Debt, Labor Markets and the Creation and Destruction of Firms

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Abstract

We analyze the financing and liquidation decisions of firms that face a labor market with search. In our model, debt facilitates the process of creative destruction (i.e., the elimination of inefficient firms and the creation of new firms) but may induce excessive liquidation and unemployment; in particular, during economic downturns. Within this setting we examine the role of monetary policy, which can reduce debt burdens during economy-wide downturns, and tax policy, which can influence the incentives of firms to use debt financing.

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1 Introduction

Economic forecasters and policymakers have long recognized that financial structure at the corporate and household level can influence macro-economic conditions. The most recent economic crisis, which was triggered in part by the substantial leverage in the real estate and banking sectors, is perhaps the most visceral illustration of this point. While financial economists have responded to this crisis with a plethora of work that examines policy issues that relate to the leverage of financial institutions, the more general issue of the interaction between corporate financing choices and macro policy has received scant attention.\footnote{See for example Brunnermeier (2009).}

To examine the interaction between corporate financing choices and macro policy we combine a corporate finance model in which debt plays a fundamental role with a model from the macro/search literature where production requires a match between workers and firms. Specifically, we follow Hart and Moore (1995) and assume that debt is chosen by investors to indirectly control managers who enjoy private control benefits and incorporate this into a macro labor search model along the lines of Pissarides (2000). The model explores how capital structure, and its effect on liquidation, affects the tightness of labor markets in both booms and recessions, and how these effects can in turn affect the emergence of new firms.

Within the context of this model we consider a number of policy issues. For example, U.S. tax policy tends to subsidize debt financing. Under what conditions is this good or bad?\footnote{The issue of the desirability of debt subsidies has been periodically raised. For instance during the Clinton and Bush administrations, the Congressional Budget Office (1997, 2005) considered proposals to eliminate the unequal treatment of debt and equity.} We also consider monetary policy, which affects the overall price level and thus influences the real value of a firm’s nominal debt obligations. Hence, it is natural to ask how expectations about monetary policy influence the capital structure choices of firms, and through this channel, how monetary policy affects the liquidation of old firms and the creation of new firms.

We start with the simplest form of our model that includes a fixed number of existing firms and an unlimited number of ex-ante identical firms that can enter the market. As
we show, in this setting there is no externality associated with debt financing, so the optimal subsidy or tax on debt is zero. However, even within the context of this simple model there can be an important role for policies that influence firms’ liquidation choices. Specifically, by generating inflation, a loose monetary policy can reduce the real value of debt during economy wide downturns and, as a result, reduce bankruptcies in bad times, when liquidations tend to be costly. In addition, since such a policy leads to higher ex-ante debt ratios, it increases bankruptcies in good times, when there would otherwise be too few liquidations.

We next consider a setting with a fixed number of entrants that may earn rents. When this is the case, there are externalities associated with firm liquidations as well as with their capital structure choices. These include negative externalities imposed on unemployed workers in the event of liquidation (i.e., liquidation causes the unemployed to have more workers to compete with for jobs), as well as positive externalities that benefit emerging new firms that need to hire labor. Depending on the magnitude of these two effects a social planner may want to use tax policy to tilt firms towards either more or less debt financing.

In addition to Hart and Moore (1995) and Pissarides (2000), which provide the basis for our model, our analysis is related to a number of papers in the literature. These include theories that consider potential negative spillovers created by debt financing. For instance, bankruptcy induced fire-sales (as discussed in Shleifer and Vishny 1992 and more recently Lorenzoni 2008) which impose negative externalities of other firms by affecting their collateral constraints (Kiyotaki and Moore 1997). In addition, our analysis of positive externalities of liquidations is related to Schumpeter’s (1939) ideas on creative destruction, and to more recent work by Kashyap et al.(2008), which examines the inability of Japanese banks to shut down failing firms in the 1990s. Finally, a contemporaneous paper by He and Matvos (2012) consider a case where debt facilitates firm exit when companies compete for survival in a declining industry and concludes that firms use less than the socially optimal amount of debt financing. To our knowledge, however, we are the first to consider the influence of debt in an economy where externalities can be imposed on workers as well as emerging new firms, and therefore the first to analyze
the effects of debt as well as the effect of policies that influence the real value of debt obligations, on the process of firm creation.

The rest of the paper is organized as follows. Section 2 presents the base model and Section 3 analyzes it. Section 4 considers the policy implications that emanate from the base model and Section 5 presents a modification of the base model and revisits the policy implications. Section 6 discusses the main conclusions. Proofs and other technical derivations are related to the appendix.

2 The model

We consider a risk-neutral economy in which the discount rate is normalized to zero. The economy consists of two productive periods \( t = 1, 2 \) and an interim period in which existing firms can be liquidated and new firms can be created. Next, we describe the agents, technology, contracting environment and labor market.

2.1 Agents: workers and two generations of firms

Old generation firms

The economy starts in the first productive period, \( t = 1 \), with a continuum of size one of workers that are each employed by a firm \( i \). These old generation firms produce in period 1 and may retain their workers and produce in period 2. If the firm fails to retain its worker (i.e., is unable to pay the worker his outside option) the firm is liquidated and does not produce in period 2.

When active in period \( t \), an old generation firm \( i \) produces a cash-flow of \( r_{it} \) that can be decomposed as follows:

\[
r_{it} = s_t + \varepsilon_i. \tag{1}
\]

The first component \( s_t \) is an aggregate productivity shock that is common to all firms in the economy. Formally \( s_t \) is an element of a sequence of two binomial variables which are positively correlated across time, where

\[
s_1 = \begin{cases} s_h & \text{with prob. } p \\ s_l & \text{with prob. } 1 - p \end{cases} \tag{2}
\]
and
\[ \Pr(s_2 = s_h|s_1 = s_l) = p - \varphi \leq \Pr(s_2 = s_h|s_1 = s_h) = p + \varphi. \] (3)

We assume that \( \varphi \leq \min\{1 - p, p\} \), which is required for the probabilities to be well defined.

The second cash-flow component \( \varepsilon_i \) is firm-specific, independent across firms, constant over time and drawn from a uniform distribution:
\[ \varepsilon_i \sim U[-\bar{\varepsilon}, +\bar{\varepsilon}]. \] (4)

**New generation firms**

*New generation firms* can enter the economy in the interim period, between production periods 1 and 2. In particular, there is an unlimited number of ex-ante identical potential entrants that can enter the economy by paying a fixed entry cost \( k > 0 \).\(^3\) This assumption implies that new generation firms enter the economy until their expected profits are zero.

After entering the economy, a new generation firm \( j \) needs to hire a worker to be productive. However, as discussed below, there are search frictions that may prevent these firms from finding a suitable worker. If a firm \( j \) succeeds in hiring a worker, it generates a cash-flow \( r_{j2} \) at the end of period 2, however, if it fails to hire a worker, firm \( j \) loses its investment \( k \) and liquidates. Analogous to old generation firms, the cash-flow of an active new generation firm \( j \) in period 2 is
\[ r_{j2} = s_2 + \varepsilon_j \]
where \( s_2 \) is the aggregate productivity shock in period 2 and \( \varepsilon_j \) is a uniformly distributed firm-specific component independent across firms, i.e., \( \varepsilon_j \sim U[-\bar{\varepsilon}, +\bar{\varepsilon}] \).

**2.2 Contracting environment**

Firms are initially controlled by *investors* who subsequently transfer control to *managers*. While this transfer of control is innocuous for new generation firms, as we show, it has important consequences for old generation firms. Specifically, we assume that managers

\(^3\)In Section 5 we consider the alternative case when there is a limited number of ex-ante identical potential entrants.
of old generation firms enjoy private benefits of control and, following Hart and Moore (1995), that because of the private benefits, an old generation firm continues to operate in period 2 as long as the manager has access to the necessary funds to retain the firm’s worker. For simplicity, we assume that any funds available beyond those needed to retain the worker are paid out to the investors.

At the beginning of period 1, investors, before transferring control to the firm’s manager, set the firm’s capital structure. In particular, we assume that old generation firm $i$ issues short-term debt with a face value $d_i \geq 0$ that matures at the end of period 1, just after the cash-flow $r_{i1}$ is realized. We assume that short-term debt is a “hard claim” which cannot be renegotiated with creditors and, because of this, the firm is forced to liquidate if it fails to meet its payment $d_i$. The firm can repay its short-term debt either from the period 1 cash-flow, $r_{i1}$, or by borrowing funds against the period 2 cash-flow, $r_{i2}$. We exclude any other financial contract and in particular, we assume that debt cannot be made contingent on specific cash-flow components $\{\varepsilon_i, s_i\}$, which we assume are not verifiable.

