time-varying risk aversion. Second, as shown above, a low EIS increases the premium on the investment shock, which helps explain cross-sectional differences in asset returns.

5 Empirical evidence

My model has several empirical implications for the cross-section of stock returns. I provide evidence that the data support these implications.

The first implication is that investment-specific technological change earns a negative risk premium. To see this, note that the pricing kernel implied by the model can be linearized as

$$\pi = a - b_Y dZ^Y - b_X dZ^X. \tag{31}$$

This is the empirical equivalent of equation (22) linearized around $\overline{\omega}$. The model implies that shocks to investment technology, dZ^Y , have a negative price of risk, or equivalently that $b_Y < 0$. A positive investment-specific shock lowers the cost of new capital and thus acts as a shock to the real investment opportunities in the economy, increasing the marginal value of wealth and therefore state prices. As a result, firms that covary positively with the investment-specific shock should have, ceteris paribus, lower average returns.

The second implication is that a portfolio that is long the value of assets in place and short the value of growth opportunities is negatively correlated with the I-shock, and should therefore earn a positive premium. Sorting stocks on book to market is well known to produce portfolios that are mispriced by the CAPM. This is the well documented value puzzle. To the extent that value firms have more assets in place and fewer growth opportunities than growth firms, sorting stocks on book to market should produce a positive spread in returns that is explained by their covariance with the I-shock.

However, to test the empirical implications of the model, it is necessary to identify investment-specific technological change in the data. One of the advantages of my model is that it provides restrictions that identify investment-specific shocks. As shown in Propositions 1 and 2, random fluctuations in the ratio of the value of the investment goods sector, $S_{I,t}$, to that of the consumption goods sector, $S_{C,t}$, are driven only by the investmenttechnology shock, dZ_t^Y , and are unrelated to the C-shock, dZ_t^X . In other words, a zeroinvestment portfolio that is long the investment goods sector and short the consumption goods sector spans the investment shock. Thus, I can investigate the cross-sectional implications of risk exposure to the investment shock by including returns to this portfolio in standard factor pricing models. Doing so enables me to identify innovations in investmentspecific shocks by the return on an investment minus consumption portfolio (IMC).

5.1 Investment minus consumption portfolio

Ideally, the distinction between firms producing investment and consumption goods would be clear and the new factor portfolio would be straightforward to obtain. However, many companies produce both types of goods. To overcome this difficulty, I use information from the U.S. Department of Commerce's National Income and Product Account (NIPA) tables to classify industries as consumption or investment good producers.² I classify industries as investment or consumption producers based on the sector to which they contribute the most value. I describe the full procedure in the Appendix. As a robustness check, I construct an investment minus consumption portfolio using the data provided by Gomes, Kogan and Yogo (2007), who use a different set of NIPA tables and industry classifications. I create this

²A similar procedure is followed by Chari, Kehoe, and McGrattan (1996), Castro, Clementi and Mac-Donald (2006) and Gomes, Kogan and Yogo (2007).

portfolio, labeled IMC_{GKY} , by subtracting from the investment portfolio the non-durables portfolio.

The composition details of the IMC portfolio are displayed in Table 5. The sector producing consumption goods is larger than the sector producing investment goods, both in number of firms and in terms of market capitalization. However, the consumption and investment portfolio have fairly similar ratios of book to market equity, debt to assets, cashflows to assets and dividend payout.

The correlation of the IMC portfolio with IMC_{GKY} and the Fama-French factors is displayed in Table 6. The two proxies are highly correlated. The IMC portfolios have negative correlation with HML and small but positive correlation with the market. The negative correlation with HML is consistent with the model, since the value of assets in place minus growth opportunities is negatively correlated with the I-shock. Finally, the IMC portfolio has a positive correlation with the SMB factor, which stems from the fact that investment firms tend to have smaller market capitalization.

Table 7 shows average returns on the three IMC portfolios along with their CAPM alpha, broken down by decade. The IMC portfolios have negative average returns (-1.63% and -2.62% per year), and negative CAPM alphas (-2.74% and -4.12%). The CAPM alpha on the IMC portfolio has a p-value of 7.3% and the alpha on the IMC_{GKY} has a p-value of 0.8%.

Figure 5(a) plots the relative valuation of the two industries versus the investmentconsumption ratio. The valuation of the investment sector relative to the consumption sector tends to be higher when the investment to consumption ratio is higher, as in the 1975-85 and 1992-2000 periods. Figure 5(b) plots the realized time series of the IMC portfolio, which is clearly heteroscedastic. Makarov and Papanikolaou (2007) exploit the heteroscedasticity in the cross-section of stock returns to identify the underlying systematic shocks. One of their factors thus identified is highly correlated with the IMC portfolio. This is important because it establishes that the IMC portfolio is responsible for a significant amount of variability in realized returns, i.e. it is a factor that explains the time-series of returns.

5.2 Cross-sectional asset pricing tests

The pricing kernel in equation (31) summarizes all the cross-sectional asset pricing implications of the model. The restrictions on the rate of return of all traded assets that is imposed by no arbitrage,

$$E[\pi R] = 1, (32)$$

can be used to estimate (31) by the generalized method of moments. Accordingly, I estimate the model using two-stage GMM with the details described in the Appendix³.

I report the mean absolute pricing error (MAPE), the sum of squared pricing errors (SSQE) and the J-test of the over-identifying restrictions of the model, namely that all the pricing errors are zero. I choose to report the sum of squared errors rather than the normalized sum of squared errors (R^2) because in the absence of a constant term it is not clear what the benchmark (i.e. the normalization term) is.

I use returns on the CRSP value-weighted portfolio and monthly non-durable consumption growth from NIPA as empirical proxies for the C-shock and focus on the period 1961-2005. I use the return on the IMC portfolio as proxy for the I-shock.⁴ I compare the performance of the two models incorporating IMC with the CAPM, the CCAPM and the Fama-French three factor model. The model implies that HML derives its pricing ability through its exposure to the investment shock. Therefore, to see if each factor has additional pricing ability in the presence of the other, I include both HML and IMC in the same specification.

In this paper, I examine whether investment-specific shocks are an important component of the pricing kernel. Therefore, I focus on the estimate of b_Y , rather than on the overall ability of the model to price each cross-section. The overall performance of the model might depend on the particular choice of proxy for the C-shock. Most importantly, because (31) must hold for *all* traded assets in the economy, estimates of b_Y should be robust to using different test assets.

5.2.1 Risk-sorted portfolios

Estimating (31) using the entire cross-section of stock returns can be problematic because covariances are measured with error. To address this, most studies focus on a particular subset of assets, mostly portfolios of stocks sorted on economically meaningful characteristics.

Following Fama and French (1992), I form portfolios of stocks sorted on their estimated covariance with the IMC portfolio. I use five years of weekly data to estimate pre-ranking covariances. At the end of every 5 year period, I sort firms into 10 value-weighted portfolios based on the estimated covariances. I examine the excess returns of these portfolios relative to their risk and show the results in Table 8. The top panel shows that there is a weakly declining pattern of average excess returns, however these portfolios have different levels of

³The estimated parameters from the first and second stage are qualitatively and quantitatively similar.

⁴Results using the IMC_{GKY} portfolios are qualitatively and quantitatively very similar and are available upon request.

risk, as measured by the standard deviation of their returns. The Sharpe Ratio of these portfolios (mean excess return divided by standard deviation) declines monotonically from 14.1% to 5.3%. Given that these portfolios have different risk, it is more informative to look at CAPM alphas, i.e. the excess return after adjusting for their risk exposure to the market portfolio. The alphas decline monotonically from 2.99% to -3.20%. The 10 minus 1 portfolio has an alpha of -6.19%, whereas the 9-2 has an alpha of -5.44%. The alphas with respect to the Fama-French 3 Factor model display the same pattern, but they are not statistically significant from zero. Finally, as predicted by the model, a two factor model that includes the market portfolio and IMC successfully prices the spread.

Nagel, Lewellen and Shanken (2006) caution that if the cross-section of test assets has a strong factor structure, then some factors may appear to price the test assets arbitrarily well if they are correlated with the common factors. This is an argument against evaluating an asset pricing model only on a set of portfolios that are known to possess a strong factor structure (i.e. the 25 ME/BM portfolios created by Fama and French) or portfolios that are only sorted on covariances with the proposed factor. To address this, I also consider portfolios of stocks sorted first by industry and then covariance with the IMC portfolio. The construction of these portfolios is standard and is described in the Appendix. I estimate (31) using the above set of test assets.

Table 9 reports estimates using 24 portfolios sorted first on industry and then on IMC covariance. The CAPM and (C)CAPM perform rather poorly in this cross-section, with risk prices on the market portfolio and consumption growth not statistically different from zero. Both models generate large absolute pricing errors, though the J-tests fail to reject each model due to the large standard error of the pricing errors. The Fama-French three factor model performs significantly better, with smaller absolute pricing errors and with half the variance of pricing errors. However, this improvement is mostly driven by the negative and statistically significant premium on SMB, whereas the premium on HML is positive but not statistically significant. The two models that include the IMC portfolio are substantially more successful than the (C)CAPM. The estimated risk prices on IMC are negative and statistically different from zero. The estimated risk price on the market is also positive and significant, but the same is not true for the price of consumption growth. Both models produce lower absolute pricing errors and smaller variation of pricing errors than the Fama-French three factor model, even though they have fewer number of factors. The estimated premium on IMC is negative at -4.26 and -4.33 respectively and statistically significant from zero. To assess the magnitude of the estimated price of risk of the IMC portfolio, I compare it with the in-sample mean-variance ratio of the IMC portfolio (-1.67), which lies within the confidence interval of the estimate. I examine whether IMC and HML capture similar pricing information. The last column includes both IMC and HML along with the market portfolio. Using this cross-section as test assets, the IMC portfolio drives out HML. The estimated premium on IMC is -4.51 with a standard error of 2.17 and the premium on HML is -0.41 with a standard error of 3.37.

