Risk-sharing or risk-taking? Hedging, margins and incentives*

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Abstract

We develop an incentive-based theory of margins in the context of a tradeoff between the benefits of hedging in terms of enhanced risk-sharing and its costs in terms of financial instability. We model hedging as the design of a contract between a protection buyer, seeking to reduce his risk exposure, and a protection seller. If the seller learns that the hedge is likely to be loss-making for her even though actual losses have not materialized yet, her incentives to control the risk of her other positions (balance sheet risk) diminish. The seller's risk-taking incentives limit hedging and generate endogenous counterparty risk. Margins improve incentives to control balance sheet risk and thus enhance the scope of hedging.

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1 Introduction

We analyze the trade-off between the benefit of hedging in terms of gains from trade and its cost in terms of financial instability.[1] The benefit arises since hedging enhances risk-sharing opportunities between agents with different risk-bearing capacities. The cost arises when hedging creates *hidden leverage* that increases risk-taking incentives. We build on this trade-off to develop an incentive-based rationale for margins and their substitutability or complementarity with capital requirements.

We model hedging as the design of an optimal contract between a risk-averse buyer of insurance who seeks protection against a risk exposure and a risk-neutral seller of insurance who provides the protection. Examples of hedging contracts are forward contracts, futures and credit default swaps (CDS).

Financial institutions selling protection are exposed to risk from their own assets and liabilities, i.e., they are exposed to balance sheet risk. Controlling balance sheet risk is costly. For example, financial institutions must devote resources to scrutinize the default risk of their borrowers or to manage their maturity mismatch.[2] Not controlling balance sheet risk (risk-taking) can lead to the failure of the protection seller and to the default on his contractual obligations. Protection buyers are therefore exposed to counterparty risk. For example, Lehman Brothers and Bear Stearns defaulted on their CDS derivative obligations because of losses incurred on their other investments, in particular sub-prime mortgages.

Our main assumption is that the care with which financial institutions manage their balance sheet risk is unobservable to outsiders and that financial institutions are protected by limited liability. This creates a moral hazard problem between the buyer and seller of protection. In this context, we show that hedging creates hidden leverage: Ex-ante, when entering the position, the hedge is neither an asset nor a liability for the protection seller. For

[1] Rajan (2006) notes that while hedging enhances risk-sharing opportunities it can also create new risk-taking venues.

[2] For example, in the wake of the 2007-09 crisis many financial institutions financed themselves through short-term debt. While such financing was relatively easy to establish, it left these institutions exposed to the risk of not being able to roll-over their liabilities (Brunnermeier and Oehmke, 2009; Acharya et al., 2009).

example, the seller of a credit default swap pays the buyer in case of credit events (default, restructuring) but collects an insurance premium otherwise. On average, for the seller to be willing to enter the position, she must at least break even. But, if negative information about the underlying position arrives after the contract is signed, the hedge is more likely to be a liability for the protection seller. For instance, after bad news about the future solvency of firms, the seller of a CDS is more likely to pay out the insurance than after good news. The implicit liability embedded in the hedging position undermines the protection seller's incentive to control her balance sheet risk.[3] This is because she bears the full cost of risk control while the benefit accrues in part to the protection buyer.[4]

Given the incentives of the protection seller, the buyer faces a trade-off between risk-sharing and risk-taking. If he wants to curb risk-taking incentives, he must reduce the hidden leverage by accepting incomplete risk-sharing. Since such under-insurance is costly, he may instead opt for full insurance, recognizing that it will encourage risk-taking and lead to counterparty risk.

Our analysis thus identifies a channel through which hedging together with asymmetric information can lead to systemic risk. In the absence of a moral hazard problem, the risk the protection buyer is hedging and the balance sheet risk of the protection seller are independent. The counterparty risk that arises endogenously when the hidden leverage leads to risk-taking creates interconnectedness: advance negative news about the risk of the protection buyer propagates to the protection seller whose default risk increases.[5]

We use the model to develop an incentive-based theory of margins. The buyer and seller of protection can agree as part of their risk-sharing transaction that the seller deposits cash on a separate account if the buyer makes a margin call. This is costly since cash

[3]Holmström and Tirole (1998) show how *liquidity* shocks can weaken incentives to exert effort. In contrast, in the present analysis, we show how *information* shocks can weaken incentives.

[4]This is in line the debt-overhang effect identified by Myers (1977). Note, however, that instead of exogenous debt, we have endogenous liabilities emerging from optimal contracting.

[5]Our approach differs from other models of systemic risk, see, e.g., Freixas, Parigi and Rochet (2000), Cifuentes, Shin and Ferrucci (2005), and Allen and Carletti (2006), since in our analysis contagion arises because of incentive problems.

deposited will earn a lower rate of return than assets kept on the balance sheet. The benefit is that these assets are ring-fenced from risk-taking. The margin thus reduces the size of the moral hazard problem between the protection buyer and seller. However, margins have a potential downside. To the extent that margins insure against counterparty risk, they make the protection buyer more willing to accept risk-taking by the protection seller. Our analysis thus supports as well as challenges the view that market infrastructures that enable implementation of margin calls can reduce the vulnerability of the financial system to systemic risk.[6]

When the failure of financial institutions affects third parties, privately optimal contracts that entail risk-taking, including those with margins, may not be socially optimal. In this case, one way to mitigate systemic risk is to impose capital requirements. Requiring a financial institution to hold capital in proportion to its hedging activities counters the hidden leverage embedded in these activities. Having more to lose in case of default ("skin in the game") reduces risk-taking incentives. By extension, financial institutions that opt out of such capital requirements should not be allowed to undertake hedging activities that are conducive to risk-taking.

The remainder of the paper is organized as follows. In Section 2, we describe the model setup. In Section 3, we analyze the benchmark case in which effort is observable and there is no moral hazard. In Section 4, we analyze the optimal contract when effort is unobservable. To develop the trade-off between risk-sharing and risk-taking, we assume temporarily that the technology to have margin is not available. In Section 5, we discuss the implementation of the optimal contract. In Section 6, we analyze the optimal contract with margins. In Section 7, we examine regulatory interventions when there is a wedge between private and social welfare. Section 8 analyzes the case when there are multiple protection sellers who can reinsure or trade with each other. Section 9 concludes. All proofs are in the appendix.

[6] See, for example, Pirrong (2009).

2 The model

There are three dates, $t = 0, 1,2$, and two agents, the protection buyer and the protection seller, who can enter a risk-sharing contract at $t = 0$.

Protection buyer. The protection buyer is risk-averse with twice differentiable concave utility function, denoted by u. At $t = 0$ he is endowed with a risky exposure of size I whose per unit return is $\tilde{\theta}$. The return is realized at $t = 2$. It can take on two values: $\bar{\theta}$ with probability π and $\underline{\theta}$ with probability $1 - \pi$. Moreover, the buyer has an amount C of cash which has zero net return. He seeks insurance against the risk $\tilde{\theta}$.

Protection seller. The protection seller is risk-neutral. At time $t = 0$ she has an amount A of assets in place which have an uncertain per unit return $\tilde{R}$ at $t = 2$ (balance sheet risk).

At $t = 1$ the protection seller has to exert costly unobservable effort e to manage the risk of her assets. To capture the moral hazard problem in the simplest possible way, we assume that the protection seller can choose between effort, $e = 1$, and no effort, $e = 0$. If she exerts effort, we assume that $A\tilde{R}(e = 1) = AR > A$. If she does not exert effort, then $A\tilde{R}(e = 0) = AR$ with probability p and $A\tilde{R}(e = 0) = 0$ with probability $1 - p$. That is, if the seller does not manage the risk of her balance sheet then she may default her obligations since she protected by limited liability (counterparty risk).

If the seller does not exert effort, she obtains a private benefit B per unit of assets on her balance sheet. That is, the cost of controlling balance sheet risk is proportional to the size of the balance sheet. Note that the impact of the seller's effort on $\tilde{R}$ does not depend on the return of the buyer's asset $\tilde{\theta}$. We assume that the opportunity cost of not exerting effort is higher than the private benefit: $(1 - p) R > B$. Hence, the protection seller prefers effort to no effort if she is solely concerned with managing the risk of her assets.

Since the seller is protected by limited liability, she does not internalize the losses that may occur to others if she does not exert effort. Even though not exerting effort can be privately optimal, it may not be socially optimal. We will discuss social optimality and the scope for regulation in Section 7. Until then, we simply speak of "optimality" when meaning private optimality.

Advance information. Information about the risk $\tilde{\theta}$ is publicly revealed at $t = 0.5$, before the seller makes her effort decision at $t = 1$. Specifically, a signal $\tilde{s}$ about the return $\tilde{\theta}$ is observed. Let λ be the probability of a correct signal:

$$\lambda = \text{prob}[\bar{s}|\bar{\theta}] = \text{prob}[\underline{s}|\underline{\theta}]$$

The probability π is updated to $\bar{\pi}$ upon observing $\bar{s}$ and to $\underline{\pi}$ upon observing $\underline{s}$, where

$$\begin{aligned} \bar{\pi} &= \text{prob}[\bar{\theta}|\bar{s}] = \frac{\text{prob}[\bar{s}|\bar{\theta}]\text{prob}[\bar{\theta}]}{\text{prob}[\bar{s}]} = \frac{\lambda\pi}{\lambda\pi + (1-\lambda)(1-\pi)} \\ \underline{\pi} &= \text{prob}[\bar{\theta}|\underline{s}] = \frac{\text{prob}[\underline{s}|\bar{\theta}]\text{prob}[\bar{\theta}]}{\text{prob}[\underline{s}]} = \frac{(1-\lambda)\pi}{(1-\lambda)\pi + \lambda(1-\pi)} \end{aligned}$$

according to Bayes' Law.

We assume that $\lambda \geq \frac{1}{2}$. If $\lambda = \frac{1}{2}$, $\bar{\pi} = \pi = \underline{\pi}$ and the signal is completely uninformative. It is as if there was no advance information about the risk θ. For $\lambda > \frac{1}{2}$, $\bar{\pi} > \pi > \underline{\pi}$, observing $\bar{s}$ increases the probability of $\tilde{\theta} = \bar{\theta}$ (good signal) whereas observing $\underline{s}$ decreases the probability of $\tilde{\theta} = \bar{\theta}$ (bad signal). If $\lambda = 1$, the signal is perfectly informative and it is as if the realization of $\tilde{\theta}$ was already observed at $t = 0.5$.

Contract. The contract specifies a transfer τ from the protection seller to the protection buyer, conditional on all contractible information (in case $\tau < 0$, the buyer pays the seller). The contract can also specify a margin α to be deposited by the protection seller as cash with a third party (see below). We assume that the realization of $\tilde{\theta}$, the return on the seller's assets $\tilde{R}$ and the advance signal $\tilde{s}$ are all publicly observable and contractible. Hence, the contract is given by $\tau = \tau(\tilde{\theta}, \tilde{R}, \tilde{s})$. The contract must also be consistent with the limited liability of the protection seller, so that $A\tilde{R} > \tau(\tilde{\theta}, \tilde{R}, \tilde{s})$. We assume $AR > I\Delta\theta$, which implies that as long as the agent exerts effort, the limited liability constraint does not bind.

Margins. A margin is a technology that allows the protection seller to liquidate a fraction α of her assets for cash and to deposit the cash on a separate account when requested to do so by the protection buyer. Thus, αA earns the risk-free rate (which we normalize to one), while $(1-\alpha)A$ continues to earn the return $\tilde{R}$ whose distribution depends on the effort choice of the protection seller. The liquidation of assets into cash and the transfer of cash into a separate account removes the fraction α of the seller's assets from her balance sheet. Those assets are ring-fenced from the seller's moral hazard problem. The cash is unaffected by the default of the seller and can be used to service the buyer's claim. At the same time, the seller no longer obtains private benefits on those assets since he no longer controls them.

We will consider two types of margins. An initial margin is a requirement to deposit cash at $t = 0$, when the protection buyer and seller enter a risk-sharing contract. A variation margin is a requirement to deposit cash after advance information about the risk θ is observed.

The sequence of events is summarized in Figure 1 below.

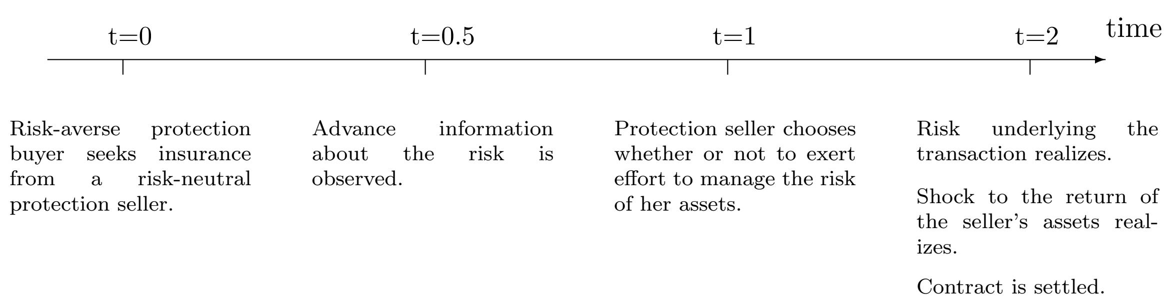

Figure 1: The timing of events

3 First-best: observable effort

In this section we consider the case where the protection buyer can observe the effort level of the protection seller so that there is no moral hazard problem. While implausible, this case will enable us to identify the inefficiencies generated by moral hazard.

Consider the case where the protection buyer instructs the protection seller to exert effort after both a good and a bad signal. In that case the seller's assets always return $\tilde{R}(1) = R$. Hence we don't need explicitly write $\tilde{R}$ when writing the variables upon which τ is contingent. Since the seller never defaults when exerting effort, the margin is never transferred to the protection buyer. Also, as will be clear below, under our assumption that $AR > I\Delta\theta$, the limited liability constraint of the agent does not bind when he exerts effort. Hence, for simplicity we neglect that constraint.