### 2.3 Labor market

To finish the description of the economy, we need to describe the labor market that allocates workers to firms during the interim period.

#### 2.3.1 Labor market and search costs

We consider a labor market with search frictions, a framework that captures the fact that it is costly for firms and workers to find a suitable match (e.g., Pissarides 2000). A setting like this is described by two main features: (i) a matching technology, which describes the likelihood of a suitable match and (ii) a sharing rule, which indicates how the matched parties share the surplus created by the newly formed relationship. The matching technology is described by the following constant returns-to-scale Cobb-Douglas
matching function:\(^4\)

$$m(a, v) = \lambda a^\alpha v^{(1-\alpha)} \quad (5)$$

where \(m\), the number of matches, is determined by \(a\), the number of workers looking for jobs, and \(v\), the number of firms searching for workers. In this function, \(0 < \alpha < 1\) is the elasticity of matches to workers seeking jobs and \(\lambda > 0\) measures the efficiency of the matching technology.

Given this matching technology, if the ratio of firms to workers is \(\theta \equiv \frac{v}{a}\), then each worker is hired with probability \(q(\theta) \equiv m(1, \theta)\), and each firm hires a worker with probability \(\frac{q(\theta)}{\theta}\).\(^5\) For future reference, we follow the literature and refer to \(\theta\) as the “labor market tightness”. Notice that some firms and workers do not find a suitable match, that is, some firms fail to hire while some workers remain unemployed. Unemployment \(u\) refers to the mass of workers that look for a job during the interim period but cannot find one, i.e., \(u = a - m(a, v)\).

When there is a match between a firm and worker, the surplus they create is allocated with a sharing rule. In particular, the workers in new generation firms receive a wage

$$w_2 = \gamma + \beta E(r_2|s_1) \quad (6)$$

where \(\gamma \geq 0\) and \(\beta \in [0, 1]\), implying that after paying wages, the new generation firm’s expected profit equals,

$$E(\pi_2) = (1 - \beta)E(r_2|s_1) - \gamma. \quad (7)$$

We assume that \(k < E(\pi_2)\), which guarantees that new generation firms enter the market during the interim period. Notice that this specification encompasses the case in which \(\beta = 0\), where workers receive a constant wage, as well as the case in which \(\gamma = 0\), where workers and firms bargain over the surplus generated by their relation, with \(\beta\) being the

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\(^4\)Petrongolo and Pissarides (2001) justify the use of a Cobb-Douglas function with constant returns to scale on the basis of its success in empirical studies.

\(^5\)Since \(m(a, v)\) features constant returns-to-scale, it follows that \(q(\theta) \equiv \frac{m(a,v)}{a} = m(1, \theta)\) and \(\frac{q(\theta)}{\theta} \equiv \frac{m(a,v)}{v} = m\left(\frac{1}{\theta}, 1\right)\). Also, we assume an interior solution which requires us to impose parametric constraints on \(\lambda\) (that is, that \(\lambda\) is small enough) such that, in equilibrium, probabilities are well defined, i.e., \(m(a, v) < \min\{a, v\}\).
workers’ bargaining power. This formulation assumes that wages are paid before the realization of $r_2$, which is without loss of generality since agents are risk neutral in the economy.

2.3.2 Workers’ retention costs

We assume that old generation firms hire and pay their period 1 workers prior to period 1.\footnote{This is done for simplicity. We also considered the case in which old generation firms also face a labor market with search costs and found qualitatively similar results.} However, to produce in period 2, an old generation firm must pay the worker’s outside option, $U$, which is his expected compensation if he quits his job and searches for an alternative job during the interim period,

$$U = q(\theta)w_2. \quad (8)$$

As discussed below, this assumption simplifies the investor’s design of the optimal capital structure of the old generation firm.\footnote{In particular this implies that the debt choices in period 1 cannot be used to extract rents from the workers in period 2.}

2.4 Timing of events

There are two production periods and an interim period with the following relevant events in each.

**Period 1 ($t = 1$):** A measure of size 1 of old generation firms employ one worker each. At the beginning of the period, each firm $i$ issues short-term debt $d_i$, and then transfers the control of its operations to a manager. At the end of the period, firm $i$ produces a cash-flow $r_{i1}$, and its short-term debt $d_i$ matures.

**Interim period:** Managers of old firms make their liquidation decisions and new generation firms invest $k$ to enter the market.

**Period 2 ($t = 2$):** Each newly created firm $j$ attempts to hire an unemployed worker. If a firm and a worker match, the firm becomes active. Old firms that are not liquidated and newly created active firms generate cash-flows \{r\_{i2}\} and \{r\_{j2}\} respectively.

The following time-line summarizes the relevant events:
3 Analysis of the model

The analysis of the model proceeds by backward induction. We start in Section 3.1 by characterizing the labor market during the interim period assuming an exogenous number of (unemployed) workers looking for jobs, i.e., the labor supply, and endogenize the labor demand, i.e., the number of firms created. In Section 3.2, we consider the firms’ liquidation decisions in the interim period as a function of the debt choices in \( t = 1 \) and the number of new firms created. Finally, in Section 3.3, we study the choice of debt by old generation firms in \( t = 1 \).

3.1 Labor market and the creation of new generation firms

We start by examining the creation of new firms in the interim period. For a given labor supply \( a(s_1) \), new firms will enter the market until the expected profit from entering \( V(s_1) \) is zero. Expressed in terms of market tightness, i.e., \( \theta_1 \equiv \theta(s_1) = \frac{v(s_1)}{a(s_1)} \), where \( v(s_1) \) is the number of new firms entering the market, the expected profit of new firms is given by:

\[
V(s_1) = -k + \frac{q(\theta_1)}{\theta_1} \left[(1 - \beta)E(r_2|s_1) - \gamma\right],
\]

where \( \frac{q(\theta_1)}{\theta_1} \) is the probability of finding a worker, and \((1 - \beta)E(r_2|s_1) - \gamma\) is the expected cash-flow net of the worker’s salary (i.e., \( E(r_2|s_1) - w_2(s_1) \)). Setting \( V(s_1) = 0 \) and under the specific Cobb-Douglas matching function (5) we have the following lemma:

**Lemma 1** Workers face a labor market tightness \( \theta^*(s_1) = \left(\frac{\lambda[(1-\beta)E(r_2|s_1) - \gamma]}{k}\right)^{1/\alpha} \) and have a reservation utility

\[
U^*(s_1) = \lambda \theta^*(s_1)^{(1-\alpha)}(\gamma + \beta E(s_2|s_1)).
\]
From the previous lemma a number of observations follow. First, the market tightness \( \theta^*(s_1) \) increases with the efficiency of the matching technology \( \lambda \), and the expected surplus generated by a match \( E(s_2|s_1) \), and decreases with the worker’s share of this surplus (i.e., decreases in both \( \beta \) and \( \gamma \)), the fixed cost of creating the firm \( k \), and the elasticity of the number of matches to the labor supply \( \alpha \). Second, workers’ reservation utility \( U^*(s_1) \) increases with matching efficiency \( \lambda \), and the expected surplus \( E(s_2|s_1) \), and decreases with the cost of entry \( k \) and the match elasticity to labor supply \( \alpha \). The effect of the worker’s compensation, however, is ambiguous and will play an important role when we consider the policy implications below.

Third, it is worth noting that since aggregate shocks are positive serially correlated, i.e., \( E(r_2|s_1 = s_h) \geq E(r_2|s_1 = s_l) \), there are intertemporal effects on \( w_2^*(s_1) \), \( U^*(s_1) \), and \( \theta^*(s_1) \). In particular, a positive aggregate shock in the first period (i.e., \( s_1 = s_h \)) leads to an increase in wages, workers’ reservation utility, and market tightness, that is, \( w_2^*(s_h) \geq w_2^*(s_l) \), \( U^*(s_h) \geq U^*(s_l) \), and \( \theta^*(s_h) \geq \theta^*(s_l) \).

Finally it is worth noting that Lemma 1 also establishes that \( \theta^*(s_1) \) and \( U^*(s_1) \) are independent of the labor supply \( a^*(s_1) \). Specifically, any labor supply effects \( a(s_1) \) are offset in equilibrium by adjustments in firm entry \( v(s_1) \) until the market tightness \( \theta^*(s_1) \) reaches its equilibrium level. This effect simplifies the analysis by isolating any supply of labor effects on the labor market conditions. In other words, since all newly created firms are ex-ante identical, shocks to the labor supply are fully accommodated by a perfectly elastic labor demand. This, in turn, implies that firm exits have no effect in market tightness or workers’ compensation.