The main results of this section can also be graphically displayed. Figure 6(b) plots the CAPM pricing errors for the above sets of portfolios against their covariance with the IMC portfolio. One can see that this sort produce a significant spread in pricing errors, and that there is a strong negative relationship between pricing errors and covariances with the IMC portfolio. This suggests that the CAPM omits a significant source of systematic risk, specifically risk associated with the IMC portfolio. Most importantly, the relationship is negative, which is consistent with a negative premium on I-shock.

5.2.2 Book to market sorted portfolios

Table 10 presents estimation results for the 25 Fama-French portfolios sorted on market capitalization (ME) and book to market equity (BM). As always, the CAPM performs poorly in pricing this cross-section, generating large pricing errors, both in terms of absolute magnitude and variation across assets. Even though the estimated price of risk on the market portfolio is statistically significant, the mean absolute pricing error is 2.8%. The CCAPM is performing somewhat better with a mean absolute pricing error of 2.1%. The estimated price of risk on consumption growth is positive, statistically significant and equal to 68.3, and its magnitude is consistent with studies on the equity premium puzzle. The Fama-French three-factor model performs significantly better, with the sum of squared pricing errors equal to 0.65 vs 3.01 for the CAPM and 1.69 for the CCAPM.

Including the IMC portfolio in the (C)CAPM dramatically improves the performance of both models. More importantly, the estimated premium on the IMC portfolio is negative at -5.6 and -9.3 respectively, and statistically different from zero. The overall performance of the model however depends on the choice of proxy for the C-shock. The model that includes the market portfolio does substantially worse, with the sum of squared errors equal to 2.22. This is partially due to the fact that IMC cannot explain the size premium. If SMB is added to the specification, then the model performs as well as the Fama-French three factor model with a sum of squared errors of 0.61, but with a substantially higher estimated premium on IMC at -16.8. The model with only consumption growth and IMC performs slightly better than the Fama-French model, generating a sum of squared errors of 0.62 with only two factors. Finally, including the IMC portfolio in the Fama-French model does not improve the performance of the model, with the estimated price of risk on IMC still negative but not statistically significant.

The Fama-French 25 portfolios are sorted on size (market equity) in addition to bookto-market. As a robustness check and to ensure that my model explains the book-to-market effect and not the size effect, I sort stocks into 25 portfolios based on BM and their estimated covariance with IMC, and report the results on Table 11. The CAPM and CCAPM perform rather poorly, generating mean absolute pricing errors of 2.3-2.4% and sum of squared errors of 2.2 and 1.8 respectively. The Fama-French model performs significantly better, generating a mean absolute pricing error of 1.2% and a sum of squared errors of 0.57. As before, including the IMC portfolio in the (C)CAPM dramatically improves the pricing performance of the model, reducing the variation in pricing errors by half to 1.06 and 0.79 respectively. Again, the estimated premium on IMC is negative at -4.5 and -6 and statistically significant. The Hansen-Jagannathan (HJ) test rejects all models at the 5% level except the one featuring consumption growth and the IMC portfolio. When including both the IMC and HML portfolio in the specification, only the premium on HML is statistically significant, suggesting again that, consistent with my model, the IMC portfolio embodies similar pricing information as HML.

I display the results of this section in the bottom panel of Figure 6. Figures 6(c) and 6(d) plot the CAPM pricing errors for the above sets of portfolios against their covariance with the IMC portfolio. As before, there is a negative relationship between the pricing errors and covariances with IMC, suggesting that the CAPM omits a systematic source of risk that may be proxied by the IMC portfolio.

5.2.3 Individual stocks

In this section, I repeat the cross-sectional tests using the entire cross-section of returns. The problem when using individual stocks is that covariances are measured with error, which biases the estimated risk premium towards zero. To alleviate measurement error, I follow the procedure of Fama and French (1992) of first sorting stocks into portfolios based on their estimated covariance with IMC, and then assigning the portfolio covariance with IMC to the individual stocks. The full procedure is described in the Appendix.

The estimation results are shown on table 12. The Panel A presents results for the entire sample. The estimated premium on IMC ranges between -3.86 and -4.91 and is statistically

significant as long as log size is included in the specification. Most importantly, the estimate of b_Y is very close to the estimates obtained using different sets of test assets.

One issue when using the entire cross-section of stock returns is that this procedure places essentially equal weight on all stocks, regardless of their market capitalization. Since there is a large number of small stocks, which tend to behave abnormally especially in January due to tax-loss selling by institutions, I repeat the procedure excluding all January observations. The results, shown on Panel B, indicate that the estimated negative premium on IMC is not due to a January-type effect. In fact, the point estimates of the premium on IMC now range between -7.25 and -8.29 and is statistically significant. On the other hand, the size effect disappears, as the estimate coefficient on log market capitalization is no longer statistically significant from zero.

5.3 Response of investment, consumption and labor Supply

Proposition 2 identifies the IMC portfolio as a proxy for the investment-specific shock. However, it is possible that this portfolio captures other sources of systematic risk or perhaps proxies for the systematic mispricing of some stocks. If the IMC portfolio is a valid proxy for investment-specific shocks, then it must satisfy the following.

First, IMC must be a factor that drives the time series of realized returns. If stocks A and B have high average returns because they are exposed to similar risk, then they must also move together. Makarov and Papanikolaou (2007) show that one of the factors driving the cross-section of industry portfolios is highly correlated (87%) with a portfolio of investment minus consumption goods industries. In their work, the factors are identified through heteroscedasticity and not by imposing any economic structure.

Second, there must be a link between IMC and aggregate price and quantity of investment. Specifically, if this shock captures investment-specific productivity shocks, then it must predict an increase in investment and a fall in the quality-adjusted price of the investment good. On the other hand, if IMC captures mostly shocks to the demand for investment then it should predict and increase in both the quantity and the quality-adjusted price of the investment good.

Third, if IMC is part of the pricing kernel, there must be a link, at some horizon, between cashflows or returns on this portfolio and aggregate consumption or leisure. Even though in the model consumption can adjust instantaneously via the allocation of labor across sectors, this is a simplification. In reality, it might take several quarters for consumption to adjust. If the IMC portfolio captures an investment-specific shock, it should predict a short run fall and a long-run increase in the growth rate of consumption. In addition, it should predict a short-run increase in labor supply.

I explore how investment and the price of new investment goods respond to returns on the IMC portfolio. In particular, I look at investment in non-residential structures and equipment and software. I use the investment quantity indexes from the Bureau of Economic Analysis (BEA). For the price of new equipment, quality adjustment is an issue. The reason is that if investment-specific shocks also represent an increase in the quality of the investment good, then its price might increase. Lack of quality-adjusted prices makes it difficult to disentangle quality-improving productivity shocks from demand-side shocks, because both could predict an increase in prices and quantities. Unfortunately, obtaining quality-adjusted prices of investment goods is somewhat problematic. According to Moulton (2001), NIPA currently incorporates hedonic methods to quality-adjust computers, semiconductors, and digital telephone switching equipment, but does not adjust other types of equipment deflators. Consequently, I use the price index for computers and peripheral equipment.

In addition, the model implies that the cross-sectional heterogeneity in expected returns between investment and consumption firms stems from the differences in correlations with the stochastic discount factor, which partially depends on consumption and leisure. Here, I look at the cumulative response of consumption and leisure on returns to the IMC portfolio. I employ the quantity indexes from BEA on consumption expenditures excluding food and energy. As measures of labor supply I use the Civilian Unemployment Index and total hours worked in the non-farm business sector.

I estimate

$$x_{t+k} - x_t = \alpha_0 + \beta_k R_{IMC,t} + \gamma_{1,k} R_{MKT,t} + \gamma_{2,k} \Delta x_t + \epsilon_t, \tag{33}$$

where x_t denotes log investment, the log relative price deflator, log consumption and hours, and $R_{IMC,t}$ and $R_{MKT,t}$ denote returns on IMC and the market portfolio respectively. Since the IMC portfolio correlates with the market portfolio, which is known to predict investment and consumption, I control for returns to the market portfolio. Figures 7(a)-(f) plot the β_k coefficients along with 90% confidence intervals. I adjust the standard errors for heteroscedasticity and serial correlation using the Newey-West (1987) procedure.

I find that investment in equipment and non-residential structures sharply increases following positive returns on the IMC portfolio. The increase in investment following positive returns on IMC is statistically significant for up to five quarters forward. At the same time, the quality adjusted price of equipment falls following positive returns on IMC, but the response is not statistically significant at the 10% level. Investment in non-residential structures also displays a strong response, which is consistent with the findings of Gort, Greenwood and Rupert (1999) who document significant technological advances in structures.