The protection buyer solves

$$\max_{\alpha,\tau(\bar{\theta},\bar{s}),\tau(\underline{\theta},\bar{s}),\tau(\bar{\theta},\underline{s}),\tau(\underline{\theta},\underline{s})} \pi\lambda u(C + I\bar{\theta} + \tau(\bar{\theta},\bar{s})) + (1-\pi)(1-\lambda)u(C + I\underline{\theta} + \tau(\underline{\theta},\bar{s})) \tag{1}$$

$$+\pi(1-\lambda)u(C + I\bar{\theta} + \tau(\bar{\theta},\underline{s})) + (1-\pi)\lambda u(C + I\underline{\theta} + \tau(\underline{\theta},\underline{s}))$$

subject to the seller's participation constraint

$$\pi\lambda[\alpha A + (1-\alpha)\,AR - \tau(\bar{\theta},\bar{s})] + \pi(1-\lambda)[\alpha A + (1-\alpha)\,AR - \tau(\bar{\theta},\underline{s})]$$
$$+ (1-\pi)\lambda[\alpha A + (1-\alpha)\,AR - \tau(\underline{\theta},\underline{s})] + (1-\pi)(1-\lambda)[\alpha A + (1-\alpha)\,AR - \tau(\underline{\theta},\bar{s})] \geq AR$$

where $0 \leq \alpha \leq 1$ is a fraction of assets to be deposited as an initial margin.[7] The expression on the right-hand side is seller's payoff if she does not enter the transaction. It is given by the return on her capital, AR.

The participation constraint can be written as

$$\alpha A\,(1-R) \geq \pi\left[\lambda\tau(\bar{\theta},\bar{s}) + (1-\lambda)\tau(\bar{\theta},\underline{s})\right] + (1-\pi)\left[\lambda\tau(\underline{\theta},\underline{s}) + (1-\lambda)\,\tau(\underline{\theta},\bar{s})\right] = E[\tau] \tag{2}$$

where the expectation is over $\tilde{\theta}$ and $\tilde{s}$. If margins are not used ($\alpha = 0$), the protection seller agrees to the contract as long as the average payment to the buyer is non-positive. If

[7]The case of a variation margin can be written analogously. Unlike an initial margin, which affects the seller's return on assets in all four states of the world, a variation margin is called after a particular signal, thus affecting only two states.

margins are used ($\alpha > 0$), the protection seller needs to be compensated for the opportunity cost of putting cash aside, $R-1$. The Proposition 1 states the solution to this maximization problem. It is easy to show that the corresponding value function is greater than what would be obtained if effort was not always requested. Thus, we can state our first result.

Proposition 1 (First-best contract) *When effort is observable, the optimal contract entails effort after both signals, provides full insurance, and is actuarially fair. Margins are not used. The transfers are given by:*

$$\begin{aligned}\tau^{FB}(\bar{\theta}, \bar{s}) &= \tau^{FB}(\bar{\theta}, \underline{s}) = -(1-\pi)I\Delta\theta = I(E[\tilde{\theta}] - \bar{\theta}) < 0 \\ \tau^{FB}(\underline{\theta}, \bar{s}) &= \tau^{FB}(\underline{\theta}, \underline{s}) = \pi I\Delta\theta = I(E[\tilde{\theta}] - \underline{\theta}) > 0\end{aligned}$$

In the first-best contract, the consumption of the protection buyer is equalized across states (full insurance). The contract does not react to the signal. Since the only effect of margins here is to tighten the seller's participation constraint, they are not used. Expected transfers are zero and there are no rents to the protection seller. The seller pays the buyer if $\tilde{\theta} = \underline{\theta}$ and vice versa if $\tilde{\theta} = \bar{\theta}$. The payments are proportional to the size of the position I and to its riskiness, measured by $\Delta\theta$.

It is optimal for the protection buyer to demand effort after both signals. He is fully insured and the seller's assets are safe so there is no counterparty risk. If there was no effort, the buyer would be exposed to counterparty risk and full insurance would no longer be possible.

Finally, note that the values of the transfers given in the proposition confirms our initial claim that, under our assumption that $AR > I\Delta\theta$, the limited liability condition does not bind.

4 Unobservable effort, no margins

4.1 Effort after both signals

We now turn to the case in which the effort level of the protection seller is not observable. In this section, we characterize the optimal contract assuming that margins are not used, $\alpha = 0$. This provides a useful benchmark to assess the effect of margins on incentives in Section 6.

We first consider a contract that induces effort of the seller after both a good and a bad signal.

As the protection buyer expects the seller to always exert effort and control the risk of her balance sheet, $\tilde{R}(1) = R$. Hence, the contract does not need to be contingent on $\tilde{R}$. Thus, the protection buyer solves (1) subject to (2) and the seller's incentive compatibility constraints. Since the signal about the risk θ is observed before the effort decision is made, the incentive constraints are conditional on the realization of the signal.

Suppose a good signal, $\tilde{s} = \bar{s}$, is observed. Then, the incentive-compatibility constraint is given by

$$\bar{\pi}[AR - \tau(\bar{\theta}, \bar{s})] + (1 - \bar{\pi})[AR - \tau(\underline{\theta}, \bar{s})] \geq$$
$$\bar{\pi}[p(AR - \tau(\bar{\theta}, \bar{s}))] + (1 - \bar{\pi})[p(AR - \tau(\underline{\theta}, \bar{s}))] + BA$$

The expression on the right-hand side is seller's (out-of-equilibrium) expected payoff if she does not exert effort. With probability $1 - p$, the seller defaults and she cannot make any positive payment to the protection buyer. The buyer, in turn, has no interest in making a payment to the seller since it would make it more difficult to satisfy the incentive constraint. Hence $\tau(\tilde{\theta}, \tilde{s}\ |\text{default}) = 0$. The incentive-compatibility constraint after a bad signal, $\tilde{s} = \underline{s}$, is derived analogously.

Simplifying the incentive constraint for each realization of the signal, we get:

$$\begin{aligned}\mathcal{P} &\geq \bar{\pi}\tau(\bar{\theta},\bar{s})+(1-\bar{\pi})\tau(\underline{\theta},\bar{s}) \\ \mathcal{P} &\geq \underline{\pi}\tau(\bar{\theta},\underline{s})+(1-\underline{\pi})\tau(\underline{\theta},\underline{s})\end{aligned}$$

where

$$\mathcal{P}=A\left(R-\frac{B}{1-p}\right) \tag{3}$$

denotes “pledgeable income” (as in Tirole, 2005). Available pledgeable income puts an upper bound on the expected transfer to the protection buyer, conditional on the observed signal. Note that $\mathcal{P}>0$ since we assumed $(1-p)\,R>B$.

It will be useful to introduce the following notation for the expected transfer conditional on the signal:

$$\bar{\tau}\equiv E[\tau|\bar{s}]=\bar{\pi}\tau(\bar{\theta},\bar{s})+(1-\bar{\pi})\tau(\underline{\theta},\bar{s}) \tag{4}$$

$$\underline{\tau}\equiv E[\tau|\underline{s}]=\underline{\pi}\tau(\bar{\theta},\underline{s})+(1-\underline{\pi})\tau(\underline{\theta},\underline{s}). \tag{5}$$

The incentive constraints become

$$\mathcal{P} \geq \bar{\tau} \tag{6}$$

$$\mathcal{P} \geq \underline{\tau} \tag{7}$$

and the participation constraint (2) becomes

$$0\geq \text{prob}[\underline{s}]\underline{\tau}+\text{prob}[\bar{s}]\bar{\tau} \tag{8}$$

Lemma 1 (First-best attainable) *When effort is not observable and the signal is informative, the first-best can be achieved if and only if the protection seller has enough pledgeable income, i.e., for* $\mathcal{P}>(\pi-\underline{\pi})I\Delta\theta=I(E[\tilde{\theta}]-E[\tilde{\theta}|\underline{s}])$.

For sufficiently high pledgeable income levels, incentive-compatibility constraints are not binding and the first-best allocation can be reached even when effort is not observable. The threshold level of pledgeable income beyond which the first-best is attainable, $(\pi - \underline{\pi})I\Delta\theta$, is proportional to the size of the position I, to its riskiness $\Delta\theta$, and to the informativeness of the signal λ (which induces a higher wedge between the prior and the updated probability). We can state the following corollary.

Corollary 1 *When the signal is uninformative, the first-best is always reached:* $\mathcal{P} > (\pi - \underline{\pi})I\Delta\theta = 0$*.*

Consider the case when the signal is informative and the pledgeable income is small enough so that the first-best is not attainable. To ensure that the protection seller always exerts effort, the optimal contract must satisfy two incentive-compatibility constraints. The next lemma states that only one of them will be binding.

Lemma 2 (Incentives given the signal) *When effort is not observable and the first-best is not attainable,* $\mathcal{P} < (\pi - \underline{\pi})I\Delta\theta$*, the incentive constraint after a good signal is slack whereas the incentive constraint after a bad signal is binding.*

Ex ante, before the signal is observed, the derivative position is neither an asset nor a liability for the protection seller. After observing a good signal about the underlying risk, the position is more likely to be an asset for the seller. He is more likely to be paid by the buyer than the other way around. Thus, good news do not generate incentive problems. Negative news, on the other hand, make it more likely that the position moves against the seller. Now it is the seller who is more likely to pay the buyer. For $\mathcal{P} < (\pi - \underline{\pi})I\Delta\theta$, this undermines her incentives to exert effort. She has to bear the full cost of effort while the benefit accrues in part to the protection buyer. This is reminiscent of the debt-overhang effect (Myers, 1977). The derivative position contains *hidden leverage* that affects seller's incentives to control her balance sheet risk when she has limited pledgeable income.

The following proposition characterizes the optimal contract with effort after both signals.

Proposition 2 (Optimal contract with risk-control) *When effort is not observable and the first-best is not attainable, $\mathcal{P} < (\pi - \underline{\pi}) I \Delta\theta$, the optimal contract that induces effort after both signals provides full insurance conditional on the signal and is actuarially fair. The transfers are given by:*

$$\begin{aligned}
\tau^{SB,e=1}(\bar{\theta}, \bar{s}) &= -(1-\bar{\pi}) I \Delta\theta - \frac{prob[\underline{s}]}{prob[\bar{s}]}\mathcal{P} = I(E[\tilde{\theta}|\bar{s}] - \bar{\theta}) - \frac{prob[\underline{s}]}{prob[\bar{s}]}\mathcal{P} < 0 \\
\tau^{SB,e=1}(\underline{\theta}, \bar{s}) &= \bar{\pi} I \Delta\theta - \frac{prob[\underline{s}]}{prob[\bar{s}]}\mathcal{P} = I(E[\tilde{\theta}|\bar{s}] - \underline{\theta}) - \frac{prob[\underline{s}]}{prob[\bar{s}]}\mathcal{P} > 0 \\
\tau^{SB,e=1}(\bar{\theta}, \underline{s}) &= -(1-\underline{\pi}) I \Delta\theta + \mathcal{P} = I(E[\tilde{\theta}|\underline{s}] - \bar{\theta}) + \mathcal{P} < 0 \\
\tau^{SB,e=1}(\underline{\theta}, \underline{s}) &= \underline{\pi} I \Delta\theta + \mathcal{P} = I(E[\tilde{\theta}|\underline{s}] - \underline{\theta}) + \mathcal{P} > 0
\end{aligned}$$

As in the first-best contract, the participation constraint binds and there are no rents to the protection seller. Expected transfers are zero so that the contract is actuarially fair.

The key difference to the first-best contract is that the transfers now depend on the signal:

$$\tau^{SB,e=1}(\tilde{\theta}, \underline{s}) < \tau^{FB}(\tilde{\theta}, \underline{s}) = \tau^{FB}(\tilde{\theta}, \bar{s}) < \tau^{SB,e=1}(\tilde{\theta}, \bar{s})$$

To preserve the seller's incentives to exert effort, the buyer must reduce the hidden leverage by accepting incomplete risk-sharing. In particular, the incentive-compatible amount of insurance is smaller following a bad signal. Hence, the protection buyer must bear signal risk. Correspondingly, the protection seller must be left with some rent after a bad signal in order to induce effort. The protection buyer "reclaims" this rent after a good signal so that the expected rent to the seller is zero.[8]

Conditional on the signal, the optimal contract provides full insurance against the underlying risk $\tilde{\theta}$:

$$\tau(\underline{\theta}, \bar{s}) - \tau(\bar{\theta}, \bar{s}) = \tau(\underline{\theta}, \underline{s}) - \tau(\bar{\theta}, \underline{s}) = I \Delta\theta > 0 \tag{9}$$

Since there is full insurance conditional on the signal, we can rewrite the objective function

[8]Interpreting signals as types, there is then cross-subsidization between types.

of the risk-averse protection buyer (1) as

$$\text{prob}[\underline{s}]u(C + IE[\theta|\underline{s}] + \underline{\tau}) + \text{prob}[\bar{s}]u(C + IE[\theta|\bar{s}] + \bar{\tau}) \quad (10)$$

where we have used (4) and (5).

Figure 2 illustrates our results so far in the contract space $(\underline{\tau}, \bar{\tau})$. The relevant part of the contract space is when $\underline{\tau} \geq 0$ (x-axis) and when $\underline{\tau} \leq 0$ (y-axis). After a bad signal the protection seller is more likely to pay the protection buyer than vice versa. The opposite holds after a good signal.

Insert Figure 2 here

The participation constraint of the protection seller (8) is a line through the origin with slope $-\frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]}$. The protection seller agrees to any contract that lies on or below this line. Contracts that lie on the line are actuarially fair since expected transfers are zero. The slope gives the "relative price" at which the risk-neutral protection seller is willing to exchange expected transfers after a good and a bad signal.

The indifference curves corresponding to (10) are decreasing, convex curves in the contract space $(\underline{\tau}, \bar{\tau})$.[9] The utility of the protection buyer increases as he moves to the north-east in the figure.

The first-best allocation is given by point A where the indifference curve of the protection buyer is tangent to the participation constraint of the protection seller. The first-best allocation achieves full insurance. In the first-best, transfers are independent from the realization of the signal for a given realization of θ, i.e., $\tau(\theta, \bar{s}) = \tau(\theta, \underline{s})$ for all θ. Using (4), (5) and (9), such transfers yield

$$\bar{\tau} = -I(E[\tilde{\theta}|\bar{s}] - E[\tilde{\theta}|\underline{s}]) + \underline{\tau} \quad (11)$$

[9]The slope of the indifference curve is given by $\frac{d\bar{\tau}}{d\underline{\tau}} = -\frac{\text{prob}[\underline{s}]\underline{u}'}{\text{prob}[\bar{s}]\bar{u}'} < 0$, where $\underline{u}' \equiv u'(C + IE[\theta|\underline{s}] + \underline{\tau})$ and $\bar{u}' \equiv u'(C + IE[\theta|\bar{s}] + \bar{\tau})$. The change in the slope is $\frac{d^2\bar{\tau}}{d\underline{\tau}^2} = -\frac{\text{prob}[\underline{s}]\underline{u}''\bar{u}'}{(\text{prob}[\bar{s}]\bar{u}')^2} > 0$.