### 3.2 Liquidation decisions of old generation firms

So far we have characterized the labor market conditions in terms of \( \theta^*(s_1) \) and \( U^*(s_1) \) for a given labor supply \( a^*(s_1) \). The labor supply corresponds to the number of workers employed at \( t = 1 \) by those firms that are liquidated, \( l^*(s_1) \):

\[
a^*(s_1) = l^*(s_1).
\]  

(11)

Hence determining \( a^*(s_1) \), requires us to characterize the liquidation decision of the
old generation firms. In our setting, managers of the old generation firms enjoy private
effects of control and choose to liquidate their firms only when they are unable to retain
their workers. Worker retention requires firms to pay the workers’ outside option, \( U^*(s_1) \)
with either internally generated or borrowed funds. Formally, since an old generation
firm  \( i \) generates a period 1 cash-flow  \( r_{1i} = s_1 + \varepsilon_i \) and expects to generate a period
2 cash-flow  \( E(r_{2i}|r_{1i}, s_1) = E(s_2|s_1) + \varepsilon_i \), the manager liquidates the firm’s operations when:\(^8\)

\[
G(\varepsilon_i, s_1, d_i) \equiv 2\varepsilon_i + s_1 + E(s_2|s_1) - d_i - U^*(s_1) < 0. \tag{12}
\]

Hence, for a certain amount of debt, \( d_i \), and an aggregate shock, \( s_1 \), the firm is liquidated
when its idiosyncratic shock is smaller than \( \varepsilon^*_d, \) where \( \varepsilon^*_d \) is such that
\( G(\varepsilon^*_d, s_1, d_i) = 0. \) This condition can be expressed as:

\[
\varepsilon^*_d = \frac{1}{2}[d_i - s_1 - E(s_2|s_1) + U^*(s_1)]. \tag{13}
\]

Given \( \varepsilon^*_d \) and the distributional assumptions on \( \varepsilon_i \), the probability that a firm \( i \) with
debt \( d_i \) is liquidated after period 1 is

\[
Pr(\varepsilon_i < \varepsilon^*_d) = \frac{d_i - s_1 - E(s_2|s_1) + U^*(s_1) + 2\varepsilon}{4\varepsilon}, \tag{14}
\]

and has the following properties:

**Proposition 1** The probability of liquidation increases if the firm employs more debt \( d_i \)
in its capital structure and, for a given realization of the aggregate shock, with workers
reservation utility in period 2, \( U^*(s_1) \).

Intuitively, managers liquidate their firms when they cannot raise the necessary funds
to retain their workers, which is more likely to occur when firms have more debt and
when the workers’ reservation utility is high.\(^9\)

\(^8\)The firm pays the initial wage to the worker at the beginning of period 1.

\(^9\)Notice that while a negative productivity shock, \( s_1 = s_l \), decreases the workers outside option,
that is, \( U^*_2(s_l) \leq U^*_2(s_h) \), it also decreases the amount of funds that a firm can raise against its future
cash-flow, \( r_{i2} \).
3.3 Debt choice of old generation firms

To close the model we need to characterize the optimal capital structure, \( i.e. \), the choice of debt, \( d_i \), made by investors to maximize firm value. To determine the optimal amount of debt, firm \( i \)'s investors solve the following problem:

\[
\max_{d_i} p \left[ \int_{\varepsilon^{s_h}_{d_i}} \frac{\varepsilon_i + E(s_2|s_h) - U^*(s_h)}{2\varepsilon} d\varepsilon_i + (1-p) \int_{\varepsilon^{s_l}_{d_i}} \frac{\varepsilon_i + E(s_2|s_l) - U^*(s_l)}{2\varepsilon} d\varepsilon_i \right]
\]

where \( \varepsilon^{s_h}_{d_i} \) and \( \varepsilon^{s_l}_{d_i} \) correspond to \( \varepsilon^{s_1}_{d_i} \) when \( s_1 = s_h \) and \( s_1 = s_l \), respectively.

In the objective function (15), the first and second terms are the expected profits in period 2 when \( s_1 = s_h \) and \( s_1 = s_l \), respectively. These profits are affected by the amount of debt because debt determines when the firm is liquidated, \( i.e. \), it changes the liquidation cut-offs \( \varepsilon^{s_h}_{d_i} \) and \( \varepsilon^{s_l}_{d_i} \).

Problem (15) yields the following f.o.c.,

\[
\frac{1}{4\varepsilon}[p(\varepsilon^{s_h}_{d_i} + E(s_2|s_h) - U^*(s_h)) + (1-p)(\varepsilon^{s_l}_{d_i} + E(s_2|s_l) - U^*(s_l)))] = 0,
\]

which, using the definition of \( \varepsilon^{s_1}_{d_i} \) in (13) and the fact that old generation firms (which are ex-ante identical) choose the same amount of debt (i.e., \( d^* = d_i^* \) for all \( i \)), can be rewritten as\(^{10}\)

\[
d^* = E(s_1) - E[E(s_2|s_1) - U^*(s_1)]
\]

where

\[
U^*(s_1) = \lambda \left[ \frac{(1 - \beta)E(s_2|s_1) - \gamma}{k} \right]^{\frac{1-\alpha}{\alpha}} (\gamma + \beta E(s_2|s_1)).
\]

As the above equation illustrates, the optimal debt level \( d^* \) increases with the expected cash-flow in period 1, (a higher expected cash-flow increases the severity of the managerial free cash-flow problem), and decreases with the expected value of the firm in period 2, (which is related to the severity of the debt overhang problem created by the issuance of debt).\(^{11}\)

\(^{10}\)This requires a parametric condition on the distribution of shocks to ensure \( d > 0 \).

\(^{11}\)Since, for any given amount of debt \( d_i \) and shock \( s_1 \), an increase in \( E(s_2|s_1) - U^*_2(s_1) \) decreases \( \varepsilon^{s_1}_{d_i} \) by only \( \frac{1}{2} [E(s_2|s_1) - U^*_2(s_1)] \), the value of the marginal firm liquidated \( i.e. \), \( \varepsilon^{s_1}_{d_i} + E(s_2|s_1) - U^*_2(s_1) \) increases by \( \frac{1}{2} [E(s_2|s_1) - U^*_2(s_1)] \).
Intuitively, the optimal amount of debt $d^*$ is chosen so that the marginal firm liquidated has an expected value of zero, that is, $d^*$ solves

$$E[\epsilon_d^{s1} + E(s_2|s_1) - U^*(s_1)] = 0. \quad (17)$$

In other words, the optimal debt choice $d^*$ equates $1 - p$ times the value lost from liquidating the (profitable) marginal firm when $s_1 = s_l$ to $p$ times the value saved from liquidating the (unprofitable) marginal firm when $s_1 = s_h$.

Finally, notice that $d^*$ determines $\epsilon_d^{s1}$, which, in turn, determines the number of workers whose employers are liquidated at the end of period:

$$l^*(s_1) = \Pr(\epsilon_i < \epsilon_d^{s1}) \quad (18)$$

From Proposition 1, it follows that the number of firms liquidated after period 1 increases when firms employ more debt and in downturns, i.e., when $s_1 = s_l$. Regarding the factors affecting the optimal choice of debt we have the following result that describes our comparative statics:

**Proposition 2** The optimal amount of debt $d^*$ increases with the efficiency of the matching technology $\lambda$ and decreases with the entry cost $k$. Furthermore, if the labor market is too tight (loose) from the social point of view, that is, there are too many (few) firms looking for workers relative to the number of unemployed workers, an increase (decrease) in $\beta$ or in $\gamma$ increases both $U^*(s_1)$ and $d^*$.

Intuitively, debt increases with the expected profitability in the second period, and hence, decreases in worker’ outside option $E(U^*(s_1))$. A more efficient matching technology $\lambda$ or a lower cost of entry decreases $k$ increases $E(U^*(s_1))$. Furthermore, a lower $\beta$ or $\gamma$ decreases wages but also promotes new entry, and if the market is too loose from the social point of view, the net effect is one of increasing $E(U^*(s_1))$.