Consumption sharply falls following positive returns on the IMC portfolio. The point estimates suggest that this fall is reversed after 10-14 quarters, but the long-run response of consumption growth is not statistically significant from zero. This is hardly surprising given that the sample contains very few long-horizon observations that are independent. On the other hand, labor supply increases following positive returns on the IMC portfolio, as evidenced by the fall in unemployment and the increase in hours worked.

The overall pattern is consistent with the spirit of the model. A positive shock to investment technology leads to a gradual reallocation of resources from the sector producing consumption goods to the sector producing investment goods. In addition, labor supply increases following a positive investment shock.

5.4 Consumption, investment and the pricing kernel

The cross-sectional asset pricing tests in Section 5.2 rely on the model's implication that a portfolio of investment minus consumption producing firms is a good proxy for the investment-specific shock. In this section, I repeat some of the cross-sectional tests, using data on the investment-output ratio and aggregate consumption. I compute the exact stochastic discount factor (SDF) that is implied by the model, i.e., equation (22), rather than the linearized version in equation (31).

Proposition 1 implies that my model features two state variables, ω and $x = X K_C^{\beta_C}$ and that consumption (C_t) and the investment-output ratio (i_t) are a function of (x, ω) ,

$$\left(\begin{array}{c} C_t\\ i_t \end{array}\right) = \Gamma \left(\begin{array}{c} x_t\\ \omega_t \end{array}\right). \tag{34}$$

I can invert the level of consumption $C(\omega_t, x_t)$ and the investment-output ratio $i(\omega_t, x_t)$ implied by the model to obtain estimates $\hat{\omega}_t$ and \hat{x}_t ,

$$\begin{pmatrix} \hat{x}_t \\ \hat{\omega}_t \end{pmatrix} = \Gamma^{-1} \begin{pmatrix} C_t \\ i_t \end{pmatrix}.$$
(35)

Given estimates $\hat{\omega}_t$ and \hat{x}_t , I compute the SDF implied by the model and then repeat some

of the cross-sectional tests in Section 5.2.

First, I use the full solution of the model, and data on the investment-output ratio and aggregate consumption, to construct estimates of $\hat{\pi}$,

$$\hat{\pi}_t = \frac{\pi(C_t, i_t)}{\frac{1}{T} \sum_{t=1}^T \pi(C_t, i_t)}.$$
(36)

If the stochastic discount factor, $\hat{\pi}$, prices assets correctly, it must be that for all i,

$$E[R_{i,t}^{e}] = -cov(R_{i,t}^{e}, \hat{\pi}_{t})$$
(37)

I compute the covariance of portfolio excess returns (R_i^e) and the implied SDF $(\hat{\pi})$. Finally, I form estimates of the average excess return on each portfolio, $\overline{R_i^e}$, and estimate using GMM,

$$\overline{R_i^e} = a + b \, \widehat{cov(R_{i,t}^e, \hat{\pi}_t)} + u_i \tag{38}$$

If the model is correct, then in the above equation a = 0, b = -1, and $u_i = 0 \quad \forall i$.

Figure 8(a) displays the time-series of the implied SDF versus the time-series of the SDF implied by the CCAPM. The SDF implied by the model is roughly three times as volatile as the SDF implied by the CCAPM with $\gamma = 2$ (16.7% vs 5.6%). I present the estimation results in Figure 8(b) for the cross-section of 25 ME/BM sorted portfolios and the 24 Industry/IMC beta sorted portfolios. The model is rejected on two grounds. First, b is significantly different than -1. Estimates of b are -4.9 and -3.9 respectively. This suggests that the estimated covariances of portfolio returns with the implied SDF, $\hat{\pi}$, display the right pattern (i.e. the are decreasing in the BM sort) but their magnitude is too small by a factor of 4. Second, the estimate of the risk-free or zero-beta rate is too high, since the estimates of a are 10% and 6.0% respectively.

Note however that even though the model is rejected on the grounds that $b \neq -1$ and $a \neq 0$, the model does fairly well in some dimensions. The R^2 in the above regression is fairly high for the book-to-market sorted portfolios (67%), and the mean absolute pricing errors are small (1.71%). For the 24 IMC/Industry portfolios the R^2 is 37% and the mean absolute pricing errors are 1.01%. I plot excess returns on these portfolios versus their covariance with the implied SDF in Figures 8(c)-(d).

Given that I use aggregate data on investment, consumption, and output to construct $\hat{\pi}$, the fact that b is larger in absolute magnitudes is hardly surprising, since stock returns generally display low correlation with macroeconomic aggregates. Nevertheless, the model-

implied SDF is an improvement over the CCAPM, which performs poorly in pricing the cross-section of asset returns, as shown in Tables 9 and 11, and is a step closer at satisfying the Hansen-Jagannathan (1991) bounds.⁵ Finally, note here that even though the 25 ME/BM portfolios have essentially a two-factor structure, it is correlation with a single factor, i.e. the implied SDF, that has the right pattern.

6 Conclusion

I extend the standard general equilibrium models used in finance to incorporate investmentspecific technological change. Investment-specific shocks act as a shock to the real investment opportunities in the economy. In equilibrium, an increase in the productivity of the capital goods sector is followed by a reallocation of resources from consumption to investment. This fall in consumption may result in a negative price of risk for investment-specific shocks. A negative price of risk for the investment shock implies that states where investment opportunities are high marginal valuation states.

In addition, the investment-specific shock affects both the relative cost of new capital and the relative profitability of firms. In particular, a positive investment-specific shock increases the value of investment good producers and growth firms relative to consumption-good and value firms. In contrast, the shock to the consumption sector affects all firms symmetrically. The presence of two shocks creates heterogeneity in expected returns and can also explain why HML is a factor in the time series of returns, or, equivalently, why value and growth stocks move together.

The data largely support the implications of the model. Using the restrictions of the model, I use a portfolio of investment minus consumption firms (IMC) as my proxy for investment-specific technological change. The IMC portfolio predicts an increase in investment and a short run fall in consumption and leisure. I find that when I control for their exposure to the market portfolio, firms with high correlation with the IMC portfolio have lower average returns. Additionally, the IMC portfolio improves on the ability of the (C)CAPM to price the cross-section of expected returns of portfolios of stocks sorted by book to market.

More than just adding an additional source of systematic risk, a model with investmentspecific technological change offers new insights about the relation of stock returns and macroeconomic sources of risk. Similar to Gomes, Kogan, and Yogo (2007), my model

 $^{^{5}}$ Hansen-Jagannathan (1991) show that an admissible SDF needs to be at least as volatile as the maximum Sharpe Ratio attainable. The Sharpe Ratio of the market portfolio in the 1962-2007 period has been close to 0.4.

recognizes an aspect of firm heterogeneity that is not directly related to accounting valuation ratios, and which implies that firms respond differently to some macroeconomic shocks.

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7 Tables and figures

Table 1: PARAMETERS

Parameter	γ	θ	ψ	ρ	β_C	β_I	λ	δ	μ_Y	σ_Y	μ_X	σ_X
Value	2	0.35	3	0.001	0.3	0.3	1.75	0.14	0.23	0.5	0	0.0325

Table 2: Unconditional Moments: Consumption, Output, Investment and Hours

			standard			С	orrelation	
	var	mean	deviation	autocorrelation		\dot{c}^u	\dot{y}^u	\dot{i}^u
Data	ċ	0.030	0.035	0.341	\dot{c}^u			
Median		0.027	0.039	0.784				
[5%, 95%]		$[0.010 \ 0.041]$	$[0.027 \ 0.062]$	$[0.614 \ 0.878]$				
	\dot{y}	0.029	0.048	0.088	\dot{y}^u	0.740		
		0.028	0.046	0.735		0.913		
		$[0.009 \ 0.042]$	$[0.033 \ 0.064]$	$[0.545 \ 0.856]$		$[0.846 \ 0.952]$		
	\dot{i}	0.052	0.166	0.133	\dot{i}^u	0.524	0.845	
		0.031	0.136	0.125		0.470	0.717	
		$[0.009 \ 0.067]$	$[0.054\ 0.261]$	$[-0.102 \ 0.3475]$		$[0.135 \ 0.685]$	$[0.430 \ 0.861]$	
	i	0.000	0.025	0.158	\dot{l}^u	0.534	0.845	0.855
		0.000	0.019	0.128		0.382	0.625	0.971
		[-0.001 0.001]	$[0.011\ 0.032]$	$[-0.109 \ 0.356]$		$[0.118 \ 0.595]$	$[0.444 \ 0.773]$	[0.901 0.992]

Table 1 shows the parameters used in calibration. Table 2 shows the unconditional moments of consumption, output, hours and investment growth generated by the model versus their empirical counterparts. I use annual data from 1948 to 2006 for real consumption growth in services and non-durables (c), real gross domestic product excluding government expenditures (y), real private nonresidential investment in equipment and software (i), and hours worked per capita (l). I simulate 50 years of data from the model, and repeat the simulations for 10,000 times. I report the median value and the 5% and 95% percentiles of the estimated moments across simulations. I report the mean, standard deviation and serial correlation for consumption, output, investment and labor supply growth. In addition, I report the correlation matrix of innovations (\dot{x}^u) in consumption, output, investment and labor supply growth, where the innovations are computed from a VAR(1) model.

