Macroeconomic Risk and Debt Overhang*

PRELIMINARY AND INCOMPLETE

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Abstract

We demonstrate the impact of macroeconomic risk on the investment decisions of firms with risky debt and the costs of debt overhang. Debt overhang stems from the transfer between equity holders and debt holders after investment is made. The cyclicality of a firm's assets in place and its growth options affects how such transfers are distributed across different aggregate states, which provides a range of new predictions that link investment decisions, debt covenants, and capital structure to macroeconomic risk. Using a calibrated dynamic real option/capital structure model, we find that the costs of debt overhang can become 3-4 times higher when macroeconomic risk is taken into account. More cyclical assets in place amplify the impact of macroeconomic risk on debt overhang, while more cyclical growth opportunities may either amplify or reduce the impact of macroeconomic risk on debt overhang. The model predicts that firms with high leverage would prefer to invest in growth options with high systematic risk, while less cyclical firms are likely to have stronger debt covenants preventing asset sales.

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1 Introduction

A fundamental question in finance is to determine the optimal investment decisions for firms. A part of this problem is pricing — the classic rule of Net Present Value (NPV) prescribes that we evaluate an investment opportunity by forecasting its future cash flows and finding an appropriate discount rate. The problem is greatly enriched by market frictions, especially informational asymmetries and agency problems, which not only can alter the cash flows from investment, but also the nature of the risks embedded in them.

Previous research has argued that debt financing distorts equityholders' investment decisions, producing substantial inefficiency. We focus on one type of such agency problems, *debt overhang*. Myers (1977) argues that, in the presence of risky debt, equityholders of a levered firm underinvest, because a fraction of the value generated by their new investment will accrue to the existing debtholders. Thus, investment decisions not only depend on the cash flows from investment, but also the transfers between different stakeholders. In this paper, we demonstrate how macroeconomic risk affects when and how such transfers occur, which links the investment decisions to the cyclicality of assets in place and growth options. Moreover, macroeconomic risk can also substantially amplify the costs of debt overhang, which will affect firms' financing decisions ex ante.

Because of the importance of agency cost of debt for the valuation, investment, and financing decisions of the firm, considerable attention has been devoted to understanding and measuring agency cost of debt. Taking into account the effects of macroeconomic risks on agency cost of debt is important for two reasons. First, recessions are times of high marginal utilities, which means that losses due to agency problems that occur in such times will affect investors more than losses that occur in booms. Thus, the distribution of agency costs across different macroeconomic states matters for their impact ex ante. In particular, the agency cost of debt is amplified if agency conflicts are more severe in bad times, and reduced if agency conflicts are more severe in good times. Second, the agency conflicts between debtholders and equityholders will naturally have a systematic component as agents endogenously respond to changing macroeconomic conditions. For example, for procyclical firms, debt tends to become more risky in bad times. Controlling for the investment opportunity, equityholders will be more concerned about the transfer to debtholders during such times, and will more likely defer investment.

Besides changing the timing of investment, equityholders can also reduce the transfer to debtholders by synchronizing the cash flows from investment with those from the assets in place. For example, if the assets in place are procyclical, then equityholders would prefer to invest in procyclical growth options. This result can explain why a highly levered firm (or bank) might not have incentives to diversify its investments or hedge its market risk exposure, but would instead load on more systematic risk. This result can also be applied to asset sales.[1]

To provide quantitative assessment of the effects, we build a dynamic capital structural model with investment decisions modeled as a real option. To incorporate macroeconomic risks, we impose a stochastic discount factor that generates time variations in the riskfree rate and the risk prices for small shocks as well as large business cycle shocks. The cash flows from assets in place and growth options are allowed to have time-varying expected growth rates, conditional volatility, and jumps that coincide with changes in macroeconomic conditions. We then calibrate the stochastic discount factor to match the business cycle dynamics of asset prices, and examine the agency costs of debt for firms with different leverage, NPV of investment, as well as the systematic risk of their assets in place and growth options.

Our estimates show that debt overhang costs are substantially higher if macroeconomic risk is taken into account. For example, in our benchmark case, the debt overhang costs for a low leverage firm are around 0.75% if macroeconomic risk is not taken into account, while these costs are 2% or 2.5% in booms and recessions respectively if macroeconomic risk is taken into account. For high leverage firms, in our benchmark case, the debt overhang costs are around 2.5% if macroeconomic risk is not taken into account, while these costs are around 5.5% or 6.5% in boom and recessions respectively.

The impact of macroeconomic risk on debt overhang costs depends on the cyclicality of cash flows from assets in place and growth opportunities. More cyclical cash flows from assets in place increase the probability that firms will underinvest during recessions, when marginal utilities are higher, amplifying thus the impact of macroeconomic risk on agency cost of debt. The effect of more cyclical cash flows from growth opportunities is ambiguous. On one hand, more cyclical cash flows from growth opportunities increase the probability that firms will underinvest during recessions, when marginal utilities are higher. On the other hand, the value lost from delaying investment in recessions is lower. In our calibrated dynamic capital structure model, we show that either of the two effects may dominate for reasonable set of parameters.

[1]As Diamond and Rajan (2010) argue, debt overhang might cause impaired banks reluctant to selling bad assets.

Another implication of the dynamic model is that the severe debt overhang in future bad times can also significantly distort investment decisions in good times. In anticipation of bad times arriving in the future, equityholders can become reluctant to invest, even though currently debt might appear to be safe. Thus, the persistence of the macro states will also affect the agency costs. The more persistent the states are, the less the effect of debt overhang in the bad states will propagate to the good states, hence the the bigger the differences in the conditional agency costs between good and bad states.

Our paper builds on a growing literature linking macroeconomic risks with corporate decisions. Almeida and Philippon (2007) use a reduced form approach to measure the ex ante costs of financial distress. They show that the NPV of distress costs rise significantly after adjusting for credit risk premium embedded in the losses. Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2009), and Chen (2009) use structural models to link capital structure decisions to macroeconomic risks. Chen (2009) also demonstrates how the optimal default decisions are affected by the business cycle fluctuations in expected growth rates, economic uncertainty, and risk prices.

Our paper also contributes to the literature on dynamic investment and financing decisions of the firm. Mello and Parsons (1992), Mauer and Triantis (1994), Leland (1998), Mauer and Ott (2000), Décamps and Faure-Grimaud (2002), Hennessy (2004), Titman, Tompaidis, and Tsyplakov (2004), Childs, Mauer, and Ott (2005), Ju and Ou-Yang (2006), Moyen (2007), Sundaresan and Wang (2006), Lobanov and Strebulaev (2007), Manso (2008), and Hackbarth and Mauer (2009) are among the papers that develop dynamic models of investment to study distortions produced by debt financing. These models, however, do not consider macroeconomic risk and its impact on the agency cost of debt.

The paper is also related to the real options literature which study dynamic investment decisions of the firm. McDonald and Siegel (1986), for example, study the timing of an irreversible investment decision. Dixit (1989) analyzes entry and exit decisions of a firm whose output price follows a geometric Brownian motion. Dixit and Pindyck (1994) provide a survey of this literature.

2 Two-period example

Before developing our dynamic model with macroeconomic risk, we present a simple two-period model that illustrates the interplay between macroeconomic conditions and debt overhang. This

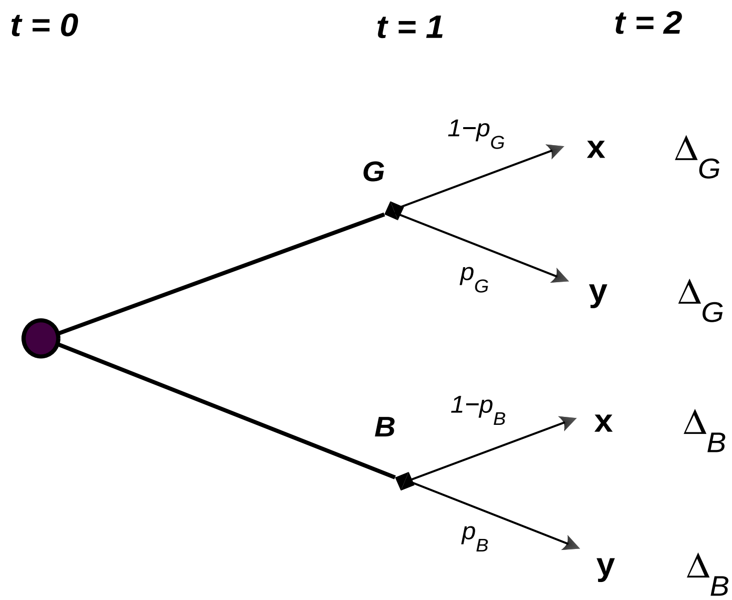

Figure 1: A Two-period Example.

simple model will help with the intuition behind the results obtained in the dynamic model with macroeconomic risk that we develop in the next section.