4 Policy implications

In the previous analysis, the amount of debt chosen by firms has an effect on both the creation and destruction of firms as well as on the number of workers that are ultimately
unemployed. Within this setting, we consider two sets of questions. First, we examine potential policy interventions that address situations of excessive or insufficient liquidation of firms after the aggregate shock is realized. These policy choices alter the real value of debt ex post, after debt choices by firms are made. Second, we examine policy instruments that affect the amount of debt chosen by firms. In contrast to ex-post interventions, these ex-ante interventions are initiated prior to $s_1$ being realized.

In what follows, we identify social welfare as the sum of firm value added, that is, the sum of firm profits and wages, and exclude the private benefits of managerial control from the social welfare function.\textsuperscript{12} It is worth noting that our social welfare function does not imply that the optimal policy maximizes employment. Since what matters is the sum of firm profits and wages, from the social point of view it may be more desirable to have fewer but more efficient firms that employ fewer workers.\textsuperscript{13}

4.1 Ex-post interventions

We start by considering the role of public policy after the aggregate shock $s_1$ is realized. As a benchmark, Proposition 3 describes the social inefficiencies of the market equilibrium after $s_1$ is realized when no public intervention takes place.

**Proposition 3** Relative to the social optimum, there are too many liquidations in recessions (when $s_1 = s_l$) and too few liquidations during booms (when $s_1 = s_h$), leading to unemployment that is too high during recessions and too low in booms.

The optimal liquidation policy leads firms to exit when their firm specific shock is sufficiently below the expected firm specific shock of new emerging firms. Since the optimal number of liquidations in this setting is independent of the state of the economy, and because defaults occur more often in recessions, there are too many liquidations in

\textsuperscript{12}Ignoring managerial private benefits would be consistent with a political system in which managers have a negligible influence in the outcome of political elections. Alternatively, it would also be consistent with a situation in which the marginal benefit of the last dollar of managerial compensation are negligible in comparison with the benefits of the marginal dollar of worker and investor compensation. See Hart (1995) p. 126-130 for a more through discussion of the conditions in which ignoring managerial private benefits for the analysis of capital structure can be justified.

\textsuperscript{13}An alternative social welfare function that either takes managerial private benefits into account or that considers an extra social cost of unemployment would imply an additional cost of debt financing but would produce qualitatively similar results.
recessions and too few in booms. Notice that this result assumes that debt cannot be made contingent on specific cash-flow components \( \{ \varepsilon_i, s_t \} \).

We now consider the possibility of public intervention that affects the amount of debt due at date 1. To start, we consider “monetary” interventions that alter the real value of debt obligations. More specifically, we abstract from institutional details of implementation and consider government interventions that change the face value of the firms’ debt after \( s_1 \) is realized but before the liquidation decisions are made. We examine two alternative cases, one in which monetary interventions are unanticipated by firms and another in which monetary interventions are fully anticipated.

We model monetary policy as a technology that changes the general price level of the economy. In our setting this implies that the face value of any debt contract due at \( t = 1 \) can be modified from \( d \) to \( d - \tau \) in real terms, at the expense of a social cost \( c(\tau) \) where \( c(0) = c'(0) = 0 \), and \( c'' > 0 \). We refer to an expansionary or inflationary monetary policy as \( \tau > 0 \) and to restrictive or deflationary policy as \( \tau < 0 \). We assume that for any \( \tau > 0 \), \( c'(\tau) \leq c'(-\tau) \), that is, the social cost of an inflationary policy is smaller than the social cost of an equivalent deflationary policy.\(^{14}\) This is consistent with the fact that on average countries exhibit some degree of inflation, albeit, in most cases moderate.

### 4.1.1 Unanticipated monetary policy

We start with the case where firms do not anticipate monetary interventions. Specifically, we examine the socially optimal price distortion, which is determined as a function of the realization of the aggregate shock \( s_1 \), i.e., \( \tau(s_1) \equiv \tau_1 \). Since we are considering the case of unanticipated monetary policy, we can solve this problem by simply taking as given the equilibrium choices of job creation \( m(1, v^*(s_1)) \), posted wages \( w^*(s_1) \) and debt choices \( d^* \) (which are oblivious to monetary interventions) and consider the socially optimal ex-post price distortion. Formally the government solves the following problem at the end of period 1:

\[
\max_{\tau_1} \int_{\varepsilon(\tau_1)}^{\tau_1} \frac{\varepsilon_i + E(s_2|s_1) - U^*(s_1)}{2\pi} d\varepsilon_i - c(\tau_1) 
\]

\(^{14}\)For instance, \( c(\tau) = \tau^2 \) for \( \tau \geq 0 \) and \( c(\tau) = \phi \tau^2 \) for \( \tau < 0 \) where \( \phi \geq 1 \).
\[
\text{s.t. } \quad \varepsilon(\tau_1) = \frac{1}{2}[d^* - \tau_1 - s_1 - E(s_2|s_1) + U^*(s_1)].
\] (20)

The following proposition characterizes the optimal unanticipated monetary policy.

**Proposition 4** It is optimal to follow an inflationary monetary policy during recessions, \( \tau^*(s_l) > 0 \), and a restrictive policy during booms, \( \tau^*(s_h) < 0 \).

Intuitively, since in the absence of a monetary intervention there is excessive liquidation in recessions and too little liquidation during booms, it is optimal to increase the real value of debt during booms and to decrease it during recessions.

### 4.1.2 Anticipated monetary policy

Consider now the case of anticipated monetary policy. In this case, firms foresee that the government will follow an inflationary (deflationary) monetary policy when the aggregate shock is \( s_1 = s_l \) (respectively \( s_1 = s_h \)) and adjust their debt choices at \( t = 1 \) accordingly. Whether firms react by increasing or decreasing their debt obligations depends on the relative cost of inflation and deflation. For instance, in our case, since inflation is less costly than deflation, that is, since \( c'(\tau) \leq c'(-\tau) \) for \( \tau > 0 \), firms have an incentive to increase their face value of debt, \( d \), ex-ante anticipating an inflationary policy ex-post by the government. The following proposition summarizes this discussion.

**Proposition 5** When the monetary policy is anticipated and firms expect on average an inflationary policy, that is, if \( E(\tau_1) > 0 \), firms increase their choice of debt beyond \( d^* \).

Note that the monetary policy \( \{\tau_l, \tau_h\} \) is determined by the overhang (when \( s_1 = s_l \)) or free-cash flow problem (when \( s_1 = s_h \)) faced by the whole economy rather than by the problem faced by any one firm. This implies that there is no time inconsistency in the government monetary policy and that the government will carry out the optimal policy anticipated by firms.
4.2 Ex-ante policy interventions: Corporate tax policy

We now analyze ex-ante policy interventions, i.e., policies chosen before the aggregate shock $s_1$ is realized. In particular, we consider whether the government should provide incentives for the use of debt, for instance, through tax policy. We start by stating the following result.

Proposition 6 Without public intervention, investors choose the socially optimal amount of debt to fund their firms at $t = 1$.

The previous proposition indicates that there is no need for public intervention ex-ante. Hence, tax incentives that distort the use of debt financing by firms reduce social welfare. This result is somewhat surprising since, as it is well-known in the search literature, in general the entry and liquidation decisions by firms need not be socially optimal. Intuitively, a firm’s entry into (exit from) the market creates a positive (negative) externality for unemployed workers and a negative (positive) externality for firms with vacancies. Stated differently, the labor market tightness $\theta_1$ can be too high or too low from a social point of view.

In our setting, however, the assumption that there is an unlimited number of ex-ante identical potential entrants implies that the market tightness is independent of how many firms are liquidated, which in turn, implies that market tightness is also independent of the leverage choices made by firms at $t = 1$. That is, $\theta_1$ does not depend on $l(s_1)$ or $d$.\textsuperscript{15} Thus since the choice of debt at $t = 1$ does not create any externalities in the labor market, the (ex-ante) social and private choices of leverage coincide.

While the assumption of unlimited entry is analytically convenient it also represents a polar case of perfect flexibility in entry. This case, however, obscures the effects that arise when firm entry reacts imperfectly (or with some delay) to firms’ liquidation decisions. In the next section, we relax this assumption and consider the alternative polar case in which there is a limited number of firms of the new generation that will enter in period

\textsuperscript{15}To illustrate, an exogenous increase in $d$ would force firms of the old generation to liquidate more often but would also encourage more firms of the new generation to enter until the initial market tightness, $\theta(s_1)$, would be restored.
2. Within this setting of limited rather than free entry, we will reexamine the effects of debt tax policy on welfare and discuss the role of government intervention.