		Х				Y	
	median	5%	95%	-	median	5%	95%
\mathbf{C}	0.765	[0.102]	0.997]	-	0.235	[0.003]	0.898]
Υ	0.242	[0.099]	0.567]		0.758	[0.433]	0.901]
\mathbf{L}	0.000	[0.000]	[0.000]		1.000	[1.000]	1.000]
Ι	0.051	[0.010]	0.871]		0.949	[0.129]	0.990]
\mathbf{Q}	0.000	[0.000]	[0.000]		1.000	[1.000]	1.000]
r_{f}	0.000	[0.000]	[0.000]		1.000	[1.000]	1.000]
MKT	0.051	[0.021]	0.102]		0.949	[0.898]	0.989]
IMC	0.000	[0.000]	0.000]		1.000	[1.000]	1.000]
VMG	0.000	[0.000]	0.000]		1.000	[1.000]	1.000]

Table 3: VARIANCE DECOMPOSITION

Table 4: Unconditional Moments: Prices and Ratios

	Model	Mode	el (simula	tion)	Data
	(inst)	median	5%	95%	Data
$ER_M - r_f$	0.041	0.048	[0.031	0.064]	0.066
$\sigma(R_M)$	0.199	0.203	[0.192]	0.213]	0.198
ER_{IMC}	-0.019	-0.020	[-0.039]	-0.015]	-0.016
$\sigma(R_{IMC})$	0.098	0.097	[0.089]	0.109]	0.099
r_{f}	0.019	0.021	[0.001	0.053]	0.017
$\sigma(r_f)$	0.075	0.058	[0.030]	0.086]	0.029
Tobin's Q	1.376	1.369	[1.181	1.546]	1.531
I/Y	0.191	0.201	[0.167]	0.241]	0.173
ER_{VMG}	0.044				
$\sigma(R_{VMG})$	0.225				

Table 3 reports the variance decomposition for the key variables in the model. Table 4 shows the unconditional moments for asset prices and other variables generated by the model versus their empirical counterparts. The moments are calculated in two ways: The first computes instantaneous moments by directly computing the unconditional moments implied by the differential equations characterizing the solution and the invariant distribution of ω as described in section 8.4.2. The second, by simulating 50 years of data from the model, and repeating the simulations for 10,000 times. I report the median value and the 5% and 95% percentiles of the estimated moments across simulations.



Table 1 plots the numerical solution of the model. The solution method is described in section 8.4.1. Without loss of generality, I evaluate the above aggregate quantities and prices and $K_C = 1$ and X = 1, and plot them as a function of ω .



Figure 2: Dynamic responses to the Investment-Specific shock Y



Figure 3: Dynamic responses to the Consumption shock X



Figure 4: Comparative Statics with respect to $\gamma, \ \theta$ and λ

Table 4 plots comparative statics with respect to the EIS (θ), risk aversion (γ) and the slope of the adjustment cost function (λ). I focus on the instantaneous volatility of investment, the instantaneous Sharpe Ratio of the IMC portfolio and the volatility of the risk-free rate. I compute the moments exactly using the differential equations characterizing the model and integration over the stationary distribution of ω , as described in section 8.4.2.

	Cons	umpti	on Por	tfolio	Inv	estmen	t port	folio
	me	ean	10%	00%	me	ean	10%	00%
	ew	VW	1070	9070	ew	VW	1070	9070
ME (\$b $)$	2.75		0.05	5.13	1.37		0.04	3.21
Book-to-Market Equity	0.93	0.56	0.28	1.74	0.89	0.59	0.29	1.60
Debt-to-Asset	0.19	0.16	0.00	0.38	0.16	0.18	0.00	0.36
Cashflows-to-Assets	0.08	0.13	0.02	0.17	0.07	0.10	0.00	0.16
Dividends-to-Earnings	0.19	0.26	0.03	0.41	0.21	0.24	0.01	0.43
Number of firms	812				437			

 Table 5: IMC PORTFOLIO: COMPOSITION

 Table 6: IMC PORTFOLIO: CORRELATIONS

1961:2005	IMC	IMC_{GKY}	MKT	SMB	HML	MOM
IMC						
IMC_{GKY}	77.5%					
MKT	22.9%	36.0%				
SMB	47.9%	49.9%	25.0%			
HML	-49.1%	-37.5%	-34.5%	-25.0%		
MOM	-1.8%	-5.6%	-5.7%	-1.4%	-13.0%	

Table 5 reports composition details for the investment minus consumption portfolio (IMC) constructed using the NIPA tables. I report the market value of equity (ME) in 2004 dollars, the book-to-market ratio (COMPUSTAT item 60 over ME), book value of debt to assets (COMPUSTAT item 9 over item 6), cashflows to assets (COMPUSTAT item 14 plus item 18 over item 6) and dividend to cashflows (COMPUSTAT item 19 plus item 21 over cashflows). I report time-series averages of the equal weighted average within each portfolio (ew), the value-weighted average (vw) and the 10% and 90% decile within each portfolio. Table 6 reports correlations with the above portfolios with the excess returns on the CRSP value-weighted index, the Fama-French (1993) factors, Carhart's momentum factor and IMC_{GKY} . IMC_{GKY} is the investment minus consumption constructed using the data provided by Gomes, Kogan and Yogo (2006). I form IMC_{GKY} by subtracting the non-durables portfolio from the investment portfolio. I report results for the whole sample (1961-2005).

Dominal	IN	IC		IMC	C_{GKY}	HI	ML
renou	E(R)	α	-	E(R)	α	E(R)	α
1961 - 1970	-2.11	-2.22	-	-0.38	-1.30	6.37	6.83
	(1.65)	(1.68)		(2.43)	(2.32)	(2.24)	(2.24)
1971 - 1980	0.98	0.63		0.80	0.22	3.95	4.51
	(1.90)	(1.87)		(2.18)	(2.05)	(3.13)	(3.07)
1981 - 1990	-5.91	-5.93		-10.70	-11.28	6.83	8.30
	(1.93)	(1.96)		(2.80)	(2.78)	(2.85)	(2.40)
1991 - 2000	1.77	-3.46		-1.05	-6.17	3.46	9.42
	(5.28)	(5.15)		(4.91)	(4.75)	(4.04)	(3.56)
2001 - 2005	-4.10	-5.27		-0.51	-1.11	9.41	9.93
	(7.14)	(5.14)		(8.14)	(5.95)	(4.87)	(3.36)
1961 - 2005	-1.63	-2.74		-2.62	-4.12	5.63	7.15
	(1.58)	(1.53)		(1.64)	(1.56)	(1.49)	(1.37)

 Table 7: SUMMARY STATISTICS

Table 7 reports average annualized excess returns, CAPM alphas and correlations with HML for the following three portfolios. Standard errors are reported in parentheses. IMC is the investment minus consumption portfolio constructed using the NIPA tables, where industries are classified as investment or consumption based on which sector they contribute the most. Sample includes data from 1961 to 2005. IMC_{GKY} is the investment minus consumption constructed using the data of Gomes, Kogan and Yogo (2006). I form IMC_{GKY} by subtracting a weighted average of the services and the non-durables portfolio from the investment portfolio. I report results for the whole sample and for each decade.





Figure 5(a) shows the ratio of market values of investment over consumption industries (left axis) and the ratio of private fixed nonresidential investment over personal consumption expenditures from the NIPA tables (right axis). Figure 5(b) shows returns for the IMC portfolio, with the solid line classifying industries according to the sector they contribute most, and the dotted line drops common industries.

IMC Beta	Lo	2	33	4	ъ	9	2	x	6	Ηi	Hi - Lo	9 - 2
					Pan	el A						
Excess Return $(\%)$	6.50	7.82	7.12	6.78	6.95	5.81	6.08	5.92	4.82	5.02	-1.48	-3.00
	(2.15)	(2.55)	(2.46)	(2.67)	(2.84)	(2.83)	(3.19)	(3.42)	(3.86)	(4.42)	(3.90)	(2.61)
σ (%)	13.27	15.73	15.16	16.47	17.50	17.41	19.64	21.04	23.79	27.23	24.03	16.08
$\mu/\sigma~(\%)$	14.14	14.36	13.57	11.89	11.47	9.64	8.94	8.12	5.85	5.32	-1.78	-5.39
β_{MKT}	0.63	0.91	0.87	0.96	1.02	1.00	1.14	1.22	1.35	1.48	0.85	0.44
	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.01)	(0.05)
$\alpha(\%)$	2.99	2.77	2.29	1.45	1.30	0.27	-0.25	-0.83	-2.66	-3.20	-6.19	-5.44
	(1.45)	(1.00)	(1.01)	(1.01)	(1.05)	(1.23)	(1.22)	(1.43)	(1.63)	(2.24)	(3.25)	(2.30)
$R^2(\%)$	57.47	84.72	83.33	86.01	85.85	83.37	85.27	84.43	81.23	74.80	31.55	18.79
					Pan	el B						
β_{MKT}	0.80	1.00	0.96	1.00	1.03	1.05	1.10	1.15	1.21	1.32	0.52	0.21
	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.01)	(0.05)
β_{SMB}	-0.34	-0.22	-0.19	-0.15	-0.06	0.08	0.07	0.21	0.26	0.49	0.83	0.48
	(0.03)	(0.03)	(0.03)	(0.03)	(0.06)	(0.04)	(0.05)	(0.05)	(0.06)	(0.01)	(0.09)	(0.08)
β_{HML}	0.33	0.15	0.16	0.02	-0.01	0.24	-0.09	-0.08	-0.29	-0.20	-0.53	-0.44
	(0.04)	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.06)	(0.06)	(0.08)	(0.01)	(0.10)	(0.10)
$\alpha(\%)$	1.31	2.16	1.50	1.56	1.46	-1.52	0.26	-0.64	-1.12	-2.67	-3.98	-3.28
	(1.17)	(0.88)	(0.88)	(1.02)	(1.08)	(1.17)	(1.18)	(1.30)	(1.52)	(2.10)	(2.81)	(2.01)
$R^2(\%)$	73.21	88.42	86.79	87.12	85.99	85.13	85.71	85.89	84.58	79.74	53.20	39.64
					Pan	el C						
β_{MKT}	0.84	1.03	0.98	1.02	1.00	1.05	1.04	1.08	1.11	1.19	0.34	0.08
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.04)
β_{IMC}	-0.50	-0.28	-0.26	-0.15	0.05	-0.11	0.25	0.34	0.57	0.71	1.21	0.86
	(0.03)	(0.03)	(0.03)	(0.03)	(0.06)	(0.04)	(0.05)	(0.04)	(0.05)	(0.01)	(0.08)	(0.06)
$\alpha(\%)$	0.37	1.28	0.93	0.69	1.56	-0.33	1.06	0.94	0.34	0.51	0.14	-0.94
	(1.03)	(0.83)	(0.85)	(0.99)	(1.10)	(1.18)	(1.10)	(1.17)	(1.20)	(1.80)	(2.27)	(1.58)
$R^2(\%)$	79.33	89.77	87.88	87.24	85.98	84.02	87.75	88.41	90.25	85.30	70.67	62.92