Thus, in the $(\underline{\tau}, \bar{\tau})$ plane, these transfers lie on the 45° line that intersects the x-axis at $\underline{\tau} = I(E[\tilde{\theta}|\bar{s}] - E[\tilde{\theta}|\underline{s}])$ and that intersects the participation constraint at point A.

Point B illustrates the optimal contract with effort. The vertical line that intersects the x-axis at $\underline{\tau} = \mathcal{P}$ represents the incentive constraint. The protection seller only exerts effort after a bad signal if the contract lies on or to the left of the line. The figure is drawn for $\mathcal{P} < I(E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}])$ so that the first-best allocation is not attainable when effort is not observable (Lemma 1). The contract achieving the highest utility for the protection buyer lies at the intersection of the incentive and the participation constraint. He is worse off than with the first-best allocation. The indifference curve passing through B lies strictly below the one passing through A.

4.2 No effort after a bad signal

The protection buyer may find the reduced risk-sharing in the contract with effort after both signals too costly. He may instead choose to accept risk-taking by the protection seller in exchange for more sharing of the risk θ. Since it is always in the interest of the seller to exert effort after a good signal, risk-taking can only occur after a bad signal. In this subsection, we characterize the optimal contract with effort after good news and no effort after bad news.

The objective function of the protection buyer is then given by:

$$\max_{\tau(\bar{\theta},\bar{s}),\tau(\underline{\theta},\bar{s}),\tau(\bar{\theta},\underline{s}),\tau(\underline{\theta},\underline{s})} \pi\lambda u(C + I\bar{\theta} + \tau(\bar{\theta},\bar{s})) + (1-\pi)(1-\lambda)u(C + I\underline{\theta} + \tau(\underline{\theta},\bar{s})) \tag{12}$$

$$+\pi(1-\lambda)[pu(C + I\bar{\theta} + \tau(\bar{\theta},\underline{s})) + (1-p)u(C + I\bar{\theta})]$$

$$+(1-\pi)\lambda[pu(C + I\underline{\theta} + \tau(\underline{\theta},\underline{s})) + (1-p)u(C + I\underline{\theta})]$$

The contract entails risk-taking following a bad signal. With probability $1-p$ the seller may default. The seller's default is a contractible event and it is (privately) optimal to set the transfer equal to zero in that case.[10]

[10]The transfer can only be from the buyer to the seller but any transfer is wasted due to the seller's default.

The incentive-compatibility constraints are given by

$$\mathcal{P} \geq \bar{\tau} \tag{13}$$

$$\mathcal{P} < \underline{\tau} \tag{14}$$

The seller exerts effort after a good signal. Following a bad signal, she prefers to run the risk of default when the expected transfers to the buyer are sufficiently high.

The seller's participation constraint is

$$\text{prob}[\bar{s}](AR - \bar{\tau}) + \text{prob}[\underline{s}][p(AR - \underline{\tau}) + AB] \geq AR$$

or, equivalently,

$$-\text{prob}[\underline{s}](1-p)\mathcal{P} \geq \text{prob}[\bar{s}]\bar{\tau} + \text{prob}[\underline{s}]p\underline{\tau} \tag{15}$$

The expected transfer from the seller to the buyer (right-hand side) is negative. If seller enters the position, she must be compensated for the potential loss of pledgeable income due to the lack of effort after bad news (left-hand side). Note that higher pledgeable income makes it more difficult for a protection seller to accept a contract with no effort. Higher returns on seller's assets AR increase the outside opportunity of the seller, and they may not materialize after entering the contract. Similarly, a smaller private benefit B reduces the value of the contract by reducing the benefit of not exerting effort after a bad signal. An implication is that apparently expensive derivative contracts sold by well established names (high $\mathcal{P}$) are an early warning of future risk-taking.[11]

The following proposition characterizes the optimal contract with effort after a good signal and no effort after a bad signal.

A positive transfer from the buyer in the default state may, however, be optimal from a social point of view. Such a transfer induces the buyer to internalize the costs stemming from the risk-taking by the seller. We return to this issue in Section 7.

[11]This is reminiscent of Biais, Rochet and Woolley (2009) although the mechanism is different. In that paper, large rents are a precursor of risk-taking. When rents become too large, investors prefer to give up on incentives and accept risk-taking. In our analysis, however, the protection seller never earns rents.

Proposition 3 (Optimal contract with risk-taking) *When effort is not observable and the first-best is not attainable, $\mathcal{P} < (\pi - \underline{\pi})I\Delta\theta$, the optimal contract with risk-taking after a bad signal provides full insurance conditional on no default and is actuarially unfair. The transfers are given by:*

$$\begin{aligned}
\tau^{e=1,e=0}(\bar{\theta}, \bar{s}) &= \tau^{e=1,e=0}(\bar{\theta}, \underline{s}) = -(1-\pi)\, I\Delta\theta \frac{1 - prob[\underline{s}|\underline{\theta}](1-p)}{1 - prob[\underline{s}](1-p)} - \frac{\mathcal{P} prob[\underline{s}](1-p)}{1 - prob[\underline{s}](1-p)} < 0 \\
\tau^{e=1,e=0}(\underline{\theta}, \bar{s}) &= \tau^{e=1,e=0}(\underline{\theta}, \underline{s}) = \pi I \Delta\theta \frac{1 - prob[\underline{s}|\bar{\theta}](1-p)}{1 - prob[\underline{s}](1-p)} - \frac{\mathcal{P} prob[\underline{s}](1-p)}{1 - prob[\underline{s}](1-p)} > 0
\end{aligned}$$

Again, there are no rents to the protection seller (the participation constraint is binding). The seller pays the buyer if $\tilde{\theta} = \underline{\theta}$ and vice versa if $\tilde{\theta} = \bar{\theta}$:

$$\tau^{e=1,e=0}(\underline{\theta}, \tilde{s}) > 0 > \tau^{e=1,e=0}(\bar{\theta}, \tilde{s})$$

There are three differences between the optimal contract with and without effort after bad news. First, the contract with risk-taking does not react to the signal:

$$\tau^{e=1,e=0}(\tilde{\theta}, \bar{s}) = \tau^{e=1,e=0}(\tilde{\theta}, \underline{s})$$

As long the protection seller does not default, the consumption of the buyer is equalized across states (as in the first-best contract). Second, unlike in the contract with effort, the buyer is now exposed to counterparty risk. He is completely uninsured in the default state. Third, the contract with no effort after a bad signal is not actuarially fair since expected transfers are not equal to zero.

Figure 3 illustrates the optimal contract without effort after a bad signal. The optimal allocation is given by point C where the participation constraint and the indifference curve that both account for the possibility of default are tangent. Compared to the case with effort, the line representing the participation constraint is less steep. The protection seller requires less compensation in terms of $\bar{\tau}$ for a higher expected transfer $\underline{\tau}$ since she sometimes defaults, in which case she does not have to make this transfer. The line also no longer intersects the y-axis at zero. Even if the protection seller made no transfer to the protection buyer after a bad signal, she would still require a transfer from him to accept the contract. She must be compensated for the loss of pledgeable income due to the lack of effort after bad news. The participation constraint without effort intersects the one with effort at point B where $\underline{\tau} = \mathcal{P}$.

Insert Figure 3 here

The utility of the protection buyer (12) in terms of expected transfers conditional on the signal is

$$\text{prob}[\bar{s}]u(C + IE[\tilde{\theta}|\bar{s}] + \bar{\tau}) + \text{prob}[\underline{s}]\left[pu(C + IE[\tilde{\theta}|\underline{s}] + \underline{\tau}) + (1-p)E[u(C + I\tilde{\theta})|\underline{s}]\right]$$

After a bad signal, the protection buyer does not receive the transfer with probability $1-p$ and is fully exposed to the risk θ conditional on the bad signal.

The possibility of default of the protection seller lowers the slope of the indifference curves of the protection buyer. For any transfer $\underline{\tau}$ he receives, he is now less willing to compensate the protection seller by paying her $\bar{\tau}$ since she may default, in which case he does not receive $\underline{\tau}$.[12]

When the protection seller exerts no effort after a bad signal, the protection buyer is exposed to counterparty risk but is fully insured against signal risk. The point C therefore lies on the 45^o line as contracts on this line do not depend on the signal (see equation (11)). Since the point C lies to the right of the vertical line through $\underline{\tau} = \mathcal{P}$, the incentive constraint (14) is satisfied.

[12]Formally, the slope of the indifference curve is now given by $\frac{d\bar{\tau}}{d\underline{\tau}} = -\frac{\text{prob}[\underline{s}]p\underline{u}'}{\text{prob}[\bar{s}]\bar{u}'}$.

4.3 Risk-sharing and risk-taking

The contract with effort after both signals entails limited risk-sharing but no risk-taking, while the contract with no effort after a bad signal entails full risk-sharing but allows risk-taking after bad news. In this section, we examine under what conditions it is privately optimal to allow risk-taking.

Proposition 4 (Endogenous counterparty risk) *Suppose effort is not observable and the first-best is not attainable, $\mathcal{P} < (\pi - \underline{\pi})I\Delta\theta$. There exists a threshold level of pledgeable income $\hat{\mathcal{P}}$ such that the contract with risk-control is optimal if and only if $\mathcal{P} \geq \hat{\mathcal{P}}$. If the probability of default is sufficiently small, $\hat{\mathcal{P}} > 0$.*

The key factor in the choice between the optimal contract with risk-control and with risk-taking is whether counterparty or signal risk is more costly for the protection buyer. For low levels of pledgeable income, the moral hazard problem is severe. Providing incentives to avoid risk-taking after a bad signal, requires a considerable reduction in hidden leverage. The buyer then has to bear a lot of signal risk. If, at the same time, default is unlikely (p is high), the counterparty risk under the risk-taking contract is small. It is then optimal for the protection buyer to allow risk-taking by the protection seller.

Counterparty risk thus arises endogenously due to moral hazard. In particular, counterparty risk occurs when the probability of default $1-p$ is *small*. Note that pledgeable income $\mathcal{P}$ is increasing in the return of seller's assets, R. Hence, privately optimal risk-sharing contracts are more likely to allow risk-taking in an environment of low returns.

5 Implementation

In this section we examine how the optimal contract $\tau(\tilde{\theta}, \tilde{s})$ can be implemented using forward contracts.

5.1 First-Best

According to Proposition 1 the optimal contract does not depend on the signal s when effort is observable. A forward written on the buyer's risk θ is therefore enough. In order for a forward to implement the payments specified by the optimal contract, we must have

$$\begin{aligned} q_\theta(F_\theta - \bar{\theta}) &= I(E[\tilde{\theta}] - \bar{\theta}) \\ q_\theta(F_\theta - \underline{\theta}) &= I(E[\tilde{\theta}] - \underline{\theta}), \end{aligned}$$

where F_θ is the price of the forward and q_θ is the quantity bought. Solving for q_θ and F_θ yields

$$\begin{aligned} q_\theta &= I \\ F_\theta &= E[\tilde{\theta}] \end{aligned}$$

The protection buyer *sells* a forward on the risk θ. The amount sold fully covers the size of the risky position I. The price of the forward is equal to the expected value of the underlying risk.

5.2 Risk-control

The optimal contract that preserves the incentives of the protection seller to control her balance sheet risk depends on the risk θ and on the advance information s about it (see Proposition 2). A "plain vanilla" forward cannot deliver such payments. Moreover, the optimal contract eliminates rents for the seller by enabling cross-subsidization across signals. With a simple forward such cross-subsidization is not feasible and the protection seller would obtain rents.

But a combination of two forwards can implement the optimal contract that preserves the protection sellers' incentives. One forward (q_θ, F_θ) is written, as before, on the risk θ.

The other forward (q_s, F_s) is written on $E[\tilde{\theta}|s]$, the expected value of the risk θ conditional on the signal s. Writing such a forward is possible since the signal s is contractible and the probability distributions for s and θ are known. Suppose, for example, that the buyer's risk θ stems from illiquid loans and the advance information consists of an indicator of business conditions, e.g., a survey of entrepreneurs. Then one forward specifies payments according to the realization of loan defaults and the other forward specifies payments according to the outcome of the survey.

In order to implement the optimal contract, the payments from the two forwards must satisfy

$$
\begin{aligned}
q_\theta(F_\theta - \bar{\theta}) + q_s(F_s - E[\tilde{\theta}|\bar{s}]) &= I(E[\tilde{\theta}|\bar{s}] - \bar{\theta}) - \frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]}\mathcal{P} \\
q_\theta(F_\theta - \underline{\theta}) + q_s(F_s - E[\tilde{\theta}|\bar{s}]) &= I(E[\tilde{\theta}|\bar{s}] - \underline{\theta}) - \frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]}\mathcal{P} \\
q_\theta(F_\theta - \bar{\theta}) + q_s(F_s - E[\tilde{\theta}|\underline{s}]) &= I(E[\tilde{\theta}|\underline{s}] - \bar{\theta}) + \mathcal{P} \\
q_\theta(F_\theta - \underline{\theta}) + q_s(F_s - E[\tilde{\theta}|\underline{s}]) &= I(E[\tilde{\theta}|\underline{s}] - \underline{\theta}) + \mathcal{P}.
\end{aligned}
$$

Solving for the quantities of the forwards yields

$$
\begin{aligned}
q_\theta &= I \\
q_s &= I\left(\frac{P}{I}\frac{1}{E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}]} - 1\right)
\end{aligned}
$$

As in the implementation of the First-Best, the protection buyer sells a forward on his risk θ and the amount of protection fully covers the size of his position. In addition, he *buys* a forward on the signal risk s. When the First-Best is not reachable, the expression is round brackets is negative since there is not enough pledgeable income to support the hidden liability embedded in the risk-sharing contract (Lemma 1).