5 Limited firm entry

5.1 Debt choices under limited firm entry

In this section, we consider a modified setting with the same features and timing as before except that now there is a limited number of potential entrants $v(s_1)$ among firms of the new generation. This specification allows for the number of entrants to depend on the aggregate shock $s_1$. In particular, we assume that there can be more entry in good than in bad economic times, i.e., $v(s_h) \geq v(s_l)$. For simplicity, we also assume that the entry cost is nil, i.e., $k = 0$. This assumption implies that all the potential entrants $v(s_1)$ enter the market during the interim period, and that each of the entrants earn rents.\(^\text{16}\)

To solve the model we proceed as follows:\(^\text{17}\)

1. For each realization of the aggregate shock $s_1$ (and given the number of entrants $v(s_1)$) we derive the workers’ outside option at $t = 2$ as a function of the number of firms liquidated after period 1, $l(s_1)$:

$$U(s_1) = \lambda \theta_1(s_1)^{(1-\alpha)}(\gamma + \beta E(s_2|s_1))$$

(21)

where

$$\theta_1(s_1) = \frac{v(s_1)}{l(s_1)}.$$  

(22)

2. We derive the number of firms liquidated $l(s_1)$ as a function of the workers’ reservation utility $U(s_1)$ and the firms’ debt choice $d$ at $t = 1$:

$$l(s_1) = \frac{d + \bar{\varepsilon} - s_1 - E(s_2|s_1) + U(s_1)}{4\bar{\varepsilon}}$$

(23)

\(^{16}\)While we analyze the polar case of a fixed number of entrants, qualitatively similar results can be derived with free-entry as long as entrants are not all identical (i.e., when potential entrants have different entry costs). In the polar case that we consider, one can think of a number of firms $v(s_1)$ that can enter without cost ($k = 0$) and then other potential entrants with a high enough cost of entry.

\(^{17}\)Please refer to the Appendix for details.
3. We solve for the optimal amount of debt taking as given the workers’ reservation utility:

\[ d = E(s_1) - E [E(s_2|s_1) - U(s_1)] \]  

(24)

Notice that while the expressions for \( U(s_1) \), \( l(s_1) \) and \( d \) show a high resemblance to the corresponding expressions (10), (18) and (16) obtained for the unlimited entry case, there are also important differences. First, technically, the model cannot be solved sequentially since now \( U(s_1) \), \( l(s_1) \), and \( d \) depend on each other, which requires the solution of three equations, (21), (23) and (24), simultaneously. Second, these expressions show that labor market tightness depends on the liquidation decision and hence on the choice of debt.

5.2 Debt subsidies with fixed firm entry

We are now ready to examine debt subsidies. We consider two cases, one in which the economy has no aggregate uncertainty in period \( t = 1 \) and then the more general case with aggregate uncertainty in period \( t = 1 \).\(^\text{18}\) Denoting \( \tilde{\beta}(s_1) \equiv \frac{u_2(s_1)}{E(s_2|s_1)} = \frac{\gamma}{E(s_2|s_1)} + \beta \), as the worker’s share of the surplus, we start with the following proposition:

**Proposition 7** Assume that there is no aggregate uncertainty at \( t = 1 \), that is, \( s_1 \) equals either \( s_l \) or \( s_h \) with probability 1, then if \( \tilde{\beta}(s_1) < \alpha \) firms have less debt than is socially optimal. Alternatively, if \( \tilde{\beta}(s_1) > \alpha \), firms have more debt than is socially optimal.

A firm’s liquidation decision imposes a negative externality on unemployed workers and a positive one on emerging new firms. These two externalities exactly offset each other when \( \tilde{\beta}(s_1) = \alpha \), that is, when the worker’s share of the surplus \( \tilde{\beta}(s_1) \) is equal to the elasticity of the matching function with respect to unemployment \( \alpha \). In such a case, the equilibrium and socially optimal market tightness coincide.\(^\text{19}\) However, if the worker’s share of the surplus is lower (higher) than the elasticity of the matching function with respect to unemployment, the equilibrium market tightness is higher (lower) than the optimal social tightness.

\(^{18}\)As it will be clear below the case of no aggregate uncertainty in period \( t = 1 \) (when either \( p = 1 \) or \( p = 0 \)) allows firms to perfectly solve their managerial agency problem, that is, at the end of period 1 there will be no privately inefficient liquidation or continuation.

\(^{19}\)See Hosios (1990) for a detailed analysis of externalities in search models.
Intuitively, a social planner would like to increase the number of workers looking for jobs—the number of liquidations—to the point where the marginal benefit in terms of additional matches with new firms is equal to the cost. Since old generation firms do not internalize the benefit that a new match has on new generation firms, (i.e., $(1 - \tilde{\beta}(s_1))E(s_2|s_1)$) if this benefit is too large, that is, if $\tilde{\beta}(s_1)$ is small enough, there is not enough liquidation and the labor market becomes too tight, that is, there are too few unemployed workers in relation to the number of vacancies. Hence if $\tilde{\beta}(s_1) < \alpha$, liquidations have a positive net externality as it loosens a labor market that is too tight from the social point of view. It is also worth noticing that the relation between $\tilde{\beta}(s_1)$ and $\alpha$ does not depend on the number of entrants, $v(s_1)$. That is $v(s_1)$ affects both the market tightness in equilibrium and the socially optimal market tightness but not whether one is smaller or larger than the other.

Notice that an increase in expected $E(s_2|s_1)$ decreases $\tilde{\beta}(s_1)$, and makes it more likely that the labor market is too tight from the social point of view. Intuitively, this occurs because an increase in $E(s_2|s_1)$ does not translate into a proportional increase in the wage $w_2(s_1)$, that is, there are real wage rigidities. The main implication that arises from this observation is that the labor market tends to be too tight during economic booms (that is, too many firms looking for workers relative to the number of unemployed workers) and too loose during recessions (that is, too many unemployed workers looking for jobs relative to the number of job vacancies). Hence, the previous proposition highlights a reason to promote debt at $t = 1$ when good economic times are expected at $t = 2$, (i.e., when the expected productivity in the economy $E(s_2|s_1)$ is high). When this is the case, the positive externalities that liquidation creates on new firms looking for workers are greater than the negative externalities that liquidation has on other unemployed workers. When bad economic times are expected (i.e., when $E(s_2|s_1)$ is low) firms can easily find workers, and hence, additional liquidations do not help these firms much while it hurts the unemployed workers who already a small probability of finding a job.

Finally for the general case with aggregate uncertainty and if $\tilde{\beta}(s_h) < \alpha < \tilde{\beta}(s_l)$, the socially optimal amount of debt will trade-off the possibility of ending up with too much debt in recessions against too little debt during booms. In addition, when there
is aggregate uncertainty there is also a general equilibrium feedback effect that firms do not internalize: firms' debt choices affect the workers' outside option in the interim period, $U$, which in turn, affects the firms' liquidation in period 1. From an ex-post point of view, there are now two reasons to increase (reduce) the value of debt during good economic times, that is to resolve the free-cash low (debt overhang) problem and to increase (decrease) the tightness of the labor market.

### 5.3 Monetary policy with fixed entry

Finally we consider the issue of monetary policy under limited firm entry. For brevity, we focus the discussion on two issues, namely on the problem of time inconsistency in monetary policy interventions and the complementarity between monetary and tax policy.

Regarding the first issue, with fixed entry policy makers face the problem of time inconsistency in monetary policy interventions (as described by Kydland and Prescott, 1977). This time inconsistency problem arises with fixed entry, because differences in the social and private benefits of liquidation choices lead firms and the government to have different preferences regarding firm debt obligations. If policymakers prefer that firms have lower debt obligations they will choose to inflate ex-post, giving firms an incentive to choose a higher debt obligation ex-ante. As we show, these choices lead to a situation with high nominal debt obligations and high inflation. Hence, when inflation has a social cost, the government would like to commit to have a specific inflation rate that is contingent on the state of the economy. Notice, even when the policy makers cannot commit, an active monetary policy can be socially beneficial by reducing firm exits in recessions, when they are especially costly, and increasing exits in booms, when they may be socially beneficial.

Finally, it should be noted that taxes and subsidies on debt play a role that can influence the incentives to inflate. In particular, by having a second channel for influencing a firm's debt choice, the analysis suggests complementary roles for tax and monetary policies. Specifically, policy makers can use taxes and subsidies to induce

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20 We refer to the appendix for the general derivations in the case of aggregate uncertainty.

21 Notice, even when the policy makers cannot commit, an active monetary policy can be socially beneficial by reducing firm exits in recessions, when they are especially costly, and increasing exits in booms, when they may be socially beneficial.