Table 8: SUMMARY STATISTICS: 10 PORTFOLIOS SORTED ON IMC BETA

Table 8 reports summary statistics of the simple monthly returns of 10 portfolios created by sorting stocks on their covariance with IMC. The pre-sorting covariances are estimated using weekly returns and 5 year windows. Portfolios are rebalanced every 5 years on June. Panel A reports mean excess returns over the 30-day T-bill rate (μ) , the standard deviation of returns (σ) and their beta and alpha with respect to the CAPM. Panel B reports beta and alpha for the Fama-French three factor model. Panel C reports beta and alpha for the model including the market portfolio and the IMC portfolio. Standard errors are shown in parenthesis.

Factor Price	CAPM	CCAPM	FF3	IMC/MKT	IMC/C	IMC/HML
Market	1.18		2.98	2.06		1.98
	(1.01)		(1.24)	(1.08)		(1.25)
SMB			-5.77			
			(2.34)			
HML			2.76			-0.41
			(2.73)			(3.37)
C_{ND}		16.40			28.25	
		(23.66)			(22.72)	
IMC				-4.26	-4.33	-4.51
				(1.66)	(1.75)	(2.17)
MAPE(%)	1.72	1.72	1.23	1.21	1.08	1.20
SSQE(%)	0.98	0.98	0.54	0.51	0.46	0.51
J-test	36.9	36.0	25.0	23.9	22.3	24.3
	(0.033)	(0.041)	(0.248)	(0.352)	(0.443)	(0.281)

Table 9: Cross-Sectional Tests: 24 portfolios sorted on Industry and IMC beta

Table 9 reports the estimation results from cross-sectional tests. I estimate the parameters of the pricing kernel,

 $\pi = a - b F.$

The set of test assets includes simple monthly returns of 24 portfolios created by sorting stocks first on 8 industries and then on their covariance with IMC. The pre-sorting covariances are estimated using weekly returns and 5 year windows. Portfolios are rebalanced every 5 years on June. The factors considered are the market portfolio, Fama and French's SMB and HML factors, the IMC portfolio, non-durable consumption growth (C_{ND}) . The sample includes monthly data from 1961 to 2005. Estimation is by two-step generalized method of moments (GMM). HAC standard errors are computed by the Newey-West estimator with 3 lags of returns. I report second stage estimates of the factor premia (b), along with standard errors, annualized mean absolute pricing errors (MAPE) and the sum of squared pricing errors (SSQE) for the first stage. Furthermore, I report the J-test that all the pricing errors are zero along with it's p-value in parenthesis.

Factor Price	CAPM	CCAPM	FF3	IMC/SMB	IMC/MKT	IMC/C	IMC/FF3
Market	2.28		4.09	3.11	2.66		4.10
	(1.04)		(1.16)	(1.28)	(1.11)		(1.23)
SMB			3.14	8.16			4.43
			(1.46)	(2.19)			(2.21)
HML			9.09				8.43
			(1.76)				(2.46)
C_{ND}		68.28				98.87	
		(22.40)				(30.58)	
IMC				-16.44	-5.60	-9.32	-3.51
				(4.27)	(2.46)	(2.90)	(4.74)
MAPE(%)	2.80	2.08	1.22	1.21	2.55	1.33	1.15
SSQE(%)	3.01	1.69	0.65	0.61	2.22	0.62	0.58
J-test	107.4	92.8	75.6	79.6	102.7	50.6	73.1
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)

Table 10: CROSS-SECTIONAL TESTS: 25 PORTFOLIOS SORTED ON ME AND BM

Table 10 reports the estimation results from cross-sectional tests. I estimate the parameters of the pricing kernel,

$\pi = a - b F.$

The set of test assets includes simple monthly returns of the 25 portfolios created by sorting stocks on first market capitalization and then on book to market using NYSE quintiles. The data come from Kenneth French's website. The factors considered are the market portfolio, Fama and French's SMB and HML factors, the IMC portfolio, non-durable consumption growth (C_{ND}) . The sample includes monthly data from 1961 to 2005. Estimation is by two-step generalized method of moments (GMM). HAC standard errors are computed by the Newey-West estimator with 3 lags of returns. I report second stage estimates of the factor premia (b), along with standard errors, annualized mean absolute pricing errors (MAPE) and the sum of squared pricing errors (SSQE) for the first stage. Furthermore, I report the J-test that all the pricing errors are zero along with it's p-value in parenthesis.

Factor Price	CAPM	CCAPM	FF3	IMC/MKT	IMC/C	IMC/HML
Market	2.77		4.64	3.65		4.71
	(1.08)		(1.29)	(1.19)		(1.25)
SMB			-0.23			
			(2.27)			
HML			8.06			7.02
			(2.36)			(2.73)
C_{ND}		80.43			152.00	
		(29.45)			(55.73)	
IMC				-4.52	-6.01	-1.63
				(2.09)	(2.87)	(2.46)
MAPE(%)	2.42	2.34	1.20	1.66	1.43	1.21
SSQE(%)	2.22	1.84	0.57	1.06	0.79	0.57
J-test	53.4	40.6	38.8	45.6	23.8	38.8
	(0.001)	(0.018)	(0.015)	(0.003)	(0.417)	(0.015)

Table 11: CROSS-SECTIONAL TESTS: 25 PORTFOLIOS SORTED ON BM AND IMC BETA

Table 11 reports the estimation results from cross-sectional tests. I estimate the parameters of the pricing kernel,

 $\pi = a - b F.$

The set of test assets includes simple monthly returns of 25 portfolios created by sorting stocks first on their book to market ratio based on NYSE breakpoints and then on their covariance with IMC. The pre-sorting covariances are estimated using weekly returns and 5 year windows. Portfolios are rebalanced every 5 years on June. The factors considered are the market portfolio, Fama and French's SMB and HML factors, the IMC portfolio, non-durable consumption growth (C_{ND}) . The sample includes monthly data from 1961 to 2005. Estimation is by two-step generalized method of moments (GMM). HAC standard errors are computed by the Newey-West estimator with 3 lags of returns. I report second stage estimates of the factor premia (b), along with t-statistics, annualized mean absolute pricing errors (MAPE) and the sum of squared pricing errors (SSQE) for the first stage. Furthermore, I report the J-test that all the pricing errors are zero along with it's p-value in parenthesis.



Figure 6: CAPM PRICING ERRORS VS COVARIANCE WITH IMC

Figure 6 plots the CAPM pricing errors of the test portfolios considered in tables 5 through 10 versus their covariances with the IMC portfolio. CAPM Pricing errors are the first stage pricing errors from the cross-sectional GMM tests. The constructions of these portfolios is standard and is described in the Appendix.