Any combination of forward prices satisfying

$$F_\theta + F_s\left(\frac{P}{I}\frac{1}{E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}]} - 1\right) = \frac{\mathcal{P}}{I}\frac{E[\tilde{\theta}]}{E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}]}$$

implements the optimal contract. When the price of the price of the forward on θ is equal to the expected value of the risk, $F_\theta = E[\tilde{\theta}]$, then the prices of the two forwards are equal, $F_s = E[\tilde{\theta}]$. If the forward on θ is cheap, $F_\theta < E[\tilde{\theta}]$, then the forward on s is expensive, $F_s > E[\tilde{\theta}]$.

5.3 Risk-taking

The optimal contract with risk-taking by the protection seller does not depend on the signal (see Proposition 3). It can therefore be implemented with a single forward provided that it has the added feature of freeing the protection buyer from the obligation to honor the forward if the seller defaults. This provision is needed since the seller's default is a contractible event and any payment from the buyer to the seller is wasted in default.

The payments from the forward must satisfy

$$\begin{aligned} q_\theta(F_\theta - \bar{\theta}) &= I(E[\tilde{\theta}] - \bar{\theta})\frac{1 - \text{prob}[\underline{s}|\underline{\theta}](1-p)}{1 - \text{prob}[\underline{s}](1-p)} - \frac{\mathcal{P}\text{prob}[\underline{s}](1-p)}{1 - \text{prob}[\underline{s}](1-p)} \\ q_\theta(F_\theta - \underline{\theta}) &= I(E[\tilde{\theta}] - \underline{\theta})\frac{1 - \text{prob}[\underline{s}|\bar{\theta}](1-p)}{1 - \text{prob}[\underline{s}](1-p)} - \frac{\mathcal{P}\text{prob}[\underline{s}](1-p)}{1 - \text{prob}[\underline{s}](1-p)}, \end{aligned}$$

which yields the following price and quantity for the forward:

$$\begin{aligned} q_\theta &= I\frac{1 - \text{prob}[\underline{s}|\underline{\theta}](1-p)}{1 - \text{prob}[\underline{s}](1-p)} \\ F_\theta &= E[\tilde{\theta}] - \frac{\mathcal{P}}{I}\frac{\text{prob}[\underline{s}](1-p)}{1 - \text{prob}[\underline{s}](1-p)} \end{aligned}$$

The amount sold by the protection buyer no longer fully covers the size of his position, $q_\theta < I$, even though he is fully hedged against the risk θ (Proposition 3). However, the buyer is exposed to counterparty risk. Moreover, he has to sell the forward at the discount relative to the expected value of the underlying risk. This discount is necessary to compensate the protection seller for the loss of (pledgeable) income in default. The discount depends on the amount of pledgeable income relative to the size of the buyer's position and the probability of seller's default.

6 Margins

In this section, we analyze the incentive effects of margins. In the case of the contract with effort, when risk-sharing is limited by the incentive constraint of the seller, we show how margins can help increase the amount of insurance for the protection buyer. In the case of the contract with risk-taking, when the buyer is exposed to counterparty risk, we examine the role of margins in providing insurance against the seller's default.

6.1 Margins and risk-control

We know that the optimal contract will provide full insurance to the protection buyer conditional on the signal. Thus, we can state his objective function in terms of the expected transfers conditional on the signal. When the protection seller exerts effort, she does not default and the margin need not be transferred to the protection buyer. His objective function therefore is given by

$$\max_{\alpha,\bar{\tau},\underline{\tau}} \text{prob}[\bar{s}]u(C + IE[\tilde{\theta}|\bar{s}] + \bar{\tau}) + \text{prob}[\underline{s}]u(C + IE[\tilde{\theta}|\underline{s}] + \underline{\tau}).$$

We also know that the incentive constraint of the seller does not bind after a good signal. Hence, the protection buyer will not make a margin call in this case. However, the buyer may want to call a margin after a bad signal when seller's incentives to exert effort may be jeopardized. The seller's participation constraint is thus given by:

$$\text{prob}[\bar{s}]AR + \text{prob}[\underline{s}]\left[\alpha A + (1-\alpha)AR\right] - E[\tau] \geq AR$$

or, equivalently,

$$E[\tau] \leq \alpha A\,(1-R)\,\text{prob}[\underline{s}] \tag{16}$$

The expression on the right-hand side is negative and represents the opportunity cost of depositing cash rather than keeping it on the balance sheet after a bad signal. The seller forgoes the net return of assets over cash, $R-1$. Placing a higher margin α makes it more difficult for the protection seller to accept the contract. The opportunity cost of the margin makes the contract actuarially unfair (expected transfers are no longer equal to zero).

The incentive-compatibility constraint after a bad signal is given by:

$$\alpha A + (1-\alpha)\,AR - \underline{\tau} \geq p\left[\alpha A + (1-\alpha)\,AR - \underline{\tau}\right] + (1-\alpha)\,BA$$

If the seller does not exert effort (right-hand side), she earns the private benefit B only on the assets she still controls. There is no private benefit associated with the cash deposited with the clearing house. Higher margins thus reduce the private benefit of risk-taking: the cash is ring-fenced from moral hazard. In case of default, the seller loses the cash deposited as it is transferred to the buyer. We can re-write the incentive constraint as

$$\alpha A + (1-\alpha)\,\mathcal{P} \geq \underline{\tau} \tag{17}$$

where $\mathcal{P}$ denotes, as before, the pledgeable income. For $A > \mathcal{P}$, the margin relaxes the incentive constraint.

Let g denote the fraction of assets that are pledgeable, $g \equiv \frac{\mathcal{P}}{A}$. The following proposition characterizes the optimal contract with margins and risk-control.

Proposition 5 (Optimal margins with risk-control) *Let $\bar{u}'(\bar{\tau})$ and $\underline{u}'(\underline{\tau})$ denote the buyer's marginal utilities conditional on the good and the bad signal, respectively. Margins are used, $\alpha^* > 0$, if and only if:*

$$\frac{\underline{u}'(\underline{\tau}(\alpha^*))}{\bar{u}'(\bar{\tau}(\alpha^*))} \geq 1 + \frac{R-1}{1-g} \tag{18}$$

with equality for $0 < \alpha^* < 1$. Margins are not used, $\alpha^* = 0$, if the reverse inequality holds in (18) at $\alpha = 0$. The expected transfers are given by:

$$\begin{aligned} \underline{\tau} &= \alpha^* A + (1-\alpha^*)\,\mathcal{P} \\ \bar{\tau} &= -\frac{prob[\underline{s}]}{prob[\bar{s}]}\left[\alpha^* AR + (1-\alpha^*)\,\mathcal{P}\right] \end{aligned}$$

The benefit of margins is improved risk-sharing via the transfers $\bar{\tau}(\alpha^*)$ and $\underline{\tau}(\alpha^*)$. The margin itself is never paid to the the protection buyer since the protection seller does not default when she exerts effort. The first-best would be obtained when $\frac{\underline{u}'(\underline{\tau})}{\bar{u}'(\bar{\tau})} = 1$ so that there is full insurance against signal risk. In the first-best,

$$\underline{\tau} - \bar{\tau} = I(E[\tilde{\theta}|\bar{s}] - E[\tilde{\theta}|\underline{s}]). \tag{19}$$

But to preserve the seller's incentives to exert effort when the first-best is not attainable, the protection buyer must bear signal risk and the left-hand side of (19) is bigger than the right-hand side. Since $\frac{\partial \bar{\tau}}{\partial \alpha^*} < 0$ and $\frac{\partial \underline{\tau}}{\partial \alpha^*} > 0$, higher margins reduce the left-hand side, moving the transfers closer to full insurance.

The cost of margins is that they tighten the participation constraint (16) and make the contract with effort actuarially unfair. The optimal margin balances enhanced insurance against signal risk and actuarial fairness. The right-hand side of (18) gives the rate at which the trade-off occurs. The numerator of the fraction, $R-1$, is the opportunity cost of foregone asset return causing the actuarial unfairness of the contract. The denominator measures the severity of the incentive problem that the margin can relax. It decreases when there is higher pledgeable income per unit of capital, g. If $g \geq 1$ so that $\mathcal{P} > A$, margins cannot relax the incentive constraint (17) and offer no benefit. We thus have the following result.

Corollary 2 *When the pledgeable income is higher than the assets in place, $g \geq 1$, margins are not used, $\alpha^* = 0$.*

Figure 4 illustrates the case with margins and risk-control.

Insert Figure 4 here

The margin affects the participation and incentive constraint but leaves the objective function of the protection buyer unchanged. The straight line from point B to point D illustrates how the margin changes the set of feasible contracts. The line is the parametric plot of the binding participation and incentive constraints as α varies from 0 to 1:

$$\begin{aligned} \text{prob}[\underline{s}]\underline{\tau} + \text{prob}[\bar{s}]\bar{\tau} &= \alpha A\,(1-R)\,\text{prob}[\underline{s}] \\ \underline{\tau} &= \alpha A + (1-\alpha)\,\mathcal{P} \end{aligned}$$

The point B represents the optimal contract with effort and no margin (see Section 4.1).

The optimal margin α^* is given by the point of tangency of the protection buyer's indifference curve to the line BD (point E).[13]

The slope of the line BD gives the relative price at which the protection seller is willing to exchange the transfers $\underline{\tau}$ and $\bar{\tau}$ when margins are used, $\alpha > 0$. The slope is steeper than in the case without margins (the line through points B and A) as long as $R > 1$. The protection seller requires more compensation in terms of $\bar{\tau}$ for a higher expected transfer $\underline{\tau}$ since depositing cash carries an opportunity cost.

[13]At the point of tangency, we have $-\frac{\text{prob}[\underline{s}]\underline{u}'}{\text{prob}[\bar{s}]\bar{u}'} = -\frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]} - \frac{(R-1)\text{prob}[\underline{s}]}{(1-g)\text{prob}[\bar{s}]}$. Multiplying both sides of the equality with $-\frac{\text{prob}[\bar{s}]}{\text{prob}[\underline{s}]}$ recovers condition (18) for $\alpha^* > 0$.

6.2 Margins and risk-taking

If the protection seller engages in risk-taking, she defaults with probability $1-p$. If she defaults, the margin is transferred to the protection buyer.

The objective function of the protection buyer is now given by

$$\max_{\alpha,\bar{\tau},\underline{\tau}} \text{prob}[\bar{s}]u(C+IE[\tilde{\theta}|\bar{s}]+\bar{\tau})+\text{prob}[\underline{s}]\left[pu(C+IE[\tilde{\theta}|\underline{s}]+\underline{\tau})+(1-p)E[u(C+I\tilde{\theta}+\alpha A)|\underline{s}]\right]$$

In case the protection seller defaults, the protection buyer obtains the cash deposit, αA.

The participation constraint of the protection seller is given by

$$\text{prob}[\underline{s}]\left[\alpha A\,(1-R)-(1-p)(\alpha A+(1-\alpha)\mathcal{P})\right]\geq \text{prob}[\bar{s}]\bar{\tau}+\text{prob}[\underline{s}]p\underline{\tau}. \tag{20}$$

The left-hand side is the sum of the opportunity cost due to the foregone asset return $R-1$ and the loss conditional on default. The right-hand side is the expected transfer from the protection seller to the buyer. It is the same as in the case without margins (see (15)).

The following proposition characterizes the optimal contract with margins and risk-taking after a bad signal.

Proposition 6 (Optimal margins with risk-taking) *Let $E\left[u'_d\right]$ and u'_{nd} denote the buyer's (expected) marginal utilities when the seller defaults and when she does not, respectively. Margins are used, $\alpha^*>0$, if and only if:*

$$(1-p)\frac{E[u'_d(\alpha^*)|\underline{s}]}{u'_{nd}(\alpha^*)}\geq R-1+(1-p)(1-g) \tag{21}$$

with equality for $0<\alpha^*<1$. Margins are not used, $\alpha^*=0$, if the reverse inequality holds in (21) at $\alpha=0$. The expected transfers are given by (19) and

$$\underline{\tau}=-\alpha^*\frac{prob[\underline{s}]\left[AR-\mathcal{P}+p(A-\mathcal{P})\right]}{1-prob[\underline{s}](1-p)}+\frac{prob[\bar{s}]I(E[\tilde{\theta}|\bar{s}]-E[\tilde{\theta}|\underline{s}])-prob[\underline{s}](1-p)\mathcal{P}}{1-prob[\underline{s}](1-p)} \tag{22}$$

As in the risk-taking contract without margins, the protection buyer gets full insurance when the seller does not default. In this case, his consumption is given by $C + I\underline{\theta} + \underline{\tau}(\alpha^*)$ where the latter is given by (22). He is, however, exposed to counterparty risk since the seller may default with probability $1 - p$ after a bad signal.

The benefit of a margin under the risk-taking contract is the insurance it provides against counterparty risk (left-hand side of (21)). The wedge between the marginal utilities under default and no default is reduced. Margins increase the buyer's expected consumption if the seller defaults, which happens with probability $1 - p$ after a bad signal. At the same time, they reduce his consumption when there is no default since $\frac{\partial \underline{\tau}}{\partial \alpha^*} < 0$. The protection buyer requires a smaller transfer after a bad signal since this is the state in which the margin may be paid to him.

The cost of margins has two components. First, there is the opportunity cost, $R - 1$. Second, there is the loss of income in case of default. The optimal margin under risk-taking balances these costs with the benefit of protecting the buyer from counterparty risk.

6.3 Margins, risk-sharing and risk-taking

Margins can be implemented as escrow accounts set up by a protection buyer or via a market infrastructure such as a central counterparty (CCP). It is privately optimal to use margins whenever $\alpha^* > 0$. When the contract with margins entails risk-control, the margin acts as a commitment device for the protection seller not to take risks once she observes negative news about her hedging position. When the contract entails risk-taking, the margin protects the buyer against the default of the seller.

The choice between the contract with margins and risk-control and the contract with margins and risk-taking depends again on whether counterparty or signal risk is more costly for the protection buyer. As in Section 4.3, the contract with risk-taking may be chosen when pledgeable income is low and the moral hazard problem is severe.

The overall effect of margins on risk-taking, and hence counterparty risk, is ambiguous.