22 Again, if there are social costs associated with firm exits, then the social planner would make debt financing more costly, which would in turn lead to less inflation in an equilibrium without central planner commitment. However, if there are social benefits associated with firm exits, then the existence of debt financing would lead to higher inflation.
firms to choose the socially optimal debt ratio ex-ante, and then use monetary policy to accommodate the realization of the aggregate shock in the interim period.\footnote{Under some conditions, this combination allows for the implementation of the optimal monetary policy without the need for commitment.}

6 Concluding remarks

Since the seminal work of Modigliani and Miller (1958), economists have examined the costs and benefits of financial leverage from the perspective of firms seeking financing. In this paper, we extend this analysis and examine how corporate financing choices influence the aggregate economy. In particular, we consider a setting where financial leverage can increase the probability of a firm liquidating following economic shocks, and within this setting we consider potential externalities. For example, corporate liquidations can have negative externalities during economic recessions, if they contribute to excess slack in the labor markets. In contrast, liquidations may have positive externalities during economic booms, if they facilitate the emergence of more productive startup companies.

The framework we develop provides intuition about the economic effects of policies that influence the magnitude of firm debt obligations. In particular, we consider inflation policy, which affects the real value of debt obligations, and show that in some situations an active policy that decreases debt obligations during economy-wide downturns can improve ex ante firm values. In addition, we identify conditions under which welfare can be improved with subsidies or taxes that alter the firms’ use of debt financing.

While we do not explore this in our paper, there are a number of other policy choices that may be evaluated within the framework of our model. For example, the U.S. government provides subsidized debt for emerging industries that may create positive externalities, like renewable energy, as well as for failing industries, like automobiles, that might otherwise create negative spillovers. Since a primary motivation for these initiatives is to create and save jobs, a model, such as ours, that explicitly considers the effect of financing on the labor market might be relevant. In addition to calculating the relevant spillovers that are a direct consequence of the policy, an evaluation of these policies should also

\footnote{Subsidies will negate the incentive to implement a deflationary policy.}
consider alternatives, like subsidized equity, as well as these policies on future decisions by the firm that may also influence the job market and the creation of new firms.

There are a number of important aspects of our analysis that merit further attention. In addition to considering the study of a richer set of policy tools, future research should also extend the scope of the model. For instance, we consider very limited dynamics (firms interact in a single period) and therefore ignore how these policy choices influence business cycles and the growth rate of the economy. An analysis of the optimal debt policy and public interventions in a more dynamic (and more complicated) setting is a challenge that is left to future work.
References


APPENDIX

Proof of Lemma 1
Note that $E(r_2|s_1) = E(s_2|s_1)$ and hence, the free-entry $V(s_1) = -k + \frac{q(\theta_1)}{\theta_1}[(1 - \beta)E(r_2|s_1) - \gamma] = 0$ implies: $\theta_1^* = \left(\frac{(1 - \beta)E(s_2|s_1) - \gamma}{k}\right)^{1/\alpha}$ and

$$U^*(s_1) = q(\theta_1^*)(\gamma + \beta E(s_2|s_1)) = \lambda^{1/\alpha} \left(\frac{(1 - \beta)E(s_2|s_1) - \gamma}{k}\right)^{1/\alpha} (\gamma + \beta E(s_2|s_1))$$

Proof of Proposition 3
From the social point of view firm $i$ should be liquidated when its expected cash-flow in period 2, $E(r_{i2}|r_{i1}, s_1)$, is lower than the workers’ outside option, $U^*(s_1)$. (Note that $U^*(s_1)$ is pinned down by the free-entry condition and hence is not affected by the liquidation decision or the amount of debt.)

$$H(\varepsilon, s_1) \equiv E(r_{i2}|r_{i1}, s_1) - U^*(s_1) = \varepsilon + E(s_{2}|s_1) - U^*(s_1) < 0.$$ (25)

In equilibrium the marginal firm liquidated is (see equation (13));

$$\varepsilon_{d, 1}^* = \frac{1}{2} [d^* - s_1 - E(s_{2}|s_1) + U^*(s_1)]$$ (26)

which implies that:

$$H(\varepsilon_{d, 1}^*, s_1) = \frac{1}{2} [d^* - s_1 + E(s_{2}|s_1) - U^*(s_1)].$$ (27)

There is too much liquidation in recessions (debt overhang) if the marginal firm liquidated in recession has a positive social value (that is, if $H(\varepsilon_{d, 1}^*, s_1) > 0$). Symmetrically, there is too little liquidation in recessions (a free cash-flow problem) if the marginal firm liquidated during booms has a negative social value (that is, $H(\varepsilon_{d, 1}^*, s_1) < 0$). Since

$$d^* = E(s_1) - E[E(s_{2}|s_1) - U^*(s_1)],$$ (28)

then $H(\varepsilon_{d, 1}^*, s_1) > 0$ and $H(\varepsilon_{d, 1}^*, s_1) < 0$ if and only if:

$$s_h - E(s_2|s_h) + U^*(s_h) > s_l - E(s_2|s_l) + U^*(s_l).$$ (29)

Since $U^*(s_h) > U^*(s_l)$, and since $s_h - E(s_2|s_h) \geq 0 \geq s_l - E(s_2|s_l)$, it follows that $H(\varepsilon_{d, 1}^*, s_1) > 0$ and $H(\varepsilon_{d, 1}^*, s_1) < 0$ ■

Proof of Proposition 4. The government solves:

$$\max_{\tau_1} \int_{\varepsilon(\tau_1, s_1)}^{\tau} \frac{H(\varepsilon, s_1)}{2\varepsilon} d\varepsilon_i - c(\tau_1)$$ (30)

s.t.: $\varepsilon(\tau_1, s_1) = \frac{1}{2} [d^* - \tau_1 - s_1 - E(s_{2}|s_1) + U^*(s_1)]$ (31)
where

\[ H(\varepsilon_i, s_1) \equiv E(r_{i2}|r_{i1}, s_1) - U^*(s_1) = \varepsilon_i + E(s_2|s_1) - U^*(s_1). \] (32)

The problem yields the following f.o.c.:

\[
\frac{1}{4\pi} H(\varepsilon(\tau_1), s_1) - c'(\tau_1) = 0. \tag{33}
\]

Consider first the case in which \( s_1 = s_h \). If \( \tau_1 = 0 \), then \( \varepsilon(1, s_h) = \varepsilon^*_{d_h} \), and from the proof of Proposition 3 above we know that \( H(\varepsilon^*_{d_h}, s_1) < 0 \). In that case, the f.o.c. evaluated at \( \tau_1 = 0 \) has a positive sign and hence there is incentives to increase \( \tau_1 \) beyond 0. (Note that \( H(\varepsilon(\tau_1), s_1) \) is linear on \( \tau_1 \) and hence the problem is well defined.)

Consider now the case in which \( s_1 = s_l \). If \( \tau_1 = 0 \), then \( \varepsilon(1, s_l) = \varepsilon^*_{d_l} \), and from the proof of Proposition 3 we know that \( H(\varepsilon^*_{d_l}, s_1) > 0 \). In that case, the f.o.c. evaluated at \( \tau_1 = 0 \) has a negative sign and hence there is incentives to decrease \( \tau_1 \) below 0. ■

**Proof of Proposition 5**

Firms solve the following optimization problem taken as given \( \{\tau_l, \tau_h\} \):

\[
\max_d E \left[ \int_{\varepsilon(\tau_1,s_1)} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\pi} d\varepsilon_i \right] 
\]

s.t.: \( \varepsilon(\tau_1,s_1) = \frac{1}{2}[d - \tau_1 - s_1 - E(s_2|s_1) + U(s_1)]. \) (34)

which yields the following f.o.c.:

\[
d = E[\tau_1 + s_1 - E(s_2|s_1) + U(s_1)] \tag{35}
\]

Since

\[
d^* = E[s_1 - E(s_2|s_1) + U(s_1)] \tag{36}
\]

then

\[
d > d^* \iff E(\tau_1) > 0. \tag{37}
\]

Given \( d \) the monetary authority solves the following problem given \( s_1 \):

\[
\max_{\tau_1} \int_{\varepsilon(\tau_1,s_1)} \frac{\varepsilon_i + E(s_2|s_1) - U^*(s_1)}{2\pi} d\varepsilon_i - c(\tau_1)
\]

s.t.: \( \varepsilon(\tau_1,s_1) = \frac{1}{2}[d - \tau_1 - s_1 - E(s_2|s_1) + U(s_1)]. \) (38)

which yields

\[
\frac{d - \tau_1 - s_1 + E(s_2|s_1) - U(s_1)}{4\pi} - c'(\tau_1) = 0. \tag{39}
\]
Next we prove that the equilibrium choices are socially optimal:

\[
\max_{d, \tau_i, \tau_1} E \left( \int_{\varepsilon(\tau_1,s_1)}^{\varepsilon_i} \frac{\varepsilon_i + E(s_2|s_1) - U^*(s_1)}{2\pi} d\varepsilon_i - c(\tau_1) \right) \\
\text{s.t.: } \varepsilon_1(\tau_1,s_1) = \frac{1}{2} [d - \tau_1 - s_1 - E(s_2|s_1) + U(s_1)].
\]

which yields the following three order conditions:

\[
d = E[\tau_1 + s_1 - E(s_2|s_1) + U(s_1)] \\
\frac{d - \tau_1 - s_1 + E(s_2|s_1) - U(s_1)}{4\pi} - c'(\tau_1) = 0 \text{ for } s_1 \in \{s_l, s_h\}.
\]

Notice that the three first order conditions that solve the social optimum are identical to the ones that solve the private optimum. (Equations 35 and 39.)