$cov(R_i, R_{MKT})$	$cov(R_i, R_{IMC})$	$\ln ME$	$\ln BM$
P	anel A: (all mont	hs)	
2.40	-3.86		
(1.92)	(2.61)		
3.16	-4.91	-0.162	
(1.78)	(2.40)	(0.044)	
1.41		-0.109	0.327
(1.64)		(0.043)	(0.070)
4.14	-4.82	-0.111	0.322
(1.76)	(2.20)	(0.042)	(0.068)
Par	nel B: (excl. Janu	ary)	
4.20	-8.29		
(1.79)	(2.26)		
4.21	-7.87	-0.008	
(1.75)	(2.15)	(0.036)	
-0.27		0.014	0.216
(1.61)		(0.038)	(0.066)
4.88	-7.25	0.010	0.208
(1.79)	(2.09)	(0.039)	(0.065)

Table 12: IMC AND EXPECTED RETURNS - ENTIRE CROSS-SECTION OF STOCKS

Table 12 reports results from Fama and MacBeth (1973) cross-sectional regressions of simple monthly returns of all NYSE, AMEX, and Nasdaq stocks on covariances and characteristics using data from 1961-2005. Covariances are estimated using a procedure similar to Fama and French (1992). Specifically, for each individual stock I estimate the covariance of its returns with IMC using 5 years of weekly log excess returns. At the end of a five year period, stocks are then sorted into 100 pre-ranking covariance centiles. I then compute the equal-weighted monthly log excess returns on these 100 portfolios over the next 5 years. This procedure is repeated every 5 years, forming a time-series of returns on these 100 portfolios. I then reestimate covariances for the portfolios formed from the pre-ranking sorts using 5 years of monthly data to obtain postranking covariances. The post-ranking covariance estimate for a given portfolio is then assigned to each stock in the portfolio. Portfolio assignments are updated every 5 years. Every month the cross-section of stock returns in excess of risk free rate is then regressed on a constant (not reported), the covariance with the excess return on the CRSP value-weighted index, the covariance with the return on the IMC portfolio, the log of market capitalization (ME) and the log of Book to Market (BM). I report the time series average of the regression coefficients along Fama-McBeth standard errors. Panel A reports results for the full sample, whereas Panels B reports results excluding the month of January.





Figure 7 plots the coefficients β_k in the regression of:

$$x_{t+k} - x_t = a + \beta_k IMC_t + \gamma_1 MKT_t + \gamma_2 \Delta x_t + e_t k$$

along with 90% confidence intervals based on HAC standard error. I compute HAC standard errors errors using Newey-West, with the truncation lag equal to the length of the overlap plus two quarters. The sample includes quarterly data in the 1951:2005 period. The quantity and price indices are from the NIPA tables in the BEA website. The price index for Information processing equipment and software is relative to the GDP deflator.

	a IN	D/IMC 24 0.060	ME/BM 25 0.101
		(0.014)	(0.016)
	q	-4.869	-3.907
		(1.759)	(0.769)
	R^{2}	36.5%	66.9%
	MAPE	1.01%	1.71%
	J-test	91.1	161.9
- 1950 1960 1970 1980 1990 2000 2010	p-value	0.00	0.00
ios sorted on Industry and IMC	(d) 25 portfolios sort 22	ed on ME/BM	
• 13	0.2 - • ¹⁵		
51 • 22 • 23 • 4562 • 33		28 ◆24 5 ◆13 ◆23 ◆13 ◆24 ◆25 ◆45 *54	
• 42 • 63 • 71 • 62 • 61 • 82 • 51	0.1 -	• 12	• 41 • 8
• • • • • • • • • • • • • • • • • • • •	0.05 -		•
• 83			

Figure 8: The Investment to GDP ratio and the Cross-Section of Stock Returns

Panels (a) and (b) plot average returns on the 24 Industry/IMC and 25 ME/BM sorted portfolios versus their covariance with the SDF implied by the model. The model implied SDF is computed in section (5.4) by inverting the observed investment to GDP ratio. I use annual data from 1929 to 2005 to construct the investment ratio (Private Non-residential Fixed Investment over Gross Domestic Product) and Consumption (Non-durable consumption). Panel (c) plots the model implied SDF versus the SDF implied by the CCAPM, and panel (d) presents estimation results of equation (38). HAC standard errors are computed by the Newey-West estimator with 1 lag of returns. I report second stage estimates of the coefficients (a,b), along with standard errors. I also report the cross-sectional R^2 and mean absolute pricing errors (MAPE) based on first-stage estimates.

8 Appendix

8.1 Proof of Proposition 1

The Hamilton-Jacobi-Bellman equation for the social planner's optimization problem is:

$$0 = \max_{L_I, L_C, i_C, N} \left\{ h(C, N, J) + (i_C - \delta) J_{K_C} K_C + J_X X \mu_X + \frac{1}{2} J_{XX} X^2 \sigma_X^2 + \mu_Y J_Y Y + \frac{1}{2} J_{YY} Y^2 \sigma_Y^2 \right\}$$

where

$$h(C, N, J) = \frac{\rho}{1 - \theta^{-1}} \left(\frac{(CN^{\psi})^{1 - \theta^{-1}}}{((1 - \gamma)J)^{\frac{\gamma - \theta^{-1}}{1 - \gamma}}} - (1 - \gamma)J \right)$$

subject to:

$$C \leq X K_C^{\beta_C} L_C^{1-\beta_C}$$

$$c(i_C) K_C \leq Y K_I^{\beta_I} L_I^{1-\beta_I}$$

$$L_C + L_I \leq 1 - N$$

Denote the lagrange multipliers of the above constraints as $(\pi, \xi \pi, w\pi)$. Replacing the labor market clearing condition $N = 1 - L_C - L_I$ in the Bellman equation, one can see that the first order condition with respect to L_C does not depend on the value function directly:

$$L_{C}^{*} = \arg \max_{L_{C}} CN^{\psi}$$

= $\arg \max_{L_{C}} (XK_{C}^{\beta_{C}}L_{C}^{1-\beta_{C}})(1-L_{I}-L_{C})^{\psi}$
= $\frac{(1-\beta_{C})(1-L_{I})}{1+\psi-\beta_{C}}$

Replacing the above in the HJB equation along with the constraint on investment:

$$0 = \max_{L_{I}} \left\{ h \left(X K_{C}^{\beta_{C}} \left(\frac{(1 - \beta_{C})(1 - L_{I})}{1 + \psi - \beta_{C}} \right)^{1 - \beta_{C}}, 1 - L_{I} - \frac{(1 - \beta_{C})(1 - L_{I})}{1 + \psi - \beta_{C}}, J \right) - \delta J_{K_{C}} K_{C} + c^{-1} (Y K_{I}^{\beta_{I}} L_{I}^{1 - \beta_{I}}) J_{K_{C}} + J_{X} X \mu_{X} + \frac{1}{2} J_{XX} X^{2} \sigma_{X}^{2} + \mu_{Y} J_{Y} Y + \frac{1}{2} J_{YY} Y^{2} \sigma_{Y}^{2} \right\}$$

We will look for a guess of the form:

$$J = \frac{(XK_C^{\beta_C})^{1-\gamma}}{1-\gamma} f(\omega) \qquad \omega \equiv \ln Y - \ln K_C$$

Using our guess, The HJB equation becomes

$$0 = \max_{L_{I}} \left\{ \rho \frac{1-\gamma}{1-\theta^{-1}} f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \left(\left(\frac{(1-\beta_{C})(1-L_{I})}{1+\psi-\beta_{C}} \right)^{1-\beta_{C}} \left(1-\frac{(1-\beta_{C})(1-L_{I})}{1+\psi-\beta_{C}} - L_{I} \right)^{\psi} \right)^{1-\theta^{-1}} + c^{-1} \left(e^{\omega} K_{I}^{\beta_{I}} L_{I}^{1-\beta_{I}} \right) \left(\beta_{C} (1-\gamma) f(\omega) - f'(\omega) \right) - u f(\omega) + (\mu_{Y} + \delta\beta_{C}) f'(\omega) + \frac{1}{2} \sigma_{Y}^{2} (f''(\omega) - f'(\omega)) \right\}.$$

where

$$u \equiv \rho \theta \frac{1-\gamma}{\theta-1} - (1-\gamma)(\mu_X - \delta \beta_C) + \frac{1}{2}\sigma_X^2 \gamma(1-\gamma) > 0$$
(39)

I solve the optimization problem numerically, as described in Section 8.4.1. It is straightforward to construct the competitive equilibrium from the solution to the planner's problem. Prices (π, ξ, w) can be recovered from the planner's first order conditions:

$$\begin{aligned} \pi_t &= \exp\left(\int_0^t h_J(C_s, N_s, J_s) \, ds\right) h_C(C_t, N_t, J_t) \\ \xi_t &= \frac{J_K(X_t, Y_t, K_{C,t})}{\pi_t} \frac{1}{c'(i_{C,t})} \\ w_t &= (1 - \beta_C) X_t K_{C,t}^{\beta_C} L_{C,t}^{-\beta_C} \end{aligned}$$

8.2 Proof of Proposition 2

Consider the value of a firm in the C-Sector. The firm buys new capital and hires labor to maximize its value

$$\pi_0 S_0^C = E_0 \int_0^\infty \max_{L_{C,s}, i_{C,s}} \pi_s \left(X_s K_{C,s}^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} - \xi_s c(i_C) K_C \right)$$

$$= E_0 \int_0^\infty \pi_s \left(\beta_C X_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - \xi_s c(i_{C,s}^*) K_{C,s} \right) ds$$

The planner's Lagrangian evaluated at the optimum can be written as:

$$\mathcal{L}_{0} = E_{0} \int_{0}^{\infty} h(C_{s}^{*}, N_{s}^{*}, J_{s}^{*}) - \pi_{s}(C_{s} - X_{s} K_{C,s}^{\beta_{C}} L_{C,s}^{*}^{1-\beta_{C}}) - \xi_{s} \pi_{s} \left(c(i_{C,s}^{*}) K_{C,s} - Y_{s} K_{I}^{\beta_{I}} L_{I}^{*1-\beta_{I}} \right) ds$$