On the one hand, margins reduce the signal risk faced by the buyer and make risk-control by the seller more attractive. On the other hand, margins protect the buyer from counterparty risk and make risk-taking by the seller more attractive. If the latter effect is small, then margins reduce the risk-taking effect of hedging. If the buyer benefits a lot from the insurance against counterparty risk, then margins lead to more risk-taking.

7 Social optimality and regulation

Whenever the privately optimal contract entails risk control by the protection seller, her assets are safe and there is no default. When the level of pledgeable income is low, the privately optimal contract, however, entails risk-taking and counterparty risk (see Proposition 4). Risk-taking by financial institutions entails costs for third parties, e.g., bankruptcy costs or disruptions in payment systems and interbank markets. To examine this possibility, suppose that the default of the seller leads to losses L. The losses L are a measure of the externality the default of a protection seller imposes on the financial system, i.e., L measures the systemic importance of a financial institution. Since the seller is protected by limited liability, she does not internalize its systemic relevance.[14]

Social (utilitarian) welfare is given by the sum of the buyer's utility and the seller's profits net of losses L. Social welfare decreases with losses and there exists a threshold level of L, L^*, such that for losses larger than L^*, social optimality requires the avoidance of risk-taking even though it may be privately optimal. The conflict between private and social optimality opens up the scope for regulating systemically relevant financial institutions that are engaged in hedging.

One way to mitigate risk-taking incentives is to impose capital requirements on protection sellers. For a given amount of hedging activities, extra capital strengthens their balance

[14] Another possibility is that L denotes the amounts of deposits if the protection seller is a commercial bank. When the bank defaults since her hedging activities, undertaken for example by its investment banking branch, induce her to take risks, deposits are either lost or, if insured, are paid by the government. In either case, there is a cost L to third parties.

sheets by increasing pledgeable income. This counters the hidden leverage embedded in hedging. Thus, requiring financial institutions to hold capital in proportion to their derivative exposures reduces risk-taking. By extension, (systemically relevant) financial institutions that opt out of the capital requirement should not be allowed to undertake such activities.

Our analysis shows that financial market infrastructures have an important effect on systemic risk. For example, the clearing of derivative contracts by a central counterparty (CCP) makes margin requirements possible. As long as margins are privately optimal, the clearing need not be mandated. As for the effect of margin requirements on risk-taking, we showed that it is ambiguous. Margins make risk-taking by the protection seller less attractive but they introduce complacency of the buyer by insuring him against seller default. If the former effect dominates, margins and capital requirements are substitutes. Market infrastructures then allow to economize on the use of costly capital. If the latter effect dominates, margins and capital requirements are complements. In that case, market infrastructures must be supported by additional regulation.

Mandating the clearing of derivatives through a CCP can also improve social welfare by penalizing the protection buyer for his complacency about seller default. We showed that it is privately optimal to have zero transfers in case the seller defaults (see the discussion following equation (12)). When the losses L incurred by third parties are high, it is socially optimal to have the buyer internalize the consequences of being complacent about the seller's risk-taking. One way to internalize the losses is to have the CCP collect a payment from the buyer even if his counterparty is no longer around.

8 Extensions

In this section, we analyze the hedging contract with multiple protection sellers. We first characterize the optimal contract when seller cannot contract each other. We then analyze reinsurance, i.e., the possibility that sellers write additional hedging contracts among themselves. We show that reinsurance is not feasible. We finally consider the possibility of retrading, i.e., sellers are able to transfer all contractual obligations vis-a-vis the buyer among themselves. We show that retrading undermines sellers' incentives to control balance sheet risk and argue that *initial* margins can restore incentives.

8.1 Multiple sellers

Suppose a protection buyer splits the hedging contract among several protection sellers. For now, we assume that protection sellers cannot reinsure or sell off the original contract (we will relax this assumption in the next sections).

We extend the benchmark model as follows: There are N identical, risk-neutral protection sellers. At time $t = 0$, each protection seller has an amount $\frac{A}{N}$ of assets in place, which have an uncertain per unit return $\tilde{R}$ at $t = 2$. If a seller does not control her balance sheet risk, she defaults with probability $1-p$. We assume that the default risk is a common or "macro" shock that is non-diversifiable across sellers. If the risk materializes (with probability $1-p$), all sellers fail at the same time when not controlling balance sheet risk. Hence, risk-taking among sellers amounts to taking perfectly correlated risks.

As before, the protection buyer solves

$$\max_{\bar{\tau}_i,\underline{\tau}_i} \text{prob}[\bar{s}]u(C + IE[\tilde{\theta}|\bar{s}] + \sum_{i=1}^{N} \bar{\tau}_i) + \text{prob}[\underline{s}]u(C + IE[\tilde{\theta}|\underline{s}] + \sum_{i=1}^{N} \underline{\tau}_i),$$

where subscript i stands for protection seller i, $i = 1, \ldots, N$. Seller i's incentive constraints are given by

$$\mathcal{P}_i \geq \bar{\tau}_i \text{ and } \mathcal{P}_i \geq \underline{\tau}_i$$

where

$$\mathcal{P}_i = \frac{A}{N}\left(R - \frac{B}{1-p}\right)$$

is seller i's pledgeable income.Seller i's participation constraint is given by

$$E[\tau_i] \leq 0$$

We can now state the following proposition.

Proposition 7 (Multiple sellers) *The optimal contract with multiple sellers, $N > 1$, that maintains their incentives to control balance sheet risk is given by $\bar{\tau}_i = \frac{\bar{\tau}}{N}$ and $\underline{\tau}_i = \frac{\underline{\tau}}{N}$, $i = 1, \ldots, N$, where $\bar{\tau}$ and $\underline{\tau}$ are the optimal expected transfers after a good and a bad signal, respectively, for $N = 1$.*

Protection sellers are risk-neutral and competitive. Summing up their (linear) participation and incentive constraints, the optimization problem with N protection sellers of size $\frac{A}{N}$ is equivalent to the problem with one seller of size A. In this sense, our model with a single protection seller is representative of an entire sector.

8.2 Reinsurance

Suppose a protection buyer splits the hedging contract among two identical, risk-neutral protection sellers as described in the previous section, i.e., $N = 2$. Each protection seller holds a contract $\left(\frac{\bar{\tau}}{2}, \frac{\underline{\tau}}{2}\right)$ (see Proposition 7). We now allow for sellers to reinsure each other after the contract with buyer τ is signed but before the signal $\tilde{s}$ about the return $\tilde{\theta}$ is observed.[15] Denote the complete reinsurance contract between the two protection seller as $\rho(\theta, s, R_1, R_2)$, where R_i is the return on the assets in place of seller i. We make the convention that $\rho > 0$ means that seller 2 is paying seller 1.

For the sellers to agree on reinsurance, there must be gains from trade. Suppose without loss of generality that seller 2 is reinsuring seller 1. After a bad signal, seller 2 therefore

[15] After the signal is observed, there is no scope for reinsurance since the position is no longer neutral. After good news, the hedge is more likely to be an asset and a protection seller does not require reinsurance, while after bad news, the hedge is more likely to be a liability and another protection seller is not willing to provide reinsurance.

expects having to pay seller 1, $\underline{\rho} > 0$. Since seller 2 will not provide reinsurance if he expects to lose money, it must be that $E[\rho] \leq 0$, and hence $\bar{\rho} < 0$. According to Proposition 7, each seller's incentive constraint after a bad signal is binding, $\frac{\mathcal{T}}{2} = \mathcal{P}_2$. An additional expected payout after a bad signal, $\underline{\rho} > 0$, induces seller 2 to shirk on her effort to control balance sheet risk. Assuming for the moment that seller 1 does not shirk, the expected gain to seller 2 from providing reinsurance then is

$$\text{prob}[\bar{s}]\,(-\bar{\rho}) + \text{prob}[\underline{s}]\left[\frac{AB}{2} - (1-p)\frac{AR}{2} + (1-p)\left(\frac{\mathcal{T}}{2}\right) + p\left(-\underline{\rho}\right)\right]$$

The first term is the payment from seller 1 after a good signal. The term is squared brackets is the gain after a bad signal. The gain has four components. First, seller 2 obtains the private benefit of no longer controlling her balance sheet risk. Second, she defaults with probability $(1-p)$ and loses her assets. However, she also gains by defaulting since she no longer has to honor the original hedging contract with the buyer (this is the third term inside the squared brackets). Finally, seller 2 does not default with probability p and makes the payment to seller 1.

The expression for $E[\rho]$ simplifies considerably since the optimal hedging contract between the buyer and a seller exactly balances the cost and benefit of shirking after a bad signal. From the binding incentive constraint, we have $\frac{\mathcal{T}}{2} = \frac{A}{2}\left(R - \frac{B}{1-p}\right)$, and thus,

$$\text{prob}[\bar{s}]\,(-\bar{\rho}) + \text{prob}[\underline{s}]p\left(-\underline{\rho}\right). \tag{23}$$

What is the expected gain from reinsurance to seller 1? Since we assumed that she does not shirk, her expected gain from reinsurance is

$$\text{prob}[\bar{s}]\,(\bar{\rho}) + \text{prob}[\underline{s}]p\left(\underline{\rho}\right). \tag{24}$$

Seller 1 receives the payment after a bad signal only if seller 2 has not defaulted, which happens with probability p.

Comparing (23) and (24), we conclude that there are no gains from reinsurance. Whenever the expected gain for seller 2 is positive, it is negative for seller 1. The conclusion extends to the case when seller 1 too shirks under reinsurance. The expressions (23) and (24) for the gains to seller 2 and seller 1 remain unchanged since we assume that protection sellers are exposed to a common macro shock if they fail to control their balance sheet risk. All sellers default with probability $1 - p$ if they shirk on risk control after observing a bad signal. The following proposition summarizes our result.

Proposition 8 (Reinsurance) *The optimal hedging contract between a buyer and multiple sellers that maintains their incentives to to control balance sheet risk leaves no room for sellers to reinsure each other.*

8.3 Retrading and initial margins

Suppose, as in the previous section, that a protection buyer splits the hedging contract among two protection sellers. We now allow seller 1 to acquire the contract held by seller 2, thus freeing seller 2 from all obligations stemming from the contract.

Before the signal $\tilde{s}$ about the return $\tilde{\theta}$ is observed, seller 2 is indifferent between selling the contract for a value of zero and keeping it since a seller's participation constraint binds (see Proposition 7). What is seller 1's expected gain from acquiring seller 2's contract at a price of zero? Prior to acquiring seller 2's contract, seller 1's incentive constraint after observing a bad signal was binding, $\frac{\underline{\tau}}{2} = \mathcal{P}_1$ (see Proposition 7). Hence, increasing her position from $\frac{\underline{\tau}}{2}$ to $\underline{\tau}$ induces seller 1 to no longer control her balance sheet risk. Her expected gain from acquiring seller 2's contract is:

$$\text{prob}[\bar{s}]\left(-\frac{\bar{\tau}}{2}\right) + \text{prob}[\underline{s}]\left[\frac{AB}{2} - (1-p)\frac{AR}{2} + (1-p)\left(\frac{\underline{\tau}}{2}\right) + p\left(-\frac{\underline{\tau}}{2}\right)\right].$$

The first term is the extra payment from the protection buyer to seller 1 after a good signal.

The term is squared brackets is the gain after a bad signal. The gain has four components. First, seller 1 obtains the private benefit of no longer controlling her balance sheet risk. Second, she defaults with probability $(1-p)$ and loses her assets. However, she also gains by defaulting since she no longer has to pay the protection buyer (this is the third term inside the squared brackets). Finally, seller 1 does not default with probability p and has to make the payment to the protection buyer that he would have obtained from seller 2 in the absence of retrading.

Using $\frac{\tau}{2} = \frac{\mathcal{P}}{2} = \frac{A}{2}\left(R - \frac{B}{1-p}\right)$ and $E\left[\frac{\tau}{2}\right] = 0$ from the binding incentive and participation constraints, seller 1's gain simplifies to

$$\text{prob}[\underline{s}]\,(1-p)\,\frac{\mathcal{P}}{2} > 0. \tag{25}$$

The expected gain from acquiring the hedging contract arises from exploiting limited liability, i.e., from taking risks after a bad signal and not having to pay the protection buyer with probability $1-p$. The following Proposition summarizes our result:

Proposition 9 (Retrading) *Retrading the protection buyer's hedging contract among the sellers undermines their incentives to control balance sheet risk.*

If protection sellers can retrade contracts, they have incentives to accumulate contracts and build up hedging positions beyond their pledgeable income. Sellers take on concentrated risks and benefit from the protection offered by limited liability.[16]

Requesting an *initial* margin restores protection sellers' incentives to control their balance sheet risk when retrading is possible. By preventing sellers from accumulating excessive hedging positions, initial margins counter sellers' desire to take on concentrated risks. The margin must be deposited *before* the signal realizes since the scope for retrading exists only then. This is in contrast to the variation margin, analyzed earlier, which is deposited after

[16] AIG's conduct prior to the 2007 financial crisis, that is, the accumulation of a vast number of CDS contracts, may have been an example of such behavior.

the signal realizes. An initial margin makes it costly for a seller to acquire another seller's contract since she has to liquidate some of her assets.

Consider again the case of two protection sellers. If both sellers retain their half of the optimal contract, $\left(\frac{\bar{\tau}}{2}, \frac{\tau}{2}\right)$, an initial margin is not required. It is only when a seller wants to accumulate a position that is larger than her pledgeable income, $\mathcal{P}_i = \frac{\mathcal{P}}{2}$, that she must put up an initial margin. We can compute the optimal amount, or the "haircut", at which a position in excess of $\left(\frac{\bar{\tau}}{2}, \frac{\tau}{2}\right)$ should be margined. The cost of liquidating assets to comply with the initial margin, denoted by α_0, must be large enough to outweigh the gain from retrading, which is given by equation (25):

$$\text{prob}[\underline{s}]\,(1-p)\,\frac{\mathcal{P}}{2} \leq \frac{A}{2}\,(R-1)\,\alpha_0,$$

In equilibrium, the condition holds as an equality to minimize the opportunity cost of the initial margin. Moreover, the size of a seller $\left(\frac{1}{2}\right)$ appears on both sides of the condition and drops out. We therefore state the following result:

Proposition 10 (Initial margin) *Initial margins maintain sellers' incentives to control balance sheet risk when retrading is possible. The optimal size of the initial margin ("haircut") is*

$$\alpha_0 = \frac{prob[\underline{s}]\,(1-p)\,g}{R-1},$$

where, as before, $g \equiv \frac{\mathcal{P}}{A}$*, is the proportion of assets that are pledgeable.*

We assumed that retrading allows protection sellers to free themselves from any contractual obligation towards the protection buyer. This is in line with the trading of contracts on an exchange, which acts as a middle man between buyers and sellers. Buyers and sellers do not contract with each other directly but instead contract with the exchange (or brokers that are members of the exchange). It is the exchange (or its members) that is liable for the contracts. It is also the exchange that can implement an initial margin to prevent the accumulation of excessive positions via trading. The size of the initial margin, or haircut, should depend not only on the (negative signal) risk of the underlying position, prob[$\underline{s}$], but also on the characteristics of traders' balance sheets, i.e., risk $(1-p)$, net return $(R-1)$ and pledgeability (g).