The economy can either be in one of two aggregate states $s \in \{G, B\}$ at $t = 1$. The price of a one-period Arrow-Debreu security that pays \$1 in state s is given by Q_s. Since the marginal utility in the bad state is higher than the marginal utility in the good state, agents will pay more for consumption in the bad state than in the good state: $Q_B > Q_G$. For simplicity, we assume that the risk-free interest rate is 0, so that $Q_G + Q_B = 1$.

At $t = 2$, the firm's assets in place produce cash flow x with probability $1 - p_s$ and y with probability p_s, where $x > y$, and the realization of cash flow in a given aggregate state is the result of firm-specific shocks in that state.

The firm has zero-coupon debt with face value $y < F \leq x$, which matures at the time cash-flows are realized. Absolute priority is satisfied. As such, if the firm does not produce enough cash flows to pay back debtholders, then debtholders seize the realized cash flows of the firm. The fact that $y < F$ makes debt risky, without which there will be no debt overhang.

Let's first assume that the equityholders of the firm can choose whether or not to undertake an investment I *after* learning the state s of the economy at $t = 1$. The investment produces an additional cash flow of $I + \Delta_s$ realized at the same time as the cash flows from assets in place. We assume that $\Delta_s > 0$ so that the investment opportunity has positive NPV.

We now derive conditions under which equityholders will undertake the available investment opportunity. The equity value of the firm when the manager makes the investment is

$$-I + (1 - p_s)(x + I + \Delta_s - F) + p_s(y + I + \Delta_s - F) \tag{1}$$

if $y + I + \Delta_s \geq F$, and

$$-I + (1 - p_s)(x + I + \Delta_s - F) \tag{2}$$

if $y + I + \Delta_s < F$. The equity value of the firm when equityholders choose not to make the investment is

$$(1 - p_s)(x - F). \tag{3}$$

It follows that equityholders will make the investment if

$$p_s \times \min(F - y, I + \Delta_s) < \Delta_s. \tag{4}$$

The left-hand side of the inequality gives the expected value of the transfer from equityholders to existing debtholders after the investment is made. Thus, equityholders will only make the investment if the expected transfer is less than the NPV of the investment, so that the "overhang-adjusted" NPV is positive.

We define the indicator function Ω_s as

$$\Omega_s \equiv \begin{cases} 0 & \text{if } \; p_s \times \min(F - y, I + \Delta_s) < \Delta_s \\ 1 & \text{otherwise.} \end{cases} \tag{5}$$

The function is equal to 1 if the equityholders do not undertake the investment opportunity, and 0 otherwise.

We next turn to the valuation of the securities of the firm and to the measurement of the agency cost of debt. To provide a benchmark, we first calculate the value V of the unlevered firm at time $t = 0$ (for which $F = 0$). If the firm is unlevered, equityholders will always make the investment and therefore

$$V = \sum_{s \in \{G,B\}} Q_s((1-p_s)x + p_s y + \Delta_s)\,. \tag{6}$$

With $F > 0$, the value of debt at the initial date is

$$D = \sum_{s \in \{G,B\}} Q_s\{(1-p_s)F + p_s((1-\Omega_s)\min(F, y + I + \Delta_s) + \Omega_s y)\}\,. \tag{7}$$

The value of equity at the initial date is:

$$\begin{aligned} E = \sum_{s \in \{G,B\}} & Q_s\{(1-p_s)(x + (1-\Omega_s)(I + \Delta_s) - F) \\ & + p_s((1-\Omega_s)\max(0, y + I + \Delta_s - F)) - (1-\Omega_s)I\}\,. \end{aligned} \tag{8}$$

The total value of the firm is thus

$$E + D = \sum_{s \in \{G,B\}} Q_s\{(1-p_s)x + p_s y + (1-\Omega_s)\Delta_s\}\,. \tag{9}$$

For the purposes of this example, we define the agency cost of debt as

$$A = V - (E + D), \tag{10}$$

the value of the unlevered firm minus the value of the levered firm.[2] Using equations (6) and (9) we obtain that

$$A = Q_G \Omega_G \Delta_G + Q_B \Omega_B \Delta_B\,. \tag{11}$$

The agency cost of debt is equal to the sum over the two states of the product of the value Q_s of 1 dollar in state s, the indicator function Ω_s which is equal to 1 when underinvestment takes place, and the losses Δ_s from underinvestment.

If there were no macroeconomic risk, then

$$Q_G = Q_B = \frac{1}{2}. \tag{12}$$

[2]In the dynamic model of state contingent agency costs of the next section, we will extend this definition to a setting with bankruptcy costs and tax benefits of debt.

To asses the impact of macroeconomic risk on the agency cost of debt we subtract the agency cost of debt when $Q_G = Q_B$ from (11) to obtain:

$$\left(\frac{1}{2} - Q_G\right)(\Omega_B\Delta_B - \Omega_G\Delta_G)\,. \tag{13}$$

Since $Q_G < \frac{1}{2}$, macroeconomic risk exacerbates the agency cost of debt if $\Omega_B\Delta_B > \Omega_G\Delta_G$. Otherwise, macroeconomic risk reduces the agency cost of debt.

We say that a variable z_s is *procyclical* (*countercyclical*) if $z_G < z_B$ ($z_G > z_B$). We say that a variable z_s is *more cyclical* than z'_s if $z_G \geq z'_G$ and $z_B \leq z'_B$.

More cyclical cash flows from assets in place, i.e. lower p_G and higher p_B, makes the condition for investment (4) easier to satisfy in state G but harder in state B. In other words, Ω_B is more likely to be 1, while Ω_G is more likely to be 0. As a result, underinvestment becomes more concentrated in the bad state, exacerbating the costs of debt overhang when macroeconomic risk is taken into account.

Next, more cyclical cash flows $I+\Delta_s$ from the investment also make the condition for investment (4) easier to satisfy in state G but harder in state B. However, it also has the additional effect of reducing the potential loss if the investment is not made in state B. Therefore, the effect of stronger cyclicality of the growth option on the costs of debt overhang is ambiguous.

So far the investment we consider is riskless – its cash flow is constant after investment is made. We now consider a risky investment opportunity that is only exposed to aggregate shocks. This is accomplished by assuming that the investment I is made at $t = 0$ as opposed to $t = 1$, while the cash flows from investment at $t = 2$ remain the same. When would equityholders make the investment? The condition is

$$Q_G p_G \min(F - y, I + \Delta_G) + Q_B p_B \min(F - y, I + \Delta_B) < Q_G\Delta_G + Q_B\Delta_B\,. \tag{14}$$

The right-hand side of the inequality gives the NPV of the investment, while the left-hand side gives the expected transfer from equityholders to debtholders. In the case where the cash flow from new investment makes the existing debt riskfree in both states, the inequality (14) simplifies to

$$Q_G p_G(F - y) + Q_B p_B(F - y) < Q_G\Delta_G + Q_B\Delta_B\,.$$

Thus, only the NPV of the investment (and not its cyclicality) matters for the investment decision in this case.

However, if the cash flow from new investment is not enough to pay off the debtholders in the states with low cash flows from assets in place, then the condition becomes

$$Q_G p_G (I + \Delta_G) + Q_B p_B (I + \Delta_B) < Q_G \Delta_G + Q_B \Delta_B \,.$$

Holding the NPV constant, making the investment opportunity more procyclical means raising Δ_G while lowering Δ_B so that $Q_G\Delta_G + Q_B\Delta_B$ is unchanged. If the assets in place are procyclical, i.e., $p_G < p_B$, then a more procyclical investment can lower the expected transfer from equityholders to debtholders, making equityholders more willing to make such an investment. In fact, the stronger the cyclicality of the investment, the better off the equityholders. Finally, it is also easy to check that when the assets in place are countercyclical, equityholders would prefer to invest in countercyclical growth options.

To summarize, our two-period model provides the following predictions:

- More cyclical assets in place make underinvestment more likely in bad times. It also raises the costs of debt overhang.

- More cyclical investment opportunities also make underinvestment more likely in bad times. The overall effect on the costs of debt overhang is ambiguous.

- Among the growth options that are not too profitable (so that debt is still risky), equityholders would prefer to invest in ones that have the same cyclicality as their assets in place.

3 A Dynamic Model of Debt Overhang

In this section, we set up a dynamic real option/captial structure model to assess the quantitative impact of macroeconomic risk on investments and the costs of debt overhang. The model is related to Mello and Parsons (1992), Sundaresan and Wang (2007), among others, but is generalized to allow for business cycle fluctuations in the cash flows from assets in place, growth options, as well as risk prices (the risk premium for bearing one unit of certain risk) in the economy.

3.1 Model Setup

The Economy Following Chen (2009), we study the investment problem in an economy with business cycle fluctuations in expected growth rates, economic uncertainty, and risk prices. For simplicity, we assume the economy has two aggregate states, $s_t = \{G, B\}$ (boom and recession).[3] The state s_t follows a Markov chain with generator matrix

$$\mathbf{\Lambda} = \begin{bmatrix} -\lambda_G & \lambda_G \\ \lambda_B & -\lambda_B \end{bmatrix}.$$

The generator matrix determines how persistent each state is. For example, the probability of the economy switching from boom (state G) to recession (state B) within time Δ is approximately $\lambda_G \Delta$, while the unconditional probability of the economy being in state G is $\frac{\lambda_B}{\lambda_G + \lambda_B}$.

We directly specify the stochastic discount factor (SDF),[4] which captures business cycle fluctuations in the risk free rate and the risk price for small and large shocks in the economy in a simple way:

$$\frac{dm_t}{m_t} = -r(s_t)\,dt - \eta(s_t)\,dW_t^m + \delta_G(s_t)\left(e^{\kappa} - 1\right) dM_t^G + \delta_B(s_t)\left(e^{-\kappa} - 1\right) dM_t^B, \tag{15}$$

with

$$\delta_G(G) = \delta_B(B) = 1, \quad \delta_G(B) = \delta_B(G) = 0,$$

where W_t^m is a standard Brownian motion that generates systematic small shocks in the economy, and $\{M_t^G, M_t^B\}$ are a pair of compensated Poisson processes with intensity $\{\lambda_G, \lambda_B\}$, which generate large shocks.

The first two terms in the stochastic discount factor process are standard. The instantaneous risk-free rate is $r(s_t)$, and the risk price for Brownian shocks is $\eta(s_t)$, both of which will change value when the state of the economy changes. The last two terms in (15) introduce jumps in the SDF that coincide with a change of state in the Markov chain specified earlier. The relative jump size is assumed to be $\kappa > 0$ ($-\kappa$) if the current state is G (B), so that the SDF jumps up when the economy moves from a boom into a recession, and jumps back down when the economy moves out

[3] It is straightforward to extend the model to allow for more aggregate states, which does not change the main insight of the paper.

[4] Chen (2009) (Proposition 1) shows that such a stochastic discount factor can be generated in a consumption-based model when the expected growth rate and volatility of aggregate consumption follow a discrete-state Markov chain, and the representative agent has recursive preferences. His calibration is based on the long-run risk model of Bansal and Yaron (2004).

of a recession. The value κ determines the risk price for the large shocks in the economy.

The Firm A firm has assets in place that generates cash flow stream $x_t y_t$, where $y_t = y(s_t)$ takes two possible values (y_G, y_B) in booms and recessions, while x_t follows

$$dx_t = \mu(s_t)x_t dt + \sigma_m(s_t)x_t dW_t^m + \sigma_f x_t dW_t^f. \tag{16}$$

This cash flow process captures the impact of the business cycle in several dimensions. First, apart from changes in y_t, $\mu(s_t)$ gives the expected growth rate of cash flow, while $\sigma_m(s_t)$ and σ_f are the systematic and idiosyncratic volatility. Second, when the economy enters into a recession, the level of cash flow jumps by a factor of y_B/y_G, which could be due to a significant change in productivity or adjustment in the amount of productive assets. This jump in cash flow is temporary. When the economy moves out of the recession, the level of cash flow jumps back by a factor of y_G/y_B.[5]

The firm has an option to make an investment. The investment requires a one-time lump-sum cost ϕ, and generates cash flow stream $h(s_t) + k(s_t)x_t y_t$. This flexible form can capture a variety of scenarios for the cyclicality of the growth option.

1. If $k = 0$ and $h > 0$ is constant, then the cash flow from investment is riskless.

2. If $k > 0$ is constant and $h = 0$, then the cash flow from investment has exactly the same growth rate, in particular, the same cash flow beta,[6] as assets in place.

3. By changing h and k, we get a range of cash flow betas for the growth option.

In addition, we can also vary the cyclicality of the growth option using mean-preserving spreads of h and k in the two states. We will investigate how these different aspects of cyclicality of the growth option affect the agency costs of debt. Finally, for simplicity, we rule out disinvestment.

The firm has debt in the form of a consol with coupon c. We first take the firm's decision for choosing the coupon c as exogenous, and focus on the effects of existing debt on investments. Then, we pin down the optimal capital structure using the tradeoff between tax benefits and costs of debt overhang. As is standard in dynamic models of capital structure, we assume that at each

[5]Hackbarth, Miao, and Morellec (2006) and Gorbenko and Strebulaev (2008) have studied the effects of temporary jumps in cash flows on the capital structure.

[6]The cash flow beta here is defined as $\beta_{CF} = \frac{Cov(g_t, r^e_{M,t})}{Var(r^e_{M,t})}$, where g_t is the growth rate of cash flow, and $r^e_{M,t}$ is the market excess return.

point in time the firm first makes the coupon payment c, then pays taxes at rate τ, and distributes all the remaining profit to its equityholders (no cash holdings). At the time of default, we assume that the absolute priority rule applies. The value of equity will be zero, and the recovery value of debt will be a fraction $\alpha(s_t)$ of the value of the unlevered asset at default.

Finally, we assume that the firm acts in the interest of its equityholders. It chooses the optimal timing of default and investment to maximize the value of equity. We also assume that the investment is entirely financed by equityholders, and there are no ex post renegotiations between bondholders and stockholders. In particular, we rule out the possibility of financing the investment with new senior debt (likely restricted by covenants in practice). As argued by Myers (1977), ex post renegotiations can be quite costly.

3.2 Model Solution

We first introduce some notations. The value of equity before investment is $e_s(x)$ in state s. The value of equity after investment is $E_s(x)$. Similarly, the value of debt before and after investment is $d_s(x)$ and $D_s(x)$, respectively.

The optimal investment policy is summarized by a pair of investment boundaries, $\{x_u^G, x_u^B\}$. For example, the firm invests when x_t is above x_u^G while the economy is in state G. The default policy is summarized by two pairs of default boundaries: $\{x_d^G, x_d^B\}$ apply before investment is made, while $\{x_D^G, x_D^B\}$ apply after investment. We first derive the value of equity and debt for given investment and default policies, and then search for the optimal policies.

While the ordering of the default and investment boundaries is endogenous, we assume the following ordering is true when presenting the model solution:

$$x_D^G < x_D^B < x_d^G < x_d^B < x_u^G < x_u^B.$$

This ordering has the intuitive implication that the firm defaults earlier and invests later in bad times. It is also satisfied (as we verify) by most of the parameter regions we consider in this paper. The solution can be easily adjusted for those cases with different ordering of the boundaries.

We will value debt and equity under the risk-neutral probability measure $\mathcal{Q}$. Under $\mathcal{Q}$, the process for x_t becomes

$$dx_t = \widetilde{\mu}(s_t)x_t dt + \sigma(s_t)\, x_t d\widetilde{W}_t, \tag{17}$$

where

$$\widetilde{\mu}(s_t) = \mu(s_t) - \eta(s_t)\,\sigma_m, \tag{18}$$

$$\sigma(s_t) = \sqrt{\sigma_m^2(s_t) + \sigma_f^2}, \tag{19}$$

and $\widetilde{W}_t$ is a standard Brownian motion under $\mathcal{Q}$. In addition, Chen (2009) shows that the generator matrix under the risk-neutral probability measure $\mathcal{Q}$ is $\widetilde{\boldsymbol{\Lambda}}$, where

$$\widetilde{\lambda}_G = \lambda_G e^{\kappa}, \quad \widetilde{\lambda}_B = \lambda_B e^{-\kappa}. \tag{20}$$

Thus, if the stochastic discount factor m_t jumps up when the economy changes from state G to B, then $\kappa > 0$, and $\widetilde{\lambda}_G > \lambda_G$, while $\widetilde{\lambda}_B < \lambda_B$. Intuitively, the jump risk premium in the model makes good (bad) state last shorter (longer) under the risk neutral probability.

3.2.1 Value of Equity

After Investment After the firm exercises the investment option, the problem becomes the same as the static capital structure model with two aggregate states, which is a special case of Chen (2009). As discussed earlier, we conjecture that the default boundaries satisfy $x_D^G < x_D^B$. Then, taking x_D^G and x_D^B as given, the value of equity can be solved in two regions: $J_1 = [x_D^G, x_D^B)$ and $J_2 = [x_D^B, \infty)$.

For $x \in J_1$, the firm has not defaulted yet in state G, but has already defaulted in state B. Thus, $E_B(x) = 0$ in this region. The Feynman-Kac formula implies that $E_G(x)$ satisfies:

$$r_G E_G(x) = (1-\tau)\left(h_G + (k_G+1)xy_G - c\right) + \widetilde{\mu}_G x E_G'(x) + \frac{1}{2}\sigma_G^2 x^2 E_G''(x) + \widetilde{\lambda}_G\left(0 - E_G(x)\right). \tag{21}$$

In Appendix B, we show that

$$E_G(x) = W_{1,1}x^{\alpha_1} + W_{1,2}x^{\alpha_2} + A_1^G x + B_1^G, \tag{22}$$

where α_1, α_2, A_1^G, and B_1^G are given in the Appendix.

Next, for $x \in J_2$, the firm is not in default yet in either state, and $E_G(x)$ and $E_B(x)$ satisfy a system of ODEs:

$$r_G E_G(x) = (1-\tau)(h_G + (k_G+1)xy_G - c) + \widetilde{\mu}_G x E_G'(x) + \frac{1}{2}\sigma_G^2 x^2 E_G''(x) + \widetilde{\lambda}_G (E_B(x) - E_G(x)), \quad (23)$$

$$r_B E_B(x) = (1-\tau)(h_B + (k_B+1)xy_B - c) + \widetilde{\mu}_B x E_B'(x) + \frac{1}{2}\sigma_B^2 x^2 E_B''(x) + \widetilde{\lambda}_B (E_G(x) - E_B(x)). \quad (24)$$

The solutions are:

$$E_G(x) = \sum_{k=1}^{4} W_{2,k} g_k^G x^{\beta_k} + A_2^G x + B_2^G, \quad (25)$$

$$E_B(x) = \sum_{k=1}^{4} W_{2,k} g_k^B x^{\beta_k} + A_2^B x + B_2^B. \quad (26)$$

The value of β_k, $\mathbf{g}_k$, $\mathbf{A}_2$, $\mathbf{B}_2$ are given in the Appendix.

In addition, we have the following boundary conditions that help pin down the values of the coefficients $\mathbf{W}$. First, due to the absolute priority rule, the value of equity at default is zero, which implies

$$\lim_{x \downarrow x_D^G} E_G(x) = 0, \quad (27)$$

$$\lim_{x \downarrow x_D^B} E_B(x) = 0. \quad (28)$$

Next, the value of $E_G(x)$ must be continuous and smooth (see, e.g., Karatzas and Shreve (1991)), which implies

$$\lim_{x \uparrow x_D^B} E_G(x) = \lim_{x \downarrow x_D^B} E_G(x) \quad (29)$$

$$\lim_{x \uparrow x_D^B} E_G'(x) = \lim_{x \downarrow x_D^B} E_G'(x) \quad (30)$$

To rule out bubbles, we also impose the following conditions:

$$\lim_{x \uparrow +\infty} \frac{E_G(x)}{x} < \infty, \quad (31)$$

$$\lim_{x \uparrow +\infty} \frac{E_B(x)}{x} < \infty. \quad (32)$$

As the Appendix shows, these boundary conditions lead to a system of linear equations for Ws, which can be solved in closed form.

Before Investment Before the investment is made, we have conjectured that $x_d^G < x_d^B < x_u^G < x_u^B$, which divide the domain for cash flow x_t into 3 relevant regions: $I_1 = [x_d^G, x_d^B)$, $I_2 = [x_d^B, x_u^G)$, and $I_3 = [x_u^G, x_u^B)$. Again, we first solve for $e_G(x)$ and $e_B(x)$ taking $x_d^G, x_d^B, x_u^G, x_u^B$ as given, then determine the optimal boundaries through a set of smooth-pasting conditions.

In region I_1, the firm has already defaulted in state B. Thus, $e_B(x) = 0$ in this region. In state G, $e_G(x)$ satisfies the same ODE as (21), except that before investment, the firm's cash flow at time t becomes $x_t y(s_t)$ instead of $h(s_t) + k(s_t) x_t y(s_t)$. The solution is

$$e_G(x) = w_{1,1} x^{\alpha_1} + w_{1,2} x^{\alpha_2} + a_1^G x + b_1^G,$$

where α_1, α_2 are the same as in the post-investment case, while a_1^G and b_1^G are given in the Appendix.

In region I_2, the firm is in default in either state, and $e_G(x)$ and $e_B(x)$ satisfy the same ODE system as (23-24), again with instantaneous profit $h(s_t) + k(s_t) x_t y(s_t)$ replaced by $x_t y(s_t)$. The solution is:

$$e_G(x) = \sum_{k=1}^{4} w_{2,k} g_k^G x^{\beta_k} + a_2^G x + b_2^G, \tag{33}$$

$$e_B(x) = \sum_{k=1}^{4} w_{2,k} g_k^B x^{\beta_k} + a_2^B x + b_2^B, \tag{34}$$

where the value of β_k and $\mathbf{g}_k$ are the same as in the post-investment case, while $\mathbf{a}_2$ and $\mathbf{b}_2$ are given in the Appendix.

In region I_3, the firm will have already made the investment in state H. In state B, $e_B(x)$ satisfies:

$$r_B e_B(x) = (1-\tau)(x y_B - c) + \widetilde{\mu}_B x e_B'(x) + \frac{1}{2}\sigma_B^2 x^2 e_B''(x) + \widetilde{\lambda}_B (E_G(x) - \phi - e_B(x)). \tag{35}$$

The last term captures the effect that the firm will invest immediately if the state changes from B to G. The solution takes the form

$$e_B(x) = w_{3,1} x^{\gamma_1} + w_{3,2} x^{\gamma_2} + a_3^B x + b_3^B + \sum_{k=1}^{4} \omega_k x^{\beta_K}, \tag{36}$$

where γ_1, γ_2, a_3^B, b_3^B, and ω are given in the Appendix.

The values of the coefficients $\{\mathbf{w}\}$ are determined by the following boundary conditions. First, the value of equity is again worthless at default:

$$\lim_{X \downarrow X_d^G} e_G(x) = 0, \tag{37}$$

$$\lim_{x \downarrow x_d^B} e_B(x) = 0. \tag{38}$$

Next, the value of $e_G(x)$ must be piecewise $\mathcal{C}^2$, so that

$$\lim_{x \uparrow x_d^B} e_G(x) = \lim_{x \downarrow x_d^B} e_G(x), \tag{39}$$

$$\lim_{x \uparrow x_d^B} e'_G(x) = \lim_{x \downarrow x_d^B} e'_G(x). \tag{40}$$

At the two investment boundaries x_u^G and x_u^B, the value-matching conditions imply

$$\lim_{x \uparrow x_u^G} e_G(x) = \lim_{x \downarrow x_u^G} E_G(x) - \phi, \tag{41}$$

$$\lim_{x \uparrow x_u^B} e_B(x) = \lim_{x \downarrow x_u^B} E_B(x) - \phi. \tag{42}$$

Finally, e_B must be piecewise $\mathcal{C}^2$,

$$\lim_{x \uparrow x_u^G} e_B(x) = \lim_{x \downarrow x_u^G} e_B(x), \tag{43}$$

$$\lim_{x \uparrow x_u^G} e'_B(x) = \lim_{x \downarrow x_u^G} e'_B(x). \tag{44}$$

Again, the boundary conditions are all linear in the coefficients $\{\mathbf{w}\}$, so we can solve for them analytically from a system of linear equations.

For a given set of default and investment boundaries, we can also price a class of contingent claims (including the consol) the same way. We summarize the results in Proposition 1 in the Appendix.

3.2.2 Optimal Default, Investment Policy, and Agency Costs

Next, we discuss the conditions that determine the optimal default and investment boundaries. Whenever the optimal default boundaries post investment $\{x_D^G, x_D^B\}$ are in the interior region (above 0), they satisfy the smooth-pasting conditions:

$$\lim_{x \downarrow x_D^G} E_G'(x) = 0, \tag{45}$$

$$\lim_{x \downarrow x_D^B} E_B'(x) = 0. \tag{46}$$

Since E_G and E_B are given in closed form, these smooth-pasting conditions render two nonlinear equations for x_D^G and x_D^B that can be solved numerically.

Similarly, the optimal investment and default boundaries $\left\{x_d^G,\ x_d^B,\ x_u^G,\ x_u^B\right\}$ satisfy 4 smooth-pasting conditions:

$$\lim_{x \downarrow x_d^G} e_G'(x) = 0, \tag{47}$$

$$\lim_{x \downarrow x_d^B} e_B'(x) = 0, \tag{48}$$

$$\lim_{x \uparrow x_u^G} e_G'(x) = \lim_{x \downarrow x_u^G} E_G'(x), \tag{49}$$

$$\lim_{x \uparrow x_u^B} e_B'(x) = \lim_{x \downarrow x_u^B} E_B'(x), \tag{50}$$

which again translate into a system of nonlinear equations in $\left\{x_d^G,\ x_d^B,\ x_u^G,\ x_u^B\right\}$.

We can measure the costs of debt overhang in the following way. The first best investment policy is achieved when the firm has no debt, i.e., $c = 0$. We denote the investment boundaries in this case as $\left\{x_u^{G*}, x_u^{B*}\right\}$. The existence of risky debt makes equity holders raise the investment thresholds, so that $x_u^G > x_u^{G*}$ and $x_u^B > x_u^{B*}$. Then, the agency costs can be measured as the difference in the value of the firm under the first best and second best investment policy. However, there is an issue with this measure of agency costs. Since the probability of default will depend on the investment policy, part of the difference in firm value comes from differences in the costs of bankruptcy, which depend on our assumption of the recovery rates.

To avoid this issue, we can instead compute the agency costs as the difference in the value of an all-equity firm under the first and second best investment policy. Let $v_s\left(x; x_u^G, x_u^B\right)$ be the value of an all-equity firm in state s with current cash flow x and investment thresholds $\{x_u^G, x_u^B\}$ (assuming the investment has not been made). Then, the costs of underinvestment are measured relative to the first best firm value:

$$ac_s(x_0) = \frac{v_s\left(x_0; x_u^{G*}, x_u^{B*}\right) - v_s\left(x_0; x_u^{G}, x_u^{B}\right)}{v_s\left(x_0; x_u^{G*}, x_u^{B*}\right)}, \quad s = G, B. \tag{51}$$

It is possible that current cash flow x_0 is higher than some of the investment thresholds under the first or second best. In that case, the firm will invest immediately, and the value of the firm is given by the value of the firm after investment minus the fixed costs of investment ϕ.

Having described the model and its solution, next we examine its quantitative implications.

4 Quantitative Analysis

4.1 Calibration

A proper calibration is key to making comparison across models with and without macroeconomic risks. We first calibrate the stochastic discount factor, which generates time-varying risk-free rate and risk prices over the business cycle. Then, we calibrate the cash flows of the firm's assets in place and growth option with various degrees of cyclicality.

To calibrate the transition probabilities (λ_G and λ_B) of the Markov chain, we estimate a two-state Hidden Markov Model for the post-war data of real consumption growth, using the EM algorithm of Hamilton (1990). The average duration of the high-growth (expansion) state is 10.5 quarters, and 2.2 quarters for the low-growth (recession) state. As a result, the unconditional probability of the expansion (recession) state is 0.83 (0.17). Next, we calibrate $r(s_t)$ in the two states to match the mean and standard deviation of real riskfree rates. Then we set $\kappa = \ln(2)$, which implies the risk-neutral probability of a jump from state G to B is twice as high as the physical probability.[7] The remaining parameters of the stochastic discount factor, the prices of Brownian shocks $\eta(s_t)$, are calibrated to match the average equity premium and the Sharpe ratio of the market portfolio, which for simplicity we assume to be a claim on the dividend stream x_t without idiosyncratic volatility. The expected growth rate $\mu(s_t)$ and systematic conditional volatility $\sigma_m(s_t)$ of x_t are calibrated to match the average growth rate and volatility of aggregate corporate profits. Finally, the idiosyncratic volatility of x_t, σ_f, is a free parameter that we use to generate variations in the Sharpe ratio of equity in the firm.

[7] This jump-risk premium is consistent with the calibration adopted in Chen (2009). Later on we examine how different values of κ affect the results.

Table 1: CALIBRATION OF THE MARKOV CHAIN MODEL

The table reports the model-generated moments of the equity market. The expressions $E(r_m - r_f)$ and $E(r_f)$ are the annualized equity premium and average risk-free rate. The expressions $\sigma(r_m)$, $\sigma(r_f)$, and $\sigma(log(P/D))$ are the annualized volatilities of the market return, risk-free rate, and the log price-dividend ratio, respectively. The variable $E(SR)$ is the average Sharpe ratio.

Variable	G	B	mean	std
A. Calibrated Parameters				
$\lambda(s_t)$	0.38	1.83	-	-
$r_f(s_t)$	1.30	-1.26	0.86	1.00
$\eta(s_t)$	0.17	0.45	0.22	0.10
$\mu(s_t)$	3.17	-4.76	1.80	3.00
$\sigma_m(s_t)$	12.27	22.84	14.10	4.00
B. Asset Pricing Implications				
$E(r_m - r_f)$	4.93	17.60	7.12	4.80
$\sigma(r_m - r_f)$	13.10	25.27	15.20	4.61
$E(r_m - r_f)/\sigma(r_m - r_f)$	0.38	0.70	0.43	0.12

The resulting values are reported in Panel A of Table 1. The mean and standard deviation are computed using the stationary distribution of the Markov chain. The asset pricing implications of the stochastic discount factor are in Panel B.

Chen, Collin-Dufresne, and Goldstein (2009) and Chen (2009) show that the Sharpe ratio of equity is a key statistic in determining whether the amount of systematic risk in a firm is reasonable in a calibration. For this reason, when comparing models with and without macroeconomic risk, we always match the average Sharpe ratio of the market portfolio (0.43) as well as that for the firm. In the benchmark case, we calibrate the idiosyncratic volatility σ_f to generate three levels of firm-level Sharpe ratio, 0.2, 0.25, and 0.3.

4.2 Debt Overhang

One factor that is important in determining the costs of debt overhang for a given investment project is the value of the growth option, which depends on the cash flows generated by the investment and the fixed costs ϕ. If the growth option is too far out of the money, the firm is unlikely to make the investment regardless of whether it has debt in place or not. In this case, the agency costs as defined by (51) will be (essentially) zero. As the value of the growth option increases,

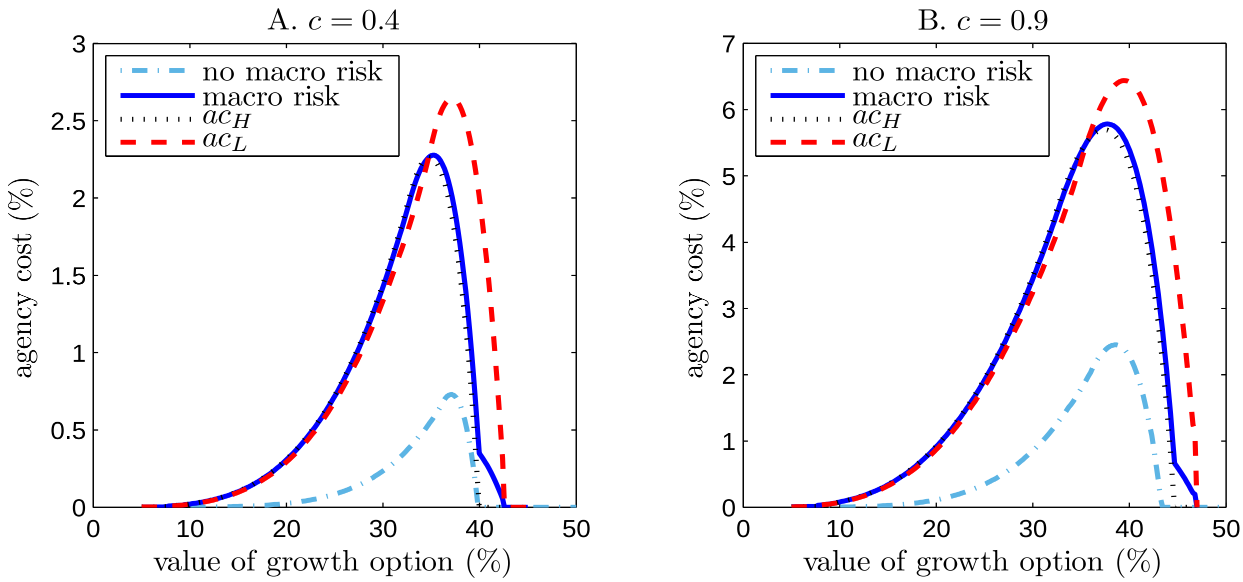

Figure 2: **Costs of Underinvestment.** This figure plots the costs of debt overhang for investments with different NPVs. ac_H and ac_L are the conditional costs of debt overhang in good and bad state. The weighted average between the two gives the solid line (macro risk).

the investment thresholds are likely to drop. If the growth option is sufficiently in the money, the optimal investment thresholds can be below x_0 both under the first best (no debt) and the second best ($c > 0$). In this case, the firm invests immediately, and there will be no difference in the actual investment thresholds under the first and second best. Then, the agency costs will again be zero.

The two extreme cases above suggest that it is important to take into account how profitable the project is when computing the costs of debt overhang. In Figure 2, we plot the costs of debt overhang for growth options with value ranging from 0 to 50% of the first best firm value. The cash flows from the investment take the form of (), with $h = 0.1$ and $k = 1$ (constant across states). We then vary the value of the growth option by changing ϕ, but recalibrate σ_f each time to fix the Sharpe ratio of equity at 0.25.

Panel A plots the agency costs for a low leverage firm ($c = 0.4$). When the value of the growth option is low, the losses from delaying investment in the project are low and therefore agency costs due to underinvestment are also low. When the value of the growth option is high, the investment opportunity is so valuable that there is very little delay in investment, and therefore agency costs due to underinvestment are also low. Agency costs due to underinvestment peak for intermediate values of the growth option. For these values, both delay in investment as well as losses from underinvestment are significant. The difference between agency costs due to underinvestment with and without taking into account macroeconomic risk is substantial. When the value of the growth option is 35% of the firm first best value, the agency cost without taking into account macroeconomic risk is close to 0.75% of the firm first best value. If we take macroeconomic risk into account, for the same value of the growth option, the agency costs of debt are approximately 2% and 2.5% of the firm first best value depending on whether the current state of the economy is B or G respectively. This shows that macroeconomic risk is indeed a key determinant of the agency costs of debt. Panel B of Figure 2 shows how agency costs of debt vary with the value of the growth option for a high leverage firm ($c = 0.9$). When the value of the growth option is 35%, agency costs of debt are 2% with no macroeconomic risk, and approximately 5% and 6% with macroeconomic risk in states B and G respectively.

The results in Figure 2 also highlight the impact of business cycle dynamics on debt overhang, which are not obvious in a static setting. The conditional agency costs in the good and bad state, ac_G and ac_B, while both significantly higher than in the case without macroeconomic risk, are not that far apart. In fact, while ac_B is higher than ac_G for sufficiently valuable growth options, the two are almost identical when the value of the growth option is low. This is due to the fact that, when in state G, equityholders are reluctant to invest because they are concerned that the state of the economy might change, which can make debt substantially more risky, and raise the amount of wealth transfer from equityholders to debtholders. Obviously, the differences between the conditional agency costs in the two state will become more significant as we make the two states more persistent.

We also examine the effects of systematic risk on the costs of debt overhang by varying the price of jump risks κ, the price of Brownian risk $\eta(s_t)$, the persistence of the recession state λ_B, and the average Sharpe ratio of equity. The results are reported in Table 2. If we increase the price of jump risk κ from $\ln(2)$ to $\ln(2.5)$, average agency costs increase from 2.04 to 3.67. A price of jump risk $\kappa = \ln(2.5)$ implies that the risk-neutral probability of a jump from state G to B is 2.5 times as high as the physical probability of a jump from state G to B. If we increase the volatility of $\eta(s_t)$ from 10% to 15% keeping its mean constant, average agency costs increase from 2.04 to 2.67. Finally, if we reduce the persistence parameter λ_B from 1.92 to 0.92, average agency costs increase from 2.04 to 4.98.

Having demonstrated the overall effect of business cycle risks on the costs of debt overhang, we

Table 2: Costs of Underinvestment: Systematic Risk

The table reports the 3-year conditional investment probabilities in the two states $(p_G^3(x_0), p_B^3(x_0))$, the conditional costs of underinvestment in the two states $(ac_G(x_0), ac_B(x_0))$, and the average costs of underinvestment at $x_0 = 1$. In the benchmark case, $y_G = y_B = 1$ (no jumps), and the volatilities of $\mu(s_t)$ and $\sigma(s_t)$ are 3.0% and 4.0%, respectively. The rest of the parameters are: $c = 0.4$, $h_G = h_B = 0.1$, $k_G = k_B = 1$, while ϕ and σ_f are calibrated to fix the value of the growth option at 35% of the first best firm value, and the average Sharpe ratio of equity at 0.25.

	invest prob (%)		agency costs (%)		average
	$p_G^3(x_0)$	$p_B^3(x_0)$	$ac_G(x_0)$	$ac_B(x_0)$	agency costs
benchmark	70.0	63.3	2.0	2.4	2.0
$\kappa = \ln(2.5)$	47.9	43.9	3.6	3.9	3.7
15% vol in $\eta(s_t)$	62.0	56.1	2.6	3.0	2.7
$\lambda_B = 0.92$	48.4	44.1	4.8	5.5	5.0

next decompose the effects into two parts, one through assets in place, the other through growth option.

4.3 Assets in Place and Growth Option

As we discussed in the static model in Section 2, the cyclicality of assets in place and growth option have different effects on the agency costs of debt. In this section, we examine these effects in the dynamic model. First, to isolate the effects from the cyclicality of assets in place, we assume that the growth option is riskless ($h(s_t) = h$, $k(s_t) = 0$). In this case, while equityholders are the ones responsible for the costs of the investment, the riskless cash flows generated by the investment will reduce the probability of default and increase the value of risky debt, especially when the cash flow from assets in place is low. The riskier the debt gets, the larger the transfer from equityholders to debtholders, which make equityholders less willing to invest, hence raising the costs of underinvestment.

Under the first best, the value of the growth option is equal to its NPV. Whenever the NPV is positive, the optimal policy is to invest immediately, i.e., the investment threshold $x_u^* = 0$. If the NPV is negative, the firm will never invest. Naturally, we focus on the former case. Specifically, we

Table 3: COSTS OF UNDERINVESTMENT: CYCLICALITY OF ASSETS IN PLACE

The table reports the 3-year conditional investment probabilities in the two states ($p_G^3(x_0), p_B^3(x_0)$), the conditional costs of underinvestment in the two states ($ac_G(x_0), ac_B(x_0)$), and the average costs of underinvestment at $x_0 = 1$. In the benchmark case, $y_G = y_B = 1$ (no jumps), and the volatilities of $\mu(s_t)$ and $\sigma(s_t)$ are 3.0% and 4.0%, respectively. The rest of the parameters are: $c = 0.4$, $h_G = h_B = 0.2$, $k_G = k_B = 0$, and $\phi = 2.5$. The idiosyncratic volatility is $\sigma_f = 22.5\%$, which imply that the average Sharpe ratio of equity in the benchmark case is 0.25.

	invest prob (%)		agency costs (%)		average
	$p_G^3(x_0)$	$p_B^3(x_0)$	$ac_G(x_0)$	$ac_B(x_0)$	agency costs
no macro risk	100		0		0
benchmark	66.0	60.6	5.6	8.0	6.0
$y_B/y_G = 0.8$	57.3	52.5	7.3	9.8	7.7
$y_B/y_G = 0.7$	52.7	48.4	8.2	10.7	8.6
4.0% vol for $\mu(s_t)$	45.5	40.3	9.7	12.6	10.2
5.0% vol for $\mu(s_t)$	29.3	25.2	13.3	16.5	13.9
5.0% vol for $\sigma_m(s_t)$	49.9	45.8	8.6	11.1	9.0
6.0% vol for $\sigma_m(s_t)$	36.0	33.3	11.2	13.8	11.7
low beta	87.4	81.3	1.7	3.4	2.0

consider a riskless project with a constant cash flow stream at the rate $h = 0.2$ and a fixed cost of investment $\phi = 2.5$. We then calibrate the idiosyncratic volatility $\sigma_f = 22.5\%$ to make the average Sharpe ratio of equity for the benchmark firm equal to 0.25.

In our model, we interpret assets in place as becoming more cyclical when either the level of cash flow or the conditional moments of its growth rates become more volatile across the two states. Thus, we consider three different ways to vary the cyclicality of assets in place: (1) increasing the size of jumps in cash flow when the state changes through a mean-preserving spread in $y(s_t)$; (2) increasing the volatility of the expected growth rate $\mu(s_t)$; and (3) increasing the volatility of the conditional volatility $\sigma_m(s_t)$.

Table 3 reports the results. First of all, without macroeconomic risk, a firm with an identical riskless project (same h and ϕ) will set the investment threshold at $x_u = 0.89$. While this implies significant delay in investment relative to the first best, the threshold is below initial cash flow $x_0 = 1$, which means the firm will make the investment immediately, and the costs of debt overhang measured at x_0 will be 0.

Next, for the benchmark firm, there are no jumps in the cash flow levels when the state changes, and the conditional mean and volatility of the growth rates of cash flow from assets in place are given in Table 1. The coupon rate is $c = 0.4$, which implies an average initial leverage of 44%. This firm delays making the investment significantly, as the probability of investment in the next 3-years is a mere 57.3% in the good state, or 52.5% in the bad state, as opposed to immediate investment under the first best. The average agency costs are 6.0% of the first best firm value, but the conditional agency costs are considerably higher in state B (8.0%) than in state G (5.6%).

As we make the assets in place more cyclical (through any one of the three approaches explained above), the probabilities of investment fall, while the average costs of debt overhang become higher. If the level of cash flow from assets in place "temporarily" jumps down by 30% when the state switches from G to B, the average costs of debt overhang are 8.6% of the first best firm value, compared to 6.0% when the cash flow does not jump. There are even more pronounced effects on the agency costs when we strengthen the cyclicality of assets in place through the conditional moments of growth rates. For example, when we increase the volatility of $\mu(s_t)$ (the expected growth rate of cash flow from assets in place) from 3.0% to 5.0% (without changing the average expected growth rate), the average agency costs of debt increase from 6.0% to 13.9%, while the probability of investment within the next 3 years fall by more than half. Bigger swings in the conditional volatility of growth rate $\sigma_m(s_t)$ across states also significantly increase the costs of debt overhang.

Finally, we examine how the cash flow beta of assets in place affects the agency costs of debt. To obtain a lower beta, we lower the systematic volatility of x ($\sigma_m(s_t)$) by 10% in both states, and then choose the idiosyncratic volatility σ_f to keep the average volatility of x unchanged. The effects of lower cash flow beta for the assets in place are very pronounced. The probability of investments in a 3-year horizon rise significantly, while the average agency costs fall from 6.0% to 2.0%.

Taken together, this evidence suggests that, keeping everything else constant, more cyclical assets in place or more systematic risk in assets in place (as measured by cash flow beta) imply bigger impact of macroeconomic conditions on the costs of debt overhang, as also suggested by the two-period model introduced in Section 2.

Table 4: Costs of Underinvestment: Cyclicality of Growth Option

The table reports the 3-year conditional investment probabilities in the two states ($p^3_G(x_0), p^3_B(x_0)$), the conditional costs of underinvestment in the two states ($ac_G(x_0), ac_B(x_0)$), and the average costs of underinvestment at $x_0 = 1$. In the benchmark case, $y_G = y_B = 1$ (no jumps), $h = 0.1$, $k = 1$. Coupon rate is $c = 0.4$. The fixed cost $\phi = 9.7$ and idiosyncratic volatility $\sigma_f = 22.1\%$ are chosen to make the value of the growth option at 35% of the first best firm value and the average Sharpe ratio of equity at 0.25 for the benchmark case.

	invest prob (%)		invest boundary		agency costs (%)		average
	$p^3_G(x_0)$	$p^3_B(x_0)$	x^G_u/x^{H*}_u	x^B_u/x^{L*}_u	$ac_G(x_0)$	$ac_B(x_0)$	agency costs
benchmark	70.0	63.3	1.33	1.37	2.0	2.4	2.0
$h_B/h_G = 0.75$	68.4	61.8	1.31	1.34	1.9	2.1	1.9
$h_B/h_G = 0.50$	66.7	60.3	1.29	1.31	1.7	1.8	1.8
$k_B/k_G = 0.75$	62.1	55.9	1.33	1.36	2.1	1.9	2.1
$k_B/k_G = 0.50$	52.9	47.5	1.32	1.34	2.0	1.5	1.9
low beta	68.9	62.5	1.83	1.97	3.7	5.1	3.9

While there is a clean positive relation between the cyclicality of assets in place and the costs of debt overhang, the case for the growth option is more complicated. On the one hand, stronger cyclicality raises the value of the growth option in the good state, but lowers it in the bad state, which has the effects of making default less likely in the good state but more likely in the bad state. This implies that for a given investment there will be more wealth transfer to the debtholders in the bad state, where such losses are valued higher, which tends to exacerbate debt overhang problem. On the other hand, a more cyclical growth option also makes the amount of cash flow from investment in the bad state lower, which means the investment will be less helpful in making the existing lenders whole in the bad state. This second effect tends to alleviate debt overhang. Finally, the losses from the same amount of delay in investment will be lower in the bad state and higher in the good state, which will also be factored into the ex ante costs of debt overhang.

We examine the magnitude of these effects in Table 4. Similar to the case of assets in place, we consider a low leverage firm ($c = 0.4$), and vary the cyclicality of the growth option in two ways.[8] The first is a mean-preserving spread in h, the part of investment cash flow independent of assets in place. Relative to the benchmark case, when h falls by 25% from state G to B, the investment thresholds in both states become higher, and the probability of making the investment within the next 3 years fall from 70.0% to 68.4% in the good state, and from 63.3% to 61.8% in the bad state. While the long run average of cash flow from investment is not affected by the mean-preserving spread in h, shifting cash flow away from the bad state makes the investment more risky. As a result, the firm becomes more reluctant to invest, both under the first and second best. What is more important is that relative to the first best, the investment thresholds under the second best are rising by a smaller amount, as is evident in the decrease of the ratio of the investment thresholds x_u^G/x_u^{H*} and x_u^B/x_u^{L*}. Thus, there is less delay in investment (measured by the investment threshold) for more cyclical investment opportunities. The costs of debt overhang also fall in both states, but more so in the bad state. On average, it falls from 2.04% to 1.92% of the first best firm value. Similar results hold when there is a 50% drop in cash flow.

Next, we consider a mean-preserving spread in k, the part of investment cash flow tied to assets in place. In this case, when the variation in k is not too big ($k_B/k_G = 0.75$), there is little effect on the delay of investment under the second best relative to the first best. However, changes in k significantly increase the difference in the value of the investment across the two states, which makes the conditional costs of debt overhang increase in the good state (from 1.97% to 2.12%), but fall in the bad state (from 2.42% to 1.92%). In this case, the average costs of debt overhang actually rise. But for a larger variation in k ($k_B/k_G = 0.5$), the costs of underinvestment become lower in both states.

Analogous to the analysis of cash flow beta for assets in place, we also examine the effects of the cash flow beta for the growth option on the agency costs. Rather than changing the systematic and idiosyncratic volatility of x, which will affect the beta of assets in place, we conduct a different experiment that only changes the beta of the growth option. Without changing the NPV of the project, we increase the riskless component (h) and lower the risky component (k) of investment, which lowers its cash flow beta. Such a change substantially increases the difference in the investment thresholds between the first and second best. The probabilities of investment in the next 3 years actually do not change much under the second best, which suggests that lower cash flow beta has a much bigger effect on the investment policy for the first best. The costs of debt overhang rise

[8] We cannot change the conditional moments for the growth rates of x in this case because it will simultaneously affect the cyclicality of assets in place.

Table 5: AGENCY COSTS UNDER OPTIMAL LEVERAGE

The table reports the 3-year conditional investment probabilities in the two states $(p_G^3(x_0), p_B^3(x_0))$, and the conditional costs of underinvestment in the two states $(ac_G(x_0), ac_B(x_0))$ conditional on optimal leverage at $x_0 = 1$.

initial state	coupon	$LH(x_0)$	$LL(x_0)$	$ac_H(x_0)$	$ac_L(x_0)$
	high value growth option				
G	0.56	48.4	51.5	1.60	1.54
B	0.51	45.6	48.6	1.44	1.39
	low value growth option				
G	0.65	55.3	58.6	0.67	0.63
B	0.59	51.9	55.1	0.59	0.55

in both states, with the average costs rise from 2.04% to 3.93% of the first best firm value. This result suggests that lower cash flow beta for the growth option increases the costs of debt overhang.

5 Optimal Leverage

6 Concluding Remarks

Using a dynamic model of capital structure with investment decisions and macroeconomic risks, we show that the agency cost of debt due to debt overhang increases substantially when macroeconomic risk is taken into account. For example, in one of our benchmark parameters, debt overhang costs are 0.75% without macroeconomic risk and 2.5% or 2.75% in booms and recessions respectively. We also show that debt overhang costs depends on the cyclicality of cash flows from assets in place and growth opportunities. More cyclical cash flows from assets in place amplify the effects of macroeconomic risk on debt overhang costs. More cyclical cash flows from growth opportunities have an ambiguous effect on debt overhang costs. We also study the impact of macroeconomic risk on optimal capital structure of the firm.

Several questions remains unanswered. For example, what is the effect of macroeconomic risk on different agency conflicts, such as asset substitution (Jensen and Meckling (1976)) or free cash-flow (Jensen (1986))? Because in bad times firms are usually closer to default, the asset substitution problem may be more prevalent in bad times. If this is indeed the case, asset substitution costs will be amplified by macroeconomic risk. On the other hand, the free cash-flow problem may be more prevalent in good times, when there is more cash available to be diverted. If this is the case, free cash-flow costs are reduced if macroeconomic risk is taken into account. A more systematic study of these and other agency conflicts seems warranted.

Appendix

A Value of Equity

After investment, for $x < J_1$, the solution to the homogeneous equation in (21) is

$$E_G(x) = W_{1,1}x^{\alpha_1} + W_{1,2}x^{\alpha_2},$$

where

$$\alpha_1, \alpha_2 = -\sigma_G^{-2}\left[\left(\widetilde{\mu}_G - \frac{\sigma_G^2}{2}\right) \pm \sqrt{\left(\widetilde{\mu}_G - \frac{\sigma_G^2}{2}\right)^2 + 2r_G\sigma_G^2}\,\right], \tag{52}$$

and it is easy to verify that the particular solution is

$$E_G(x) = A_1^G x + B_1^G,$$

where

$$A_1^G = \frac{(1-\tau)(k_G+1)\,y_G}{r_G + \widetilde{\lambda}_G - \widetilde{\mu}_G}, \tag{53}$$

$$B_1^G = \frac{(1-\tau)(h_G - c)}{r_G + \widetilde{\lambda}_G}. \tag{54}$$

For $x < J_2$, the homogeneous equations from the ODE system (23-24) can be formulated as a quadratic eigenvalue problem (see Chen (2009) for details), and the solution is given by (25-26), where β_k, g_k are the $k-th$ eigenvalue and (part of the) eigenvector for the following standard eigenvalue problem:

$$\begin{bmatrix} 0 & \mathbf{I} \\ -\left(2\mathbf{\Sigma}_X^{-1}\left(\tilde{\mathbf{\Lambda}} - \mathbf{r}^n\right)\right) & -\left(2\mathbf{\Sigma}_X^{-1}\tilde{\theta}_X - \mathbf{I}\right) \end{bmatrix} \begin{bmatrix} \mathbf{g}_k \\ \mathbf{h}_k \end{bmatrix} = \beta_k \begin{bmatrix} \mathbf{g}_k \\ \mathbf{h}_k \end{bmatrix}, \tag{55}$$

where $\mathbf{I}$ is a 2×2 identity matrix, $\mathbf{r}^n = \mathbf{diag}\left([r_G^n, r_B^n]'\right)$, $\tilde{\theta}_X = \mathbf{diag}\left([\widetilde{\mu}_G, \widetilde{\mu}_B]'\right)$, and $\mathbf{\Sigma}_X = \mathbf{diag}\left(\left[\sigma_G^2, \sigma_B^2\right]'\right)$. Barlow, Rogers, and Williams (1980) show that there are exactly 2 eigenvalues with negative real parts, and 2 with positive real parts.

Next, one can verify that the particular solutions will be in the form $\mathbf{A}_2 x + \mathbf{B}_2$, where $\mathbf{A}_2 =$

$\left[A_2^G, A_2^B\right]'$ and $\mathbf{B}_2 = \left[B_2^G, B_2^B\right]'$ are given by

$$\mathbf{A}_2 = (1-\tau)\left(\mathbf{r} - \widetilde{\mu} - \widetilde{\mathbf{\Lambda}}\right)^{-1}(\mathbf{k}_1 + 1)\,\mathbf{y}, \tag{56}$$

$$\mathbf{B}_2 = (1-\tau)\left(\mathbf{r} - \widetilde{\mathbf{\Lambda}}\right)^{-1}(\mathbf{k}_0 - c\mathbf{1}). \tag{57}$$

The coefficients $\{W_{1,k}, W_{2,k}\}$ are determined by the boundary conditions (29-32) for given default boundaries $\left\{x_D^G, x_D^B\right\}$.

Before investment, for $x < I_1$, we can solve for $e_G(x)$ the same way as for $E_G(x)$. The particular soution is $a_1^G x + b_1^G$, where

$$a_1^G = \frac{(1-\tau)\,y_G}{r_G + \widetilde{\lambda}_G - \widetilde{\mu}_G}, \tag{58}$$

$$b_1^G = -\frac{(1-\tau)\,c}{r_G + \widetilde{\lambda}_G}. \tag{59}$$

Similarly, for $x < I_2$, the particular solution will be in the form $\mathbf{a}_2 x + \mathbf{b}_2$, where

$$\mathbf{a}_2 = (1-\tau)\left(\mathbf{r} - \widetilde{\mu} - \widetilde{\mathbf{\Lambda}}\right)^{-1}\mathbf{y}, \tag{60}$$

$$\mathbf{b}_2 = -(1-\tau)\,c\left(\mathbf{r} - \widetilde{\mathbf{\Lambda}}\right)^{-1}\mathbf{1}. \tag{61}$$

Finally, for $x < I_3$, the solution to the homogeneous equation in (35) is

$$e_B(x) = w_{3,1}x^{\gamma_1} + w_{3,2}x^{\gamma_2},$$

where

$$\gamma_1, \gamma_2 = -\sigma^{-2}(L)\left[\left(\widetilde{\mu}_B - \frac{\sigma_B^2}{2}\right) \pm \sqrt{\left(\widetilde{\mu}_B - \frac{\sigma_B^2}{2}\right)^2 + 2r_B\sigma_B^2}\right], \tag{62}$$

and we can verify that the coefficients for the particular solution satisfy:

$$a_3^B = \frac{(1-\tau)\, y_B + \widetilde{\lambda}\,(L)\, A_2^G}{r_B + \widetilde{\lambda}\,(L) - \widetilde{\mu}_B}, \tag{63}$$

$$b_3^B = \frac{-(1-\tau)\, c + \widetilde{\lambda}_B \left(B_2^G - \phi\right)}{r_B + \widetilde{\lambda}\,(L)}, \tag{64}$$

$$\omega_k = \frac{\widetilde{\lambda}_B W_{2,k} g_k^G}{r_B + \widetilde{