The envelope theorem implies that

$$\frac{\partial \mathcal{L}}{\partial K_C} = \frac{\partial J}{\partial K_C}$$

also note that

$$\frac{\partial K_{C,s}}{\partial K_{C,0}} K_{C,0} = K_{C,s}$$

Therefore

$$\frac{\partial J}{\partial K_{C,0}} K_{C,0} = E_0 \int_0^\infty \pi_s \left(\beta_C X_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - \xi_s c(i_{C,s}^*) K_{C,s} \right) \, ds$$

Similarly, the value of a firm in the investment sector is:

$$\pi_0 S_0^I = E_0 \int_0^\infty \pi_s \left(\xi_s Y_s K_I^{\beta_I} (L_{I,s})^{1-\beta_I} - w_s L_{I,s} \right) \, ds$$

$$\pi_0 S_0^I = E_0 \int_0^\infty \pi_s \left(\xi_s \beta_I Y_s K_I^{\beta_I} (L_{I,s}^*)^{1-\beta_I} \right) \, ds$$

Moreover, because

$$\frac{\partial J_0}{\partial Y_0} Y_0 = E_0 \int_0^\infty \pi_s \left(\xi_s Y_s K_{I,s}^{\beta_I} (L_{I,s}^*)^{1-\beta_I} \right) \, ds$$

this implies

$$\pi_0 S_0^I = \beta_I J_Y Y_0$$

therefore

$$\frac{S_0^I}{S_0^C} = \frac{\beta_I f'(\omega)}{\beta_C (1-\gamma) f(\omega) - f'(\omega)}$$

Also, $\frac{S_t^I}{S_t^C}$ is increasing in ω if

$$\frac{f''}{f'} > \frac{f'}{f}.$$

To see that this is the case, note that the above condition implies that $\frac{f'}{f}$ is strictly decreasing. In the regions where f'' < 0, the above inequality holds because the LHS is positive while the RHS is negative, so we only need to focus on the case where f'' > 0, where both sides are negative. Now let's consider the cases where:

1. case $\frac{f^{\prime\prime\prime}}{f^{\prime\prime}} > \frac{f^{\prime\prime}}{f^{\prime}}$

In this case $\frac{f''}{f'}$ is a decreasing function. Meanwhile, the slope of $\frac{f'}{f}$ depends on whether $\frac{f''}{f'} > \frac{f'}{f}$ or $\frac{f''}{f'} < \frac{f'}{f}$. We can exclude the case where

$$\frac{f^{\prime\prime}}{f^\prime} < \frac{f^\prime}{f}$$

since that would mean that $\frac{f'}{f}$, which asymptotes to 0 as $\omega \to -\infty$ is increasing with ω . Moreover, the two curves cannot cross, since if $\frac{f'}{f}$ is below $\frac{f''}{f'}$ that it is decreasing and if $\frac{f'}{f}$ is above then it is increasing. Thus the only possibility is $\frac{f''}{f'} > \frac{f'}{f}$.

2. case $\frac{f'''}{f''} < \frac{f''}{f'}$

In this case $\frac{f''}{f'}$ is an increasing function. On the other hand, if $\frac{f''}{f'} > \frac{f'}{f}$ then $\frac{f'}{f}$ is decreasing. But we can rule out this case because $\lim_{\omega \to -\infty} \frac{f'}{f} = 0$ and the curves cannot intersect. We can also rule out the other case where $\frac{f''}{f'} > \frac{f'}{f}$, because this would imply that they are both increasing functions but by $\lim_{\omega \to -\infty} \frac{f'}{f} = 0$ and $\frac{f'}{f} \in (1 - \gamma, 0)$ this is not possible.

From the above, the only possibility then is that $\frac{f''}{f'} > \frac{f'}{f}$.

8.3 Proof of Proposition 3

Consider the value of a firm in the C-sector that plans to not invest in the future

$$\pi_0 S_{C,0}^V = E_0 \int_0^\infty \pi_s \left(X_s (K_{C,0} e^{-\delta s})^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} \right) \, ds$$

its labor decision yields

$$(1 - \beta_C) X_s (K_{C,0} e^{-\delta s})^{\beta_C} L_{C,s}^{1 - \beta_C} = w_s L_{C,s}$$

Let $\Lambda_{s,t} = \exp(\int_t^s i_{C,u}^* du)$. Now consider a firm who follows the optimal investment policy, its first order condition is

$$(1 - \beta_C) X_s K_{C,s}{}^{\beta_C} L_{C,s}^*{}^{1 - \beta_C} = w_s L_{C,s}^*$$

dividing through, this yields

$$L_{C,s} = L_{C,s}^* \Lambda_{s,0}^{-1}$$

which implies

$$\begin{split} \pi_0 S_0^V &= E_0 \int_0^\infty \pi_s X_s \, K_{C,0}^{\beta_C} e^{-\delta\beta_C \, s} \hat{L}_{C,0}^{1-\beta_C} \, ds \\ \pi_0 S_0^V &= E_0 \int_0^\infty \pi_s \beta_C X_s \, K_{C,s}^{\beta_C} L_{C,0}^{1-\beta_C} \Lambda_{s,0}^{-1} \, ds \\ \pi_0 S_0^V &= E_0 \int_0^\infty \exp\left(\int_0^s h_J(C,N,J) - i_{C,u} \, du\right) \beta_C h_C(C,N,J) C ds \\ &= \frac{X_0^{1-\gamma} K_0^{\beta_C(1-\gamma)}}{1-\gamma} \times \\ E_0 \int_0^\infty \exp\left(\int_0^s \hat{\rho}_u du\right) \rho \beta_C (1-\gamma) L_{C,s}^{(1-\beta_C)(1-\theta^{-1})} N_s^{\psi(1-\theta^{-1})} f(\omega_s)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \, ds \\ &= \frac{X_0^{1-\gamma} K_{C,0}^{\beta_C(1-\gamma)}}{1-\gamma} g(\omega_0) \end{split}$$

The Feynman-Kac theorem implies that $g(\omega_t)$ can be computed as the solution to the ODE:

$$0 = \rho\beta_C(1-\gamma)L_C(\omega)^{(1-\beta_C)(1-\theta^{-1})}N(\omega)^{\psi(1-\theta^{-1})}f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} + \hat{\rho}(\omega)g(\omega) + \mathcal{D}_{\omega}g(\omega)$$

where

$$\hat{\rho}(\omega_u) = h_J(C_u, N_u, J_u) + (\beta_C(1-\gamma) - 1)i_{C,u} - \delta\beta_C(1-\gamma) + (1-\gamma)\left(\mu_X - \frac{1}{2}\sigma_X^2\right) + \frac{1}{2}\left(1-\gamma\right)^2 \sigma_X^2$$

8.4 Numerical Solution

8.4.1 Markov-Chain Approximation

The solution method closely follows Kushner and Dupuis (1993). For exposition purposes consider the case where $\gamma = \theta^{-1}$ and $\psi = 0$. In this case the HJB equation becomes:

$$0 = \min_{l} \left\{ (1-l)^{(1-\gamma)(1-\beta_{C})} - (u+\beta_{C}(\gamma-1)c^{-1}(e^{\omega}l^{1-\beta_{I}}))f(\omega) + f'(\omega)(\mu_{Y}+\delta - \frac{1}{2}\sigma_{Y}^{2} - c^{-1}(e^{\omega}l^{1-\beta_{I}})) + \frac{1}{2}\sigma_{Y}^{2}f''(\omega) \right\}$$

and ω follows

$$d\omega = \left(\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega}l^{*1-\beta_I})\right)dt + \sigma_Y dZ_t^Y$$

I discretize the state space, creating a 1200 point grid for ω and f with $h=\Delta\omega$. Then the following approximations can be used

$$f'(\omega_n) \approx \frac{f_{n+1} - f_{n-1}}{2h}$$
 and $f''(\omega_n) \approx \frac{f_{n+1} + f_{n-1} - f_n}{h^2}$.

I then approximate the HJB equation as

$$f_n = \min_l \left\{ e^{-\beta(\omega_n; l)\Delta t^h} \left[p_-(\omega_n; l) f_{n-1} + p_+(\omega_n; l) f_{n+1} \right] + (1-l)^{(1-\gamma)(1-\beta_C)} \Delta t^h \right\}$$
(40)

where

$$\beta(\omega_n; l) = u + \beta_C(\gamma - 1)c^{-1} e^{\omega_n} l^{1 - \beta_I} \qquad p_-(\omega_n; l) = \frac{1}{2} + h \frac{\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega_n} l^{1 - \beta_I})}{2\sigma_Y^2} \\ p_+(\omega_n; l) = \frac{1}{2} - h \frac{\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega_n} l^{1 - \beta_I})}{2\sigma_Y^2} \quad \Delta t^h = \frac{h^2}{\sigma_Y^2}$$

and I have used the approximation $\frac{1}{1+\beta(\omega_n;l)\Delta t^h} \approx e^{-\beta(\omega_n;l)\Delta t^h}$. This corresponds to an Markov Chain approximation to ω , where

$$p(\omega = \omega_n + h|\omega = \omega_n) = p_+(\omega_n; l)$$
 and $p(\omega = \omega_n - h|\omega = \omega_n) = p_-(\omega_n; l)$

are the transition probabilities and the time interval is Δt^h . The markov chain is locally consistent because

$$E(\Delta\omega_n|\omega_n) = (\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega_n}l^{1-\beta_I}))\Delta t^h$$
$$E((\Delta\omega_n - E\Delta\omega_n)^2|\omega_n) = \sigma_Y^2\Delta t^h + o(\Delta t^2)$$

Note that care must be taken when choosing h to ensure that the probabilities are non-negative for all admissible controls $l \in [0, 1]$ at all points in the grid. Alternative differencing schemes that produce positive probabilities can also be used.