9 Conclusion

We consider a situation in which a protection seller offers a hedging contract to a protection buyer. We show how this contract, designed to facilitate risk-sharing, can generate incentives for risk-taking. When the position of the protection seller becomes loss-making, this creates hidden leverage, discouraging the mitigation of risks in the seller's core business. Such elevated moral hazard raises the default risk of the protection seller and, correspondingly, the counterparty risk for the protection buyer. Thus hedging can lead to systemic risk, in the form of propagation of risk from derivative positions to the traditional business of financial institutions.

We show that for well-capitalized institutions, margin requirements mitigate this problem, by reducing the severity of the moral hazard problem. Initial margins act to discourage retrading and accumulation of excessive positions, while variation margins serve to discourage risk-taking incentives for a given position. Therefore, the establishment of CCPs can be part of an appropriate regulatory response. But, our analysis implies that poorly capitalized firms should be banned from the sale of such protection, even in markets with CCPs and margin requirements.

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Appendix

Proof of Proposition 1

Let μ denote the Lagrange multiplier on the participation constraint (2). Let μ_0 and μ_1 be the Lagrange multipliers on the feasibility constraints $\alpha \geq 0$ and $\alpha \leq 1$. The first-order conditions with respect to transfers $\tau(\bar{\theta}, \bar{s})$, $\tau(\underline{\theta}, \bar{s})$, $\tau(\bar{\theta}, \underline{s})$, $\tau(\underline{\theta}, \underline{s})$ and margin α are given by:

$$\begin{aligned}
\pi\lambda u'(C + I\bar{\theta} + \tau(\bar{\theta}, \bar{s})) - \mu\pi\lambda &= 0 \\
(1-\pi)(1-\lambda)u'(C + I\underline{\theta} + \tau(\underline{\theta}, \bar{s})) - \mu(1-\pi)(1-\lambda) &= 0 \\
\pi(1-\lambda)u'(C + I\bar{\theta} + \tau(\bar{\theta}, \underline{s})) - \mu\pi(1-\lambda) &= 0 \\
(1-\pi)\lambda u'(C + I\underline{\theta} + \tau(\underline{\theta}, \underline{s})) - \mu(1-\pi)\lambda &= 0 \\
\mu A\,(1-R) + \mu_0 - \mu_1 &= 0
\end{aligned}$$

It follows that the marginal utility of the buyer of insurance is equalized across $(\tilde{\theta}, \tilde{s})$ states (full insurance) and that the participation constraint is binding:

$$\bar{u}'(\tau(\bar{\theta}, \bar{s})) = \bar{u}'(\tau(\bar{\theta}, \underline{s})) = \underline{u}'(\tau(\underline{\theta}, \underline{s})) = \underline{u}'(\tau(\underline{\theta}, \bar{s})) = \mu > 0 \tag{A.1}$$

Since $\mu > 0$ and $1 - R < 0$, it must be that $\mu_0 > 0$ and $\mu_1 = 0$. Hence, $\alpha = 0$ must hold in the optimum and margins are not used.

The optimal transfers are obtained by using the fact that the participation constraint is binding and that consumption is the same across $(\tilde{\theta}, \tilde{s})$ states.

Proof of Lemma 1

Let $\mu_{\bar{s}}$ and $\mu_{\underline{s}}$ denote the Lagrange multipliers on the incentive compatibility constraints (6) and (7), respectively (μ again denotes the multiplier on the participation constraint (2)). The first-order conditions with respect to transfers $\tau(\bar{\theta}, \bar{s})$, $\tau(\underline{\theta}, \bar{s})$, $\tau(\bar{\theta}, \underline{s})$ and $\tau(\underline{\theta}, \underline{s})$ are given by:

$$\begin{aligned}
\pi\lambda u'(C + I\bar{\theta} + \tau(\bar{\theta}, \bar{s})) - \mu_{\bar{s}}\bar{\pi} - \mu\pi\lambda &= 0 \\
(1-\pi)(1-\lambda)u'(C + I\underline{\theta} + \tau(\underline{\theta}, \bar{s})) - \mu_{\bar{s}}(1-\bar{\pi}) - \mu(1-\pi)(1-\lambda) &= 0 \\
\pi(1-\lambda)u'(C + I\bar{\theta} + \tau(\bar{\theta}, \underline{s})) - \mu_{\underline{s}}\underline{\pi} - \mu\pi(1-\lambda) &= 0 \\
(1-\pi)\lambda u'(C + I\underline{\theta} + \tau(\underline{\theta}, \underline{s})) - \mu_{\underline{s}}(1-\underline{\pi}) - \mu(1-\pi)\lambda &= 0
\end{aligned}$$

We re-write the first-order conditions as

$$\bar{u}'(\tau(\bar{\theta},\bar{s})) = \mu + \mu_{\bar{s}}\frac{\bar{\pi}}{\pi\lambda} \tag{A.2}$$

$$\underline{u}'(\tau(\underline{\theta},\bar{s})) = \mu + \mu_{\bar{s}}\frac{1-\bar{\pi}}{(1-\pi)(1-\lambda)} \tag{A.3}$$

$$\bar{u}'(\tau(\bar{\theta},\underline{s})) = \mu + \mu_{\underline{s}}\frac{\underline{\pi}}{\pi(1-\lambda)} \tag{A.4}$$

$$\underline{u}'(\tau(\underline{\theta},\underline{s})) = \mu + \mu_{\underline{s}}\frac{1-\underline{\pi}}{(1-\pi)\lambda} \tag{A.5}$$

where we use a shorthand $\bar{u}'(\tau(\bar{\theta},\tilde{s}))$ to denote marginal utility in state $\bar{\theta}$ conditional on the signal $\tilde{s}$ and, similarly, $\underline{u}'(\tau(\underline{\theta},\tilde{s}))$ to denote marginal utility in state $\underline{\theta}$ conditional on the signal $\tilde{s}$.

Since

$$\begin{aligned}
\frac{\bar{\pi}}{\pi\lambda} &= \frac{\text{prob}[\bar{\theta}|\bar{s}]}{\text{prob}[\bar{\theta}\cap\bar{s}]} = \frac{1}{\text{prob}[\bar{s}]} \\
\frac{1-\bar{\pi}}{(1-\pi)(1-\lambda)} &= \frac{\text{prob}[\underline{\theta}|\bar{s}]}{\text{prob}[\underline{\theta}\cap\bar{s}]} = \frac{1}{\text{prob}[\bar{s}]} \\
\frac{\underline{\pi}}{\pi(1-\lambda)} &= \frac{\text{prob}[\bar{\theta}|\underline{s}]}{\text{prob}[\bar{\theta}\cap\underline{s}]} = \frac{1}{\text{prob}[\underline{s}]} \\
\frac{1-\underline{\pi}}{(1-\pi)\lambda} &= \frac{\text{prob}[\underline{\theta}|\underline{s}]}{\text{prob}[\underline{\theta}\cap\underline{s}]} = \frac{1}{\text{prob}[\underline{s}]}
\end{aligned}$$

holds, it follows that there is full risk-sharing conditional on the signal:

$$\begin{aligned}
\bar{u}'(\tau(\bar{\theta},\bar{s})) &= \underline{u}'(\tau(\underline{\theta},\bar{s})) \\
\bar{u}'(\tau(\bar{\theta},\underline{s})) &= \underline{u}'(\tau(\underline{\theta},\underline{s}))
\end{aligned}$$

As in the first-best case, we therefore have

$$\tau(\underline{\theta},\bar{s}) - \tau(\bar{\theta},\bar{s}) = \tau(\underline{\theta},\underline{s}) - \tau(\bar{\theta},\underline{s}) = I\Delta\theta > 0 \tag{A.6}$$

It follows that, conditional on the signal, the transfer to the buyer when the asset return is low is higher than when the asset return is high, $\tau(\underline{\theta},\tilde{s}) > \tau(\bar{\theta},\tilde{s})$.

Next, we show that the participation constraint must bind. Suppose not, i.e. $\mu = 0$. Then, equations (A.2) and (A.3) imply that $\mu_{\bar{s}} > 0$. Similarly, (A.4) and (A.5) imply that $\mu_{\underline{s}} > 0$. Both incentive constraints bind so that $\mathcal{P} = \bar{\tau} = \underline{\tau}$. Since the participation constraint is slack, it must be that

$$\begin{aligned}
0 &> E[\tau] \equiv \text{prob}[\bar{s}]\bar{\tau} + \text{prob}[\underline{s}]\underline{\tau} \\
&= \mathcal{P}\left(\text{prob}[\bar{s}] + \text{prob}[\underline{s}]\right) \\
&= \mathcal{P}
\end{aligned}$$

which contradicts $\mathcal{P} > 0$. Hence, the participation constraint binds, $E[\tau] = 0$.

It follows that at least one incentive constraint must be slack. If not, then $\bar{\tau} = \underline{\tau} = \mathcal{P} > 0$, which contradicts $E[\tau] = 0$.

Suppose both incentive constraints are slack, $\mu_{\bar{s}} = \mu_{\underline{s}} = 0$. Then, we obtain full insurance as in (A.1) and the contract is given by proposition 1 (first-best). The conditions under which the incentive constraints are indeed slack are given by:

$$\begin{aligned} \mathcal{P} &> \bar{\pi}\tau^{FB}(\bar{\theta}, \bar{s}) + (1 - \bar{\pi})\tau^{FB}(\underline{\theta}, \bar{s}) = (\pi - \bar{\pi})I\Delta\theta \\ \mathcal{P} &> \underline{\pi}\tau^{FB}(\bar{\theta}, \underline{s}) + (1 - \underline{\pi})\tau^{FB}(\underline{\theta}, \underline{s}) = (\pi - \underline{\pi})I\Delta\theta \end{aligned}$$

When the signal is informative, $\lambda > \frac{1}{2}$, we have $\bar{\pi} > \pi > \underline{\pi}$. The result in the lemma follows.

Proof of Lemma 2

We have shown above that at least one incentive constraint must be slack. They cannot both be slack since we assume that $\mathcal{P} < (\pi - \underline{\pi})I\Delta\theta$. We now show that it is the incentive constraint following a *bad* signal that is binding. Suppose not, so that $\mathcal{P} = \bar{\tau} > 0 > \underline{\tau}$ where the last inequality follows from $E[\tau] = 0$. Then, $\mu_{\underline{s}} = 0$ and $\mu_{\bar{s}} \geq 0$ and equations (A.2) through (A.5) yield

$$\bar{u}'(\tau(\bar{\theta}, \underline{s})) = \underline{u}'(\tau(\underline{\theta}, \underline{s})) = \mu \leq \bar{u}'(\tau(\bar{\theta}, \bar{s})) = \underline{u}'(\tau(\underline{\theta}, \bar{s}))$$

Comparing the first with the third term and the second with the fourth term yields

$$\begin{aligned} \tau(\bar{\theta}, \underline{s}) &\geq \tau(\bar{\theta}, \bar{s}) \\ \tau(\underline{\theta}, \underline{s}) &\geq \tau(\underline{\theta}, \bar{s}) \end{aligned}$$

Using $\tau(\underline{\theta}, \tilde{s}) > \tau(\bar{\theta}, \tilde{s})$ (equation (A.6)) and $\bar{\pi} > \underline{\pi}$, we can write

$$\begin{aligned} 0 &< \bar{\tau} \equiv \bar{\pi}\tau(\bar{\theta}, \bar{s}) + (1 - \bar{\pi})\tau(\underline{\theta}, \bar{s}) \\ &< \underline{\pi}\tau(\bar{\theta}, \bar{s}) + (1 - \underline{\pi})\tau(\underline{\theta}, \bar{s}) \\ &\leq \underline{\pi}\tau(\bar{\theta}, \underline{s}) + (1 - \underline{\pi})\tau(\underline{\theta}, \underline{s}) \equiv \underline{\tau} \end{aligned}$$

But $\underline{\tau} < 0$, a contradiction. Hence, only the incentive constraint after a bad signal binds.

Proof of Proposition 2

The optimal contract is given by the binding incentive constraint following a bad signal:

$$\mathcal{P} = \underline{\tau},$$

the binding participation constraint

$$\text{prob}[\bar{s}]\bar{\tau} + \text{prob}[\underline{s}]\underline{\tau} = 0,$$

and full risk-sharing conditional on the signal (A.6).

Proof of Proposition 3

Let $\mu_{\bar{s}}$ denote the Lagrange multiplier on the incentive constraint following a good signal and let μ denote the multiplier on the participation constraint (15). The first-order conditions with respect to transfers $\tau(\bar{\theta}, \bar{s})$, $\tau(\underline{\theta}, \bar{s})$, $\tau(\bar{\theta}, \underline{s})$ and $\tau(\underline{\theta}, \underline{s})$ are:

$$\bar{u}'(\tau(\bar{\theta}, \bar{s})) = \mu + \frac{\mu_{\bar{s}}}{\text{prob}[\bar{s}]} \tag{A.7}$$
$$\underline{u}'(\tau(\underline{\theta}, \bar{s})) = \mu + \frac{\mu_{\bar{s}}}{\text{prob}[\bar{s}]} \tag{A.8}$$
$$\bar{u}'(\tau(\bar{\theta}, \underline{s})) = \mu \tag{A.9}$$
$$\underline{u}'(\tau(\underline{\theta}, \underline{s})) = \mu \tag{A.10}$$

The last two conditions imply that the participation constraint binds as $\mu > 0$. Moreover, we again have full sharing of the $\tilde{\theta}$ risk conditional on the signal, except for a default state:

$$\begin{aligned} \bar{u}'(\tau(\bar{\theta}, \bar{s})) &= \underline{u}'(\tau(\underline{\theta}, \bar{s})) \\ \bar{u}'(\tau(\bar{\theta}, \underline{s})) &= \underline{u}'(\tau(\underline{\theta}, \underline{s})) \end{aligned}$$

and hence

$$\tau(\underline{\theta}, \tilde{s}) - \tau(\bar{\theta}, \tilde{s}) = I\Delta\theta > 0 \tag{A.11}$$

Next, we show that the incentive constraint after a good signal (13) is slack, implying $\mu_{\bar{s}} = 0$. Suppose that the constraint is not slack and $\mathcal{P} = \bar{\tau} < \underline{\tau}$. Since $\mathcal{P} > 0$, both expected transfers are positive, which violates the participation constraint.

Since incentive constraints are slack, the first-order conditions become

$$\bar{u}'(\tau(\bar{\theta}, \bar{s})) = \bar{u}'(\tau(\bar{\theta}, \underline{s})) = \underline{u}'(\tau(\underline{\theta}, \underline{s})) = \underline{u}'(\tau(\underline{\theta}, \bar{s})) = \mu \tag{A.12}$$

Conditional on no default, there is full insurance, as in the first-best case (A.1):

$$\tau(\tilde{\theta}, \bar{s}) = \tau(\tilde{\theta}, \underline{s}) \tag{A.13}$$

The buyer is, however, exposed to counterparty risk.

The optimal contract with no effort after a bad signal is given by (A.12) and the binding participation constraint. We now check under what condition the incentive constraint following a bad signal (14) is indeed slack. Starting with the binding participation constraint and using (A.11) and (A.13), we get

$$\begin{aligned} -\text{prob}[\underline{s}](1-p)\mathcal{P} &= \text{prob}[\bar{s}][\bar{\pi}\tau(\bar{\theta}, \bar{s}) + (1-\bar{\pi})\tau(\underline{\theta}, \bar{s})] + \text{prob}[\underline{s}]p[\underline{\pi}\tau(\bar{\theta}, \underline{s}) + (1-\underline{\pi})\tau(\underline{\theta}, \underline{s})] \\ &= \text{prob}[\bar{s}][\tau(\underline{\theta}, \bar{s}) - \bar{\pi}I\Delta\theta] + \text{prob}[\underline{s}]p[\tau(\underline{\theta}, \underline{s}) - \underline{\pi}I\Delta\theta] \\ &= \tau(\underline{\theta}, \underline{s})[\text{prob}[\bar{s}] + \text{prob}[\underline{s}]p] - I\Delta\theta[\text{prob}[\bar{s}]\bar{\pi} + \text{prob}[\underline{s}]p\underline{\pi}] \end{aligned}$$

Using Bayes' Rule and simplifying, we arrive at

$$-\text{prob}[\underline{s}](1-p)\mathcal{P} = \tau(\underline{\theta},\underline{s})[1-\text{prob}[\underline{s}](1-p)] - I\Delta\theta\pi[1-\text{prob}[\underline{s}|\bar{\theta}](1-p)]$$

Hence,

$$\tau(\underline{\theta},\underline{s}) = \pi I\Delta\theta\frac{1-\text{prob}[\underline{s}|\bar{\theta}](1-p)}{1-\text{prob}[\underline{s}](1-p)} - \mathcal{P}\frac{\text{prob}[\underline{s}](1-p)}{1-\text{prob}[\underline{s}](1-p)} \quad \text{(A.14)}$$

Since $\underline{\tau} = \tau(\underline{\theta},\underline{s}) - \underline{\pi}I\Delta\theta$, for the incentive constraint after a bad signal to be slack, it must be that

$$\mathcal{P} < \tau(\underline{\theta},\underline{s}) - \underline{\pi}I\Delta\theta$$

Substituting for $\tau(\underline{\theta},\underline{s})$ and simplifying yields

$$\mathcal{P} < (\pi-\underline{\pi})I\Delta\theta - (1-p)I\Delta\theta[\pi\text{prob}[\underline{s}|\bar{\theta}] - \underline{\pi}\text{prob}[\underline{s}]]$$

or, equivalently,

$$\mathcal{P} < (\pi-\underline{\pi})I\Delta\theta$$

This is the same condition as in Lemma 1. The incentive constraint after a bad signal is slack when the first-best is not attainable.

Proof of Proposition 4

The proof proceeds in three steps. First, we show that the expected utility of the contract with effort after both signals is increasing in $\mathcal{P}$:

$$\begin{aligned}\frac{\partial EU^{e=1}}{\partial\mathcal{P}} &= -\frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]}\left[\pi\lambda\bar{u}'(\tau(\bar{\theta},\bar{s})) + (1-\pi)(1-\lambda)\underline{u}'(\tau(\underline{\theta},\bar{s}))\right] \\ &\quad +\pi(1-\lambda)\bar{u}'(\tau(\bar{\theta},\underline{s})) + (1-\pi)\lambda\underline{u}'(\tau(\underline{\theta},\underline{s})) \\ &= \text{prob}[\underline{s}]\left[\bar{u}'(\tau(\bar{\theta},\underline{s})) - \bar{u}'(\tau(\bar{\theta},\bar{s}))\right] > 0\end{aligned}$$

since $\tau(\bar{\theta},\underline{s}) < \tau(\bar{\theta},\bar{s})$ due to the signal risk.

Second, we show that the expected utility of the contract with no effort following a bad signal is decreasing in $\mathcal{P}$:

$$\begin{aligned}\frac{\partial EU^{e=1,e=0}}{\partial\mathcal{P}} &= -\frac{\text{prob}[\underline{s}](1-p)}{1-\text{prob}[\underline{s}](1-p)}\left[\pi\lambda\bar{u}'(\tau(\bar{\theta},\bar{s})) + (1-\pi)(1-\lambda)\underline{u}'(\tau(\underline{\theta},\bar{s}))\right. \\ &\quad \left. + \pi(1-\lambda)p\bar{u}'(\tau(\bar{\theta},\underline{s})) + (1-\pi)\lambda p\underline{u}'(\tau(\underline{\theta},\underline{s}))\right] \\ &= -\frac{\text{prob}[\underline{s}](1-p)}{1-\text{prob}[\underline{s}](1-p)}\left[\pi(\lambda+p(1-\lambda))\bar{u}'(\tau(\bar{\theta},\bar{s}))\right. \\ &\quad \left. +(1-\pi)((1-\lambda)+p\lambda)\underline{u}'(\tau(\underline{\theta},\bar{s}))\right] < 0\end{aligned}$$

Third, we provide sufficient condition for $EU^{e=1}(\mathcal{P}=0) < EU^{e=1,e=0}(\mathcal{P}=0)$ so that no effort after a bad signal is optimal for low $\mathcal{P}$.

We have

$$\begin{aligned} EU^{e=1}(\mathcal{P}=0) &= [\pi\lambda+(1-\pi)(1-\lambda)]\,u\left(C+I\underline{\theta}+\bar{\pi}I\Delta\theta\right)+[\pi(1-\lambda)+(1-\pi)\lambda]\times \\ &\quad u\left(C+I\underline{\theta}+\underline{\pi}I\Delta\theta\right) \\ &= \text{prob}[\bar{s}]u\left(C+I\underline{\theta}+\bar{\pi}I\Delta\theta\right)+\text{prob}[\underline{s}]u\left(C+I\underline{\theta}+\underline{\pi}I\Delta\theta\right) \\ &= \text{prob}[\bar{s}]u(C+IE[\tilde{\theta}|\bar{s}])+\text{prob}[\underline{s}]u(C+IE[\tilde{\theta}|\underline{s}]) \end{aligned} \tag{A.15}$$

and

$$\begin{aligned} EU^{e=1,e=0}(\mathcal{P}=0) &= \left(\text{prob}[\bar{s}]+p\text{prob}[\underline{s}]\right)u\left(C+I\underline{\theta}+\pi\frac{1-\text{prob}[\underline{s}|\bar{\theta}](1-p)}{1-\text{prob}[\underline{s}](1-p)}I\Delta\theta\right) \\ &\quad +(1-p)\left[\pi(1-\lambda)u(C+I\bar{\theta})+(1-\pi)\lambda u(C+I\underline{\theta})\right] \\ &= \left(\text{prob}[\bar{s}]+\text{prob}[\underline{s}]p\right)u\left(C+I\hat{E}[\tilde{\theta}]\right) \\ &\quad +\text{prob}[\underline{s}](1-p)\left[\underline{\pi}u(C+I\bar{\theta})+(1-\underline{\pi})u(C+I\underline{\theta})\right] \end{aligned} \tag{A.16}$$

where

$$\hat{E}[\tilde{\theta}]=\hat{\pi}\bar{\theta}+(1-\hat{\pi})\underline{\theta}$$

and

$$\hat{\pi}=\pi\frac{1-\text{prob}[\underline{s}|\bar{\theta}](1-p)}{1-\text{prob}[\underline{s}](1-p)}$$

Note that

$$\bar{\pi}>\hat{\pi}>\pi>\underline{\pi} \tag{A.17}$$

for $p\in(0,1)$. Note that $\bar{\pi}=\hat{\pi}$ for $p=0$ and that $\hat{\pi}=\pi$ for $p=1$. The first two inequalities follow from the fact that prob$[\underline{s}]>$prob$[\underline{s}|\bar{\theta}]$ for $\lambda>\frac{1}{2}$ (informative signal). Hence,

$$\frac{1-\text{prob}[\underline{s}|\bar{\theta}](1-p)}{1-\text{prob}[\underline{s}](1-p)}\geq 1$$

Combining (A.15) and (A.16), we have that no effort after a bad signal dominates effort (when $\mathcal{P}=0$) if and only if

$$\begin{aligned} &\text{prob}[\bar{s}]u(C+IE[\tilde{\theta}|\bar{s}])+\text{prob}[\underline{s}]u(C+IE[\tilde{\theta}|\underline{s}]) \\ &\qquad <\left(\text{prob}[\bar{s}]+\text{prob}[\underline{s}]p\right)u(C+I\hat{E}[\tilde{\theta}])+\text{prob}[\underline{s}](1-p)EU\left((C+I\tilde{\theta})|\underline{s}\right) \end{aligned}$$

where

$$EU\left((C+I\tilde{\theta})|\underline{s}\right)=\underline{\pi}u(C+I\bar{\theta})+(1-\underline{\pi})u(C+I\underline{\theta})$$

After collecting terms, we have

$$\begin{aligned} &\text{prob}[\bar{s}]\left[u(C+IE[\tilde{\theta}|\bar{s}])-u(C+I\hat{E}[\tilde{\theta}])\right]+\text{prob}[\underline{s}]\left[u(C+IE[\tilde{\theta}|\underline{s}])-EU\left((C+I\tilde{\theta})|\underline{s}\right)\right] \\ &\qquad <\text{prob}[\underline{s}]p\left[u(C+I\hat{E}[\tilde{\theta}])-EU\left((C+I\tilde{\theta})|\underline{s}\right)\right] \end{aligned}$$

All the differences in the square brackets are positive. The first one due to (A.17), the second one due to the concavity of u, and the third one due to both the concavity of u and (A.17).

Rearranging, we arrive at

$$\frac{\text{prob}[\bar{s}]}{\text{prob}[\underline{s}]}\frac{u(C+IE[\tilde{\theta}|\bar{s}])-u(C+I\hat{E}[\tilde{\theta}])}{u(C+I\hat{E}[\tilde{\theta}])-EU\left((C+I\tilde{\theta})|\underline{s}\right)}+\frac{u(C+IE[\tilde{\theta}|\underline{s}])-EU\left((C+I\tilde{\theta})|\underline{s}\right)}{u(C+I\hat{E}[\tilde{\theta}])-EU\left((C+I\tilde{\theta})|\underline{s}\right)}<p \quad \text{(A.18)}$$

It is clear that the left-hand side is strictly positive so that seller's effort dominates when p is small. The left-hand is, however, also strictly smaller than one so that no effort after a bad signal dominates when p is large.[17]

The condition

$$\frac{\text{prob}[\bar{s}]}{\text{prob}[\underline{s}]}\frac{u(C+IE[\tilde{\theta}|\bar{s}])-u(C+I\hat{E}[\tilde{\theta}])}{u(C+I\hat{E}[\tilde{\theta}])-EU\left((C+I\tilde{\theta})|\underline{s}\right)}+\frac{u(C+IE[\tilde{\theta}|\underline{s}])-EU\left((C+I\tilde{\theta})|\underline{s}\right)}{u(C+I\hat{E}[\tilde{\theta}])-EU\left((C+I\tilde{\theta})|\underline{s}\right)}<1$$

simplifies to

$$\text{prob}[\bar{s}]u(C+IE[\tilde{\theta}|\bar{s}])+\text{prob}[\underline{s}]u(C+IE[\tilde{\theta}|\underline{s}])<u(C+I\hat{E}[\tilde{\theta}])$$

By concavity,

$$\text{prob}[\bar{s}]u(C+IE[\tilde{\theta}|\bar{s}])+\text{prob}[\underline{s}]u(C+IE[\tilde{\theta}|\underline{s}])<u(C+IE[\tilde{\theta}])$$

and so the condition holds when

$$u(C+IE[\tilde{\theta}])\leq u(C+I\hat{E}[\tilde{\theta}])$$

which is always the case due to (A.17).

Hence, whenever $EU^{e=1}(\mathcal{P}=0)<EU^{e=1,e=0}(\mathcal{P}=0)$ holds, the privately optimal contract entails no effort after a bad signal for low levels of pledgeable income $\mathcal{P}$. For levels of $\mathcal{P}\geq\hat{\mathcal{P}}$ where $\hat{\mathcal{P}}$ is given by $EU^{e=1}\left(\hat{\mathcal{P}}\right)=EU^{e=1,e=0}\left(\hat{\mathcal{P}}\right)$ and $\hat{\mathcal{P}}<(\pi-\underline{\pi})I\Delta\theta$, the optimal contract is the one with effort. For $\mathcal{P}>(\pi-\underline{\pi})I\Delta\theta$, the first-best is reached.

Proof of Proposition 5

Let g denote the per unit size of pledgeable income so that $g\equiv\frac{\mathcal{P}}{A}$. Let μ and $\mu_{\underline{s}}$ denote the Lagrange multipliers on the participation and incentive-compatibility constraints (16) and (17), respectively. Furthermore, let μ_0 and μ_1 be the Lagrange multipliers on the feasibility constraints $\alpha\geq0$ and $\alpha\leq1$. The first-order conditions with respect to expected transfers

[17]Note that this inequality is evaluated at $\mathcal{P}=0$ and $\mathcal{P}$ is a function of p. There is, however, an open set of parameters for which no effort after a bad signal dominates.

$\bar{\tau}$, $\underline{\tau}$ and margin α are:

$$\bar{u}'(\bar{\tau}) = \mu \quad \text{(A.19)}$$

$$\underline{u}'(\underline{\tau}) = \mu + \frac{\mu_{\bar{s}}}{\text{prob}[\underline{s}]} \quad \text{(A.20)}$$

$$\mu_{\bar{s}}(A-\mathcal{P}) + \mu_0 = \mu A(R-1)\,\text{prob}[\underline{s}] + \mu_1 \quad \text{(A.21)}$$

where $\bar{u}'(\bar{\tau})$ and $\underline{u}'(\underline{\tau})$ denote the marginal utility conditional on the good and the bad signal, respectively.

The first condition implies that $\mu > 0$ and the participation constraint binds. Substituting (A.19) and (A.21) into (A.20), we arrive at:

$$\frac{\underline{u}'(\underline{\tau})}{\bar{u}'(\bar{\tau})} = 1 + \frac{A(R-1)}{(A-\mathcal{P})} + \frac{\mu_1 - \mu_0}{\bar{u}'(\bar{\tau})\text{prob}[\underline{s}](A-\mathcal{P})}$$

When margins are not used, $\mu_1 = 0$. If they are used, then $\mu_0 = 0$ and equation (18) follows. Moreover, equation (A.21) implies that the incentive-compatibility constraint binds, $\mu_{\bar{s}} > 0$ (since $R > 1$ and $A > \mathcal{P}$). Then,

$$\underline{\tau} = \alpha A + (1-\alpha)\mathcal{P}$$

and

$$\bar{\tau} = -\alpha\frac{\text{prob}[\underline{s}]A(R-1) + \text{prob}[\underline{s}](A-\mathcal{P})}{\text{prob}[\bar{s}]} - \mathcal{P}\frac{\text{prob}[\underline{s}]}{\text{prob}[\bar{s}]}$$

For $g \geq 1$, we have that $\mathcal{P} \geq A$. In this case, the incentive constraint in (17) cannot be relaxed using margins and $\alpha^* = 0$. To see this, note that the right-hand side of (18) is smaller or equal to 1 for $g \geq 1$. However, the left-hand side is bigger or equal to 1 (since $\bar{u}'(\bar{\tau}) \leq \underline{u}'(\underline{\tau})$). Hence, $\alpha^* = 0$.

Proof of Proposition 6

Let μ denote the Lagrange multiplier on the participation constraint (20). Furthermore, let μ_0 and μ_1 be the Lagrange multipliers on the feasibility constraints $\alpha \geq 0$ and $\alpha \leq 1$. The first-order conditions with respect to expected transfers $\bar{\tau}$, $\underline{\tau}$ and margin α are:

$$\bar{u}'(\bar{\tau}) = \mu$$

$$\underline{u}'(\underline{\tau}) = \mu$$

$$\text{prob}[\underline{s}](1-p)AE[u_d'|\underline{s}] + \mu_0 = \mu\text{prob}[\underline{s}]\left[A(R-1) + (1-p)(A-\mathcal{P})\right] + \mu_1$$

where $E[u_d'|\underline{s}]$ denotes the expected marginal utility in case the protection seller defaults:

$$E[u_d'|\underline{s}] \equiv \underline{\pi}u'(C + I\bar{\theta} + \alpha A) + (1-\underline{\pi})u'(C + I\underline{\theta} + \alpha A)$$

The first two first-order conditions yield $\bar{u}'(\bar{\tau}) = \underline{u}'(\underline{\tau}) \equiv u_{nd}'$. Plugging into the third first-order condition gives (21). Since the contract does not depend on the signal, the transfers satisfy (19). Combining (19) and the participation constraint (20) yields (22).

Proof of Proposition 7

Consider the contract with risk-control after both signals. Suppose, contrary to the claim in the proposition, that there exists a contract $(\bar{\tau}_i', \underline{\tau}_i') \neq (\bar{\tau}_i, \underline{\tau}_i)$, $i = 1, \ldots, N$, which satisfies participation and incentive constraints of each protection seller and yields a higher utility for the protection buyer.

Since $\bar{\tau}_i' \leq \mathcal{P}_i$ and $\underline{\tau}_i' \leq \mathcal{P}_i$ with $\mathcal{P}_i = \frac{\mathcal{P}}{N}$ holds for each i, we have $\sum_{i=1}^{N} \bar{\tau}_i' \leq \mathcal{P}$ and $\sum_{i=1}^{N} \underline{\tau}_i' \leq \mathcal{P}$. Similarly, $E[\tau_i'] \leq 0$ for each i implies that $\sum_{i=1}^{N} E[\tau_i'] \leq 0$. Let $\sum_{i=1}^{N} \bar{\tau}_i' \equiv \bar{\tau}'$ and $\sum_{i=1}^{N} \underline{\tau}_i' \equiv \underline{\tau}'$. Then, we have that

$$\text{prob}[\bar{s}]u(C + IE[\tilde{\theta}|\bar{s}] + \bar{\tau}') + \text{prob}[\underline{s}]u(C + IE[\tilde{\theta}|\underline{s}] + \underline{\tau}') > \text{prob}[\bar{s}]u(C + IE[\tilde{\theta}|\bar{s}] + \bar{\tau}) + \text{prob}[\underline{s}]u(C + IE[\tilde{\theta}|\underline{s}] + \underline{\tau})$$

But this contradicts the optimality of $\bar{\tau}$ and $\underline{\tau}$ for $N = 1$.

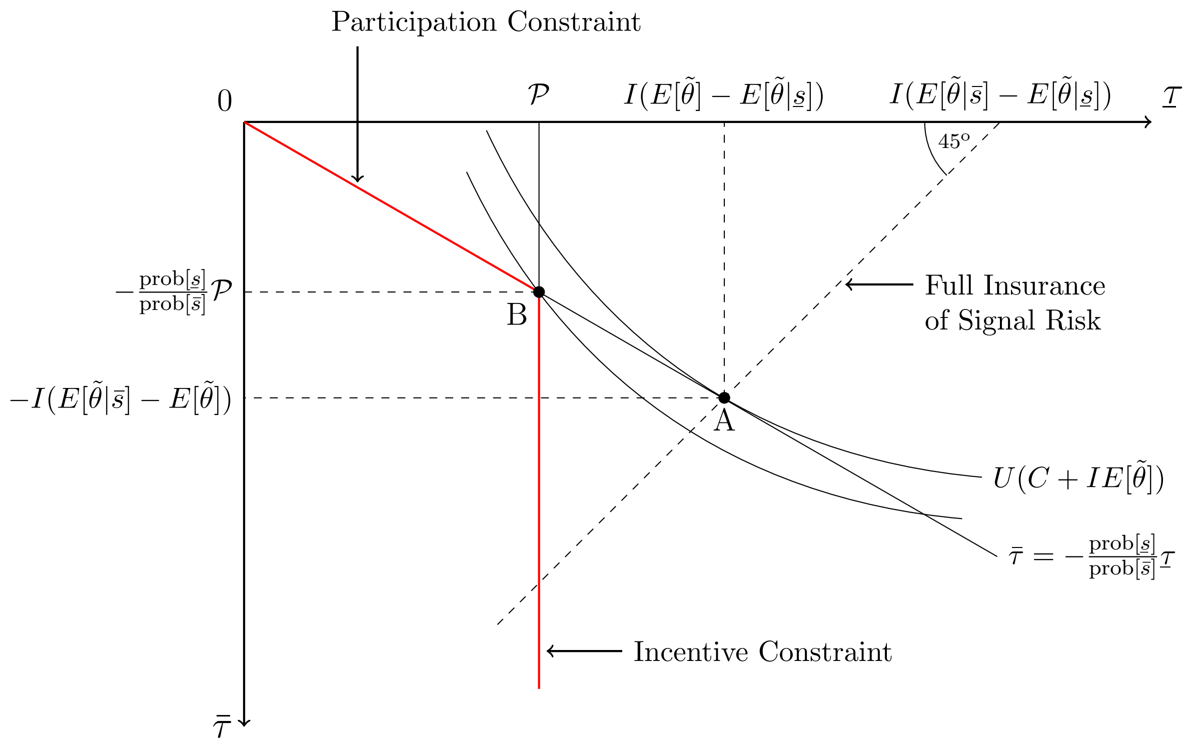

Figure 2: Optimal contracts when effort is observable (A) and when it is not, yet the protection seller exerts effort after a bad signal (B) (no counterparty risk)

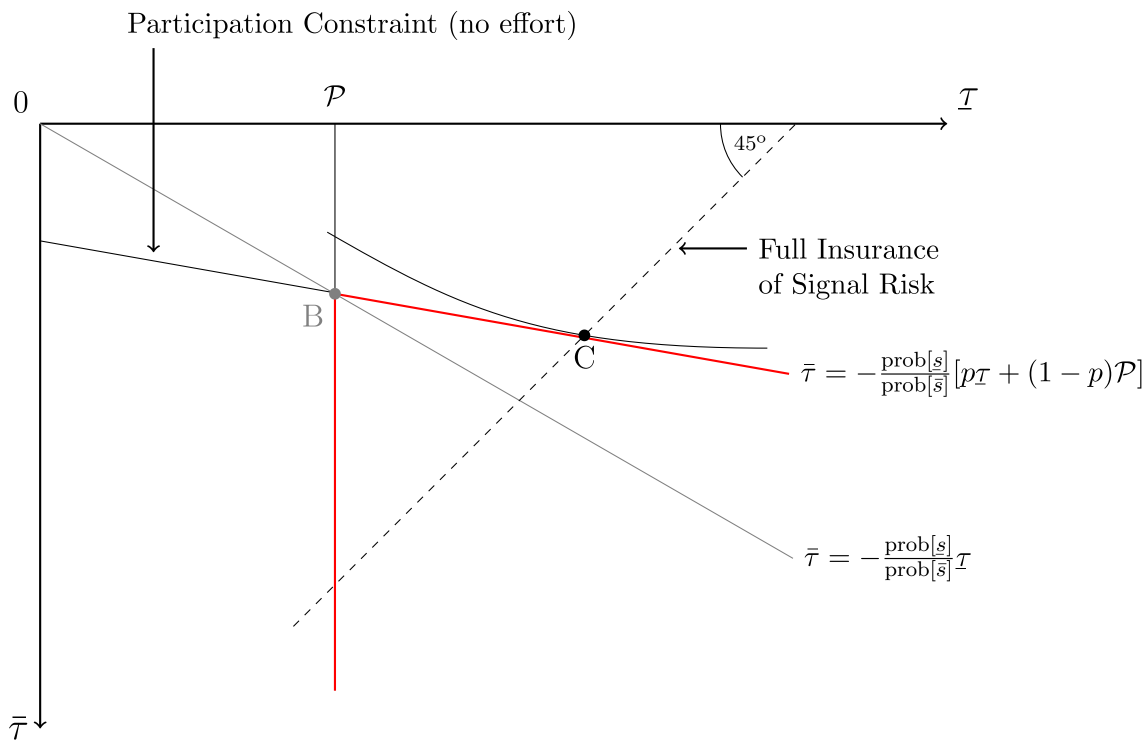

Figure 3: Optimal contract with no effort by the protection seller after a bad signal (counterparty risk)

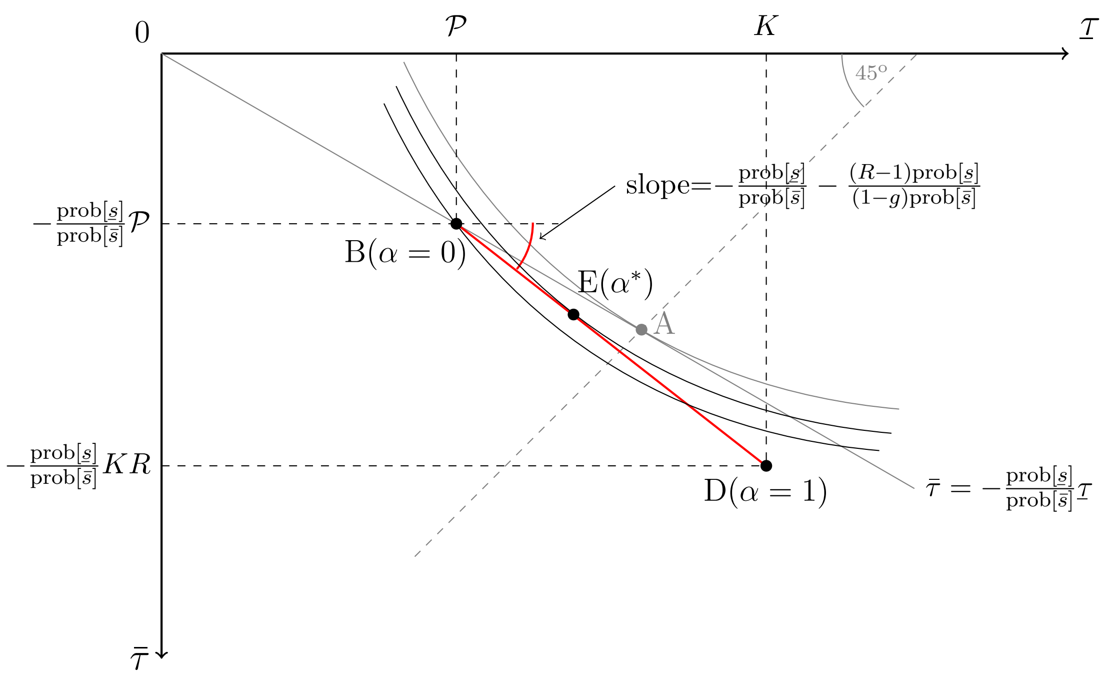

Figure 4: Margin with risk-control