**Proof of Proposition 6**

The social planner solves the following optimization problem:

\[
\max_d E \left[ \int_{\varepsilon_{d_i}^1}^{\varepsilon_i} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\pi} d\varepsilon_i \right] + E[U(s_1)]
\]

\[\text{s.t. } \varepsilon_{d_i}^1 = \frac{1}{2} [d_i - s_1 - E(s_2|s_1) + U^*(s_1)].\]

Since the reservation utility \( U^* = q_2(\theta_2^*) (\gamma + \beta E(s_2|s_1)) \) and \( \theta^* = (\frac{\Lambda[1-\beta E(s_2|s_1)-\gamma]}{\kappa})^{1/\alpha} \), then \( E(U(s_1)) \) does not depend on \( d \) and hence the solution the above problem coincides with the private optimum, that is: \( d^* = E[s_1 - E(s_2|s_1) + U^*(s_1)] \)

**Proof of Proposition 7**

Assume that there is no aggregate uncertainty at \( t = 1 \) that is \( s_1 \) equals either \( s_h \) or \( s_l \) with probability one. (Note that still \( E(s_2|s_1 = s_h) \geq E(s_2|s_1 = s_l) \).

Then we have the following equilibrium:

\[
d^* = s_1 - E(s_2|s_1) + U^*(s_1) = s_1 - E(s_2|s_1) + \lambda \left( \frac{v(s_1)}{l^*(s_1)} \right)^{1-\alpha} [\gamma + \beta E(s_2|s_1)]
\]

The social planner solves the following problem

\[
\max_d E \left[ \int_{\varepsilon_{d_i}^1}^{\varepsilon_i} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\pi} d\varepsilon_i + U(s_1) + v(s_1) \frac{q(\theta_1)}{\theta_1} \right] [(1 - \beta) E(s_2|s_1) - \gamma]
\]
where
\[ U(s_1) = \lambda(\theta_1)^{1-\alpha} [\gamma + \beta E(s_2|s_1)] \quad ; \quad \varepsilon_d^1 = \frac{1}{2}[d - s_1 - E(s_2|s_1) + U^*(s_1)] \] (47)

\[ \frac{q(\theta_1)}{\theta_1} = \lambda(\theta_1)^{-\alpha} \quad ; \quad \theta_1 = \frac{v(s_1)}{l(s_1)} \] (48)

\[ l(s_1) = \Pr(\varepsilon_i < \varepsilon_d^1) = \frac{d + 2\varepsilon - s_1 - E(s_2|s_1) + U^*(s_1)}{4\varepsilon} \] (49)

The derivative of the social planner's objective function (SPOF) w.r.t. debt is:
\[
\frac{\partial \text{SPOF}}{\partial d} = \frac{-1}{4\varepsilon} (\varepsilon_d^1 + E(s_2|s_1) - U(s_1)) \left( 1 + \frac{\partial U(s_1)}{\partial d} \right) \\
+ (1 - \Pr(\varepsilon_i \geq \varepsilon_d^1)) \frac{\partial U(s_1)}{\partial d} \\
+ \lambda(\theta_1)^{-\alpha} \frac{\partial \theta_1}{\partial d} \left[ (1 - \alpha) \gamma + \beta E(s_2|s_1) - \gamma \right] \\
\] (50)

Since
\[
\frac{\partial U(s_1)}{\partial d} = (1 - \alpha) \lambda(\theta_1)^{-\alpha} \frac{\partial \theta_1}{\partial d} [\gamma + \beta E(s_2|s_1)] \] (51)

then we can rewrite \( \frac{\partial \text{SPOF}}{\partial d} \) as:
\[
\frac{\partial \text{SPOF}}{\partial d} = \frac{-1}{4\varepsilon} \left[ 1 + \frac{\partial U(s_1)}{\partial d} \right] \left( \varepsilon_d^1 + E(s_2|s_1) - U(s_1) \right) \\
+ l(s_1) \lambda(\theta_1)^{-\alpha} \frac{\partial \theta_1}{\partial d} \left[ (1 - \alpha) [\gamma + \beta E(s_2|s_1)] - \alpha [(1 - \beta) E(s_2|s_1) - \gamma] \right] \\
\] (52)

At the private optimum
\[
d^* = U^*(s_1) = \lambda \left( \frac{v(s_1)}{l^*(s_1)} \right)^{1-\alpha} [\gamma + \beta E(s_2|s_1)] \] (53)

and
\[
\varepsilon_d^{1*} = \frac{1}{2}[d^* - s_1 - E(s_2|s_1) + U^*(s_1)] = U^*(s_1) - s_1. \] (54)

Hence evaluating \( \frac{\partial \text{SPOF}}{\partial d} \) at the private optimum (PO):
\[
\left| \frac{\partial \text{SPOF}}{\partial d} \right|_{\text{PO}} = l^*(s_1) \lambda(\theta^*)^{-\alpha} \left| \frac{\partial \theta_1}{\partial d} \right|_{\text{PO}} \left[ (1 - \alpha) [\gamma + \beta E(s_2|s_1)] - \alpha [(1 - \beta) E(s_2|s_1) - \gamma] \right] \] (55)
Next we show that $\frac{\partial \theta_1}{\partial d} < 0$:

$$\frac{\partial \theta_1}{\partial d} = -\frac{v(s_1)}{(l(s_1))^2} \frac{\partial l(s_1)}{\partial d} = \frac{-1}{2\bar{\varepsilon}v(s_1)} \left( 1 + \frac{\partial U(s_1)}{\partial d} \right) =$$

$$= \frac{-1}{2\bar{\varepsilon}v(s_1)} \left[ 1 + (1 - \alpha)\lambda \theta_1^{-\alpha} \frac{\partial \theta}{\partial d} \left[ \gamma + \beta E(s_2 s_1) \right] \right]$$

and solving for $\frac{\partial \theta_1}{\partial d}$:

$$\frac{\partial \theta_1}{\partial d} = \frac{-1}{2\bar{\varepsilon}v(s_1) + (1 - \alpha)\lambda \theta_1^{-\alpha} [\gamma + \beta E(s_2 s_1)]} < 0 \quad (54)$$

Hence

$$\left| \frac{\partial \text{SPOF}}{\partial d} \right|_{\text{PO}} > 0 \Leftrightarrow (1 - \alpha) [\gamma + \beta E(s_2 s_1)] - \alpha [(1 - \beta) E(s_2 s_1) - \gamma] < 0 \Leftrightarrow (1 - \alpha)\tilde{\beta} E(s_2 s_1) - \alpha (1 - \tilde{\beta}) E(s_2 s_1) < 0 \Leftrightarrow \tilde{\beta} < \alpha$$

AGGREGATE UNCERTAINTY & LIMITED ENTRY

Equilibrium Debt
A worker’s outside option at $t = 2$ if he quits and looks for a job at $t = 2$ is:

$$U(s_1) = \lambda(\theta_1)^{1-\alpha} [\gamma + \beta E(s_2 s_1)] \lambda\left(\frac{v_1}{l_1}\right)^{1-\alpha} [\gamma + \beta E(s_2 s_1)] \quad (55)$$

where

$$l_1 \equiv l(s_1) ; v_1 \equiv v(s_1) \quad (56)$$

The firm liquidates if the manager cannot retain the worker, that is, if

$$G(\varepsilon_i, s_1, d_i) \equiv 2\varepsilon_i + s_1 + E(s_2 s_1) - d_i - U(s_1) < 0 \quad (57)$$

which implies that the marginal firms liquidated, $\varepsilon_{d_i}^*$, is determined by the following equation,

$$G(\varepsilon_{d_i}^*, s_1, d_i) \equiv 0 \quad (58)$$

which boils down to

$$\varepsilon_{d_i}^* = \frac{1}{2}[d_i - s_1 - E(s_2 s_1) + U(s_1)]. \quad (59)$$

and since there is a continuum $[0,1]$ of firms at $t = 1$, then:

$$l_1 = \Pr(\varepsilon_i < \varepsilon_{d_i}^*) = \frac{d_i + 2\bar{\varepsilon} - s_1 - E(s_2 s_1) + U(s_1)}{4\bar{\varepsilon}} \quad (60)$$
The debt choice at \( t = 0 \) solves:

\[
\max_{d_i} E \left[ \varepsilon_i + E(s_2|s_1) - U(s_1) \right] d \varepsilon_i
\]

which yields the following f.o.c.:

\[
-\frac{1}{4 \varepsilon} E[\varepsilon_d^* + E(s_2|s_1) - U^*(s_1)] = 0,
\]

which yields the following amount of debt

\[
d^* = E [(s_1) - E(s_2|s_1) + E(U^*(s_1))].
\]

**Social Optimum Debt**

The social planner solves the following problem:

\[
\max_d E \left[ \int_{\varepsilon_{d_i}^1}^{\varepsilon_i} \varepsilon_i + E(s_2|s_1) - U(s_1) \frac{d \varepsilon_i}{2 \varepsilon} + E(U(s_1)) + \right.
\]

\[
+ E \left[ v_1 q(\theta_1) \left[ (1 - \beta) E(s_2|s_1) - \gamma \right] \right]
\]

s.t.

\[
\varepsilon_d^* = \frac{1}{2} [d - s_1 - E(s_2|s_1) + U^*_2(s_1)].
\]

\[
U(s_1) = \lambda(\theta_1)^{1-\alpha} [\gamma + \beta E (s_2|s_1)]
\]

\[
\frac{q(\theta_1)}{\theta_1} = \lambda \theta_1^{-\alpha} \quad \theta_1 = \frac{v_1}{l_1}
\]

\[
l_1 = \Pr(\varepsilon_i < \varepsilon_d^*)
\]

Note that unlike individual firms the social planner internalizes the effect that the choice of debt has on \( \theta_1 \), and hence on \( \varepsilon_d^* \) and \( U(s_1) \). Deriving the social planner’s objective function (SPOF) w.r.t. debt:

\[
\frac{\partial \text{SPOF}}{\partial d} = -E \left[ (1 + \frac{\partial U(s_1)}{\partial d}) \varepsilon_d^* + E(s_2|s_1) - U^*(s_1) \right] + E \left[ \frac{\partial U(s_1)}{\partial d} \right] l_1
\]

\[
+ E \left[ \frac{\partial q(\theta_1)}{\partial \theta_1} \right] v_1 [(1 - \beta) E (s_2|s_1) - \gamma]
\]

Notice that

\[
\frac{\partial U(s_1)}{\partial d} = -(1 - \alpha) [\gamma + \beta E(s_2|s_1)] \lambda(\theta_1)^{-\alpha} \frac{\partial \theta_1}{\partial d}
\]
\[
\frac{\partial \left( \frac{q(\theta_1)}{\theta_1} \right)}{\partial d} = -\alpha \lambda \theta_1^{-\alpha - 1} \frac{\partial \theta_1}{\partial d}
\]

so we can rewrite \( \frac{\partial \text{SPOF}}{\partial d} \) as:

\[
\frac{\partial \text{SPOF}}{\partial d} = -E \left[ \frac{1}{4\varepsilon} \left[ \varepsilon_{d}^{s1} + E(s_2|s_1) - U^*(s_1) \right] \right] + \\
E \left[ l_1 \lambda(\theta_1)^{-\alpha} E(s_2|s_1) \frac{\partial \theta_1}{\partial d} \right] \left[ (1 - \alpha)\bar{\beta}(s_1) - \alpha(1 - \bar{\beta}(s_1)) \right]
\]

which evaluated at the private optimal (PO), that is,

\[
-\frac{1}{4\varepsilon} E[\varepsilon_{d}^{s1} + E(s_2|s_1) - U^*(s_1)] = 0
\]

gives:

\[
\left| \frac{\partial \text{SPOF}}{\partial d} \right|_{PO} = -E \left[ \left| \frac{\partial \text{SPOF}}{\partial d} \right|_{PO} \left( \frac{\varepsilon_{d}^{s1} + E(s_2|s_1) - U^*(s_1)}{4\varepsilon} \right) \right] + \\
E \left[ l_1^* \lambda(\theta_1)^{-\alpha} E(s_2|s_1) \left| \frac{\partial \theta_1}{\partial d} \right|_{PO} \right] \left[ (1 - \alpha)\bar{\beta}(s_1) - \alpha(1 - \bar{\beta}(s_1)) \right]
\]

Hence there are two effects determining if at the equilibrium the social planner has incentives to increase or decrease debt, that is, whether \( \left| \frac{\partial \text{SPOF}}{\partial d} \right|_{PO} \): the "feedback effect" (in the first line) and the "search externality effect" (in the second line).

1. The "feedback effect":

\[
- E \left[ \left| \frac{\partial \text{SPOF}}{\partial d} \right|_{PO} \left( \frac{\varepsilon_{d}^{s1} + E(s_2|s_1) - U^*(s_1)}{4\varepsilon} \right) \right]
\]

When firms choose the amount of debt, \( d^* \), they do not take into account that \( d^* \) affects \( U^*(s_1) \) and hence \( \varepsilon_{d}^{s1} \). The marginal firm destroyed in good times has a value:

\[
\varepsilon_{d}^{*h} + E(s_2|s_h) - U_2^*(s_h)
\]

and the marginal firm destroyed in bad times has a value:

\[
\varepsilon_{d}^{*l} + E(s_2|s_l) - U_2^*(s_l)
\]

Since \( d^* \) affects \( U^*(s_1) \) and hence \( \varepsilon_{d}^{s1} \) the net effect depends on whether \( d \) moves the outside option more in good or bad times multiplied by the value of the marginal firm destroyed, \( \varepsilon_{d}^{*l} + E(s_2|s_1) - U_2^*(s_1) \), which is positive in bad times and negative in good times.

2. The "search externality effect":

\[
E \left[ l_1^* \lambda(\theta_1)^{-\alpha} E(s_2|s_1) \left| \frac{\partial \theta_1}{\partial d} \right|_{PO} \right] \left[ (1 - \alpha)\bar{\beta}(s_1) - \alpha(1 - \bar{\beta}(s_1)) \right]
\]
First notice that $\frac{\partial \theta_1}{\partial d} < 0$:

\[
\frac{\partial \theta_1}{\partial d} = -\frac{v_1}{(l_1)^2} \frac{\partial l_1}{\partial d} = \frac{-1}{2\varepsilon v_1} \left( 1 + \frac{\partial U(s_1)}{\partial d} \right) = \\
= \frac{-1}{2\varepsilon v_1} \left[ 1 + (1 - \alpha)\lambda \theta_1^{-\alpha} [\gamma + \beta E(s_2|s_1)] \frac{\partial \theta_1}{\partial d} \right]
\]

and solving for $\frac{\partial \theta_1}{\partial d}$,

\[
\frac{\partial \theta_1}{\partial d} = \frac{-1}{2\varepsilon v_1 + (1 - \alpha)\lambda \theta_1^{-\alpha} [\gamma + \beta E(s_2|s_1)]} < 0.
\] (75)

Hence the sign depends on $\left( (1 - \alpha)\lambda \theta_1^{-\alpha} - \alpha(1 - \lambda \theta_1^{-\alpha}) \right)$. For instance consider the case in which $\lambda \theta_1^{-\alpha} < \lambda \theta_1^{-\alpha} < \alpha$. In that case the search externality would tend push things towards increasing debt since the labor market would tend to be too tight. Alternatively, if $\alpha < \lambda \theta_1^{-\alpha} < \lambda \theta_1^{-\alpha}$, the search externality would tend push things towards decreasing debt. Finally, if $\lambda \theta_1^{-\alpha} < \alpha < \lambda \theta_1^{-\alpha}$ the search externality induces increasing debt in good time and decreasing debt in bad time, hence, ex-ante, whether search externality induces an increase or decrease in debt depends on which one of the two effects dominates.