Using an initial guess for f, say f^i , one can numerically compute the minimum in(40). Then, given l_n^{*i} , one can start from n = 0 and recursively compute the update on f using the Gauss-Seidel algorithm:

$$f_n^{i+1} = e^{-\beta(\omega_n; l_n^{*\,i})\Delta t^h} \left[p_-(\omega_n; l_n^{*\,i}) f_{n-1}^{i+1} + p_+(\omega_n; l_n^{*\,i}) f_{n+1}^i \right] + (1 - l_n^{*\,i})^{(1-\gamma)(1-\beta_C)} \Delta t^h$$
(41)

I impose a reflecting barrier on ω at the boundaries of the grid. This reduces to $f_0 = f_1$ and $f_N = f_{N-1}$, since there is no discounting at the boundary and

$$p(\omega = \omega_0 + h|\omega = \omega_0) = 1$$
$$p(\omega = \omega_N - h|\omega = \omega_N) = 1$$
$$\Delta t^h(\omega_N) = \Delta t^h(\omega_0) = 0$$

Finally, because the minimum in (40) is costly to compute, I iterate a couple of times on (41) before updating the policy function.

8.4.2 Computing unconditional moments

I compute unconditional moments in two ways. The first way yields exact computation of the instantaneous moments of the model. The initial step is to compute the conditional moments M_t from the solution of the model and its higher derivatives. The second step involves the computation of the stationary distribution of ω . The invariant distribution, $p(\omega)$, is the solution to the Kolmogoroff Forward equation:

$$0 = -(\mu_Y - \frac{1}{2}\sigma_Y^2 + \delta - i_C(\omega))p'(\omega) + i'_C(\omega)p(\omega) + \frac{1}{2}\sigma_Y^2 p''(\omega)$$

subject to

$$\int_{-\infty}^{\infty} p(\omega) d\omega = 1$$

The last step involves computing the unconditional moments $\overline{M} = E(M(\omega))$ as

$$\overline{M} = \int M(\omega) p(\omega) d\omega$$

In general, these are very different than the conditional moments evaluated at the mean state of nature, i.e. $M(E(\omega))$, which is what one obtains when log-linearizing the model around $E(\omega)$. In particular, the risk premia are often off by a factor of two and three. This point is also raised by Campanale, Castro and Clementi (2007). This method yields exact estimates of the moments instantaneous implied by the model. However, since one typically estimates the moments of macroeconomic variables using lower frequency data, I also simulate 100,000 samples of the model, each of length 50 years, and compute the median and the 5% and 95% percentile estimated moment.

8.5 Data

8.5.1 IMC

I use the 1997 BEA Standard Make and Use Tables at the detailed level. I use the standard make (table 1) and use (table 2) tables. The uses tables enumerates the contribution of each IO commodity code to Personal Consumption Expenditures (IO code F01000) and Gross Private Fixed Investment (IO code F02000). I use the make tables along with the NAICS-IO map to construct a mapping between 6-digit NAICS Codes to IO commodity codes. I then use the uses table to create a map from IO codes to Investment or Consumption. Because some industries contribute to both PCE and GPFI, I follow two schemes to create a unique link. The first assigns industries to the sector they contribute the most value in terms of producer's prices excluding transportation costs. The second scheme classifies industries as consumption or investment if they contribute only in one sector.

I use COMPUSTAT to create a PERMNO-NAICS link and form value weighted portfolios using simple returns on all common stocks traded on NYSE, AMEX and Nasdaq. I construct two portfolios, using the first (IMC) and the second (IMCX) classification scheme. Examples of Investment industries are

IO Code	Description
213111	Drilling oil and gas wells
333111	Farm machinery and equipment manufacturing
333295	Semiconductor machinery manufacturing
334111	Electronic computer manufacturing
334220	Broadcast and wireless communications equipment
336120	Heavy duty truck manufacturing

Examples of Consumption industries are

IO Code	Description
1111B0	Grain farming
221100	Power generation and supply
311410	Frozen food manufacturing
312110	Soft drink and ice manufacturing
325611	Soap and other detergent manufacturing
334300	Audio and video equipment manufacturing

The full list of IO codes and their assignments into industries is available from the author upon request.

8.5.2 10 IMC sorted portfolios

The portfolios are formed from ordinary common equity, traded in NYSE, AMEX, or Nasdaq, in the CRSP Monthly Stock Database. For every stock, I estimate pre-ranking covariances on IMC based on a 5-year window using weekly log excess returns. I sort stocks into IMC covariance deciles. I construct value-weighted portfolios using simple monthly returns, tracking the portfolios for 5 years until next re-balancing.

8.5.3 24 IND/IMC sorted portfolios

The portfolios are formed from ordinary common equity, traded in NYSE, AMEX, or Nasdaq, in the CRSP Monthly Stock Database. For every stock, I estimate pre-ranking covariances on IMC based on a 5-year window using weekly log excess returns. Stocks are sorted into eight industries based on their two-digit SIC codes: (1) nondurables manufacturing, (2) durables manufacturing, (3) other manufacturing, (4) nondurables retail, (5) durables retail, (6) services, (7) finance, and (8) natural resource. Within each industry, stocks are then sorted into three portfolios based on their IMC covariance using breakpoints of 30th and 70th percentiles. I construct value-weighted portfolios using simple monthly returns, tracking the portfolios for 5 years until next re-balancing.

8.5.4 25 BM/IMC sorted portfolios

The portfolios are formed from ordinary common equity, traded in NYSE, AMEX, or Nasdaq, in the CRSP Monthly Stock Database. For every stock, I estimate pre-ranking covariances on IMC based on a 5-year window using weekly log excess returns. Using book to market equity from COMPUSTAT and the pre-ranking covariances, including only stocks that have full observations, I first sort stocks into BM quintiles using NYSE breakpoints and then into IMC covariance quintiles. Portfolios are formed in June every 5 years. I construct value-weighted portfolios using simple monthly returns, tracking the portfolios for 5 years until next re-balancing.

8.5.5 25 ME/BM portfolios

The data come from Kenneth French's web page.

8.5.6 Macroeconomic Data

Monthly data come from the website of the St Louis Fed. Quarterly quantity data and price deflators come from the website of the Bureau of Economic Analysis, specifically from the NIPA tables 5.3.3, 5.3.4, 2.3.3, 1.1.3 and 1.1.4.

8.6 Empirical Methodology

8.6.1 GMM Cross-Sectional tests

I estimate the linear factor model using two-step GMM. As Cochrane (2001) illustrates, if one is interested in estimating a linear model for the SDF of the following form:

$$m = a - bF$$

One can test the moment restriction

$$E[mR^e] = 0. (42)$$

Letting the vector of unknown parameters $\theta = [b, \mu_F]$ and the data $X_t = [R_t^e, F_t]$, where F is the factors and R^e are excess returns. If one is using excess returns, then the mean of the pricing kernel is unidentified. If the model is correct, then (42) must hold at the true parameter values:

$$E[g(X,\theta_0)] = 0$$

where

$$g(X,\theta) = \left[\begin{array}{c} R_t^e - R_t^e (F_t - \mu_F)'b\\ F_t - \mu_F \end{array}\right]$$

I use the first stage weighting matrix

$$W = \left[\begin{array}{cc} kI_N & 0\\ 0 & \hat{\Sigma}_{ff}^{-1} \end{array} \right]$$

Following Gomes, Kogan and Yogo (2006) I pick $k = \det(\hat{\Sigma}_R)^{-1/N}$. The first stage weighting matrix puts equal weight in each of the N asset pricing restrictions. I compute the spectral density matrix for the second stage using the Newey-West estimator with 3 lags of returns.

I use the covariance rather than the beta representation because I am interested in the marginal ability of IMC to price each cross-section. As Cochrane 2001 illustrates, one can test whether a factor is priced, given the other factors in the specification by $b \neq 0$.

8.6.2 Fama-Mcbeth Cross-Sectional tests

I run Fama and MacBeth (1973) cross-sectional regressions of simple monthly returns of all NYSE, AMEX, and Nasdaq stocks on covariances and characteristics. Covariances are estimated using a procedure similar to Fama and French (1992). Specifically, for each individual stock I estimate the covariance of its returns with IMC using 5 years of weekly log excess returns. At the end of a five year period, stocks are then sorted into 100 pre-ranking covariance centiles. I then compute the equal-weighted monthly log excess returns on these 100 portfolios over the next 5 years. This procedure is repeated every 5 years, forming a time-series of returns on these 100 portfolios. I then re-estimate covariances for the portfolios formed from the pre-ranking sorts using 5 years of monthly data to obtain post-ranking covariances. The post-ranking covariance estimate for a given portfolio is then assigned to each stock in the portfolio. Portfolio assignments are updated every 5 years. Every month the cross-section of stock returns in excess of risk free rate is then regressed on a constant, the covariance with the excess return on the CRSP value-weighted index, the covariance with the return on the IMC portfolio, the log of market capitalization (ME) on December of year t-1 and the log of Book to Market (BM) of year t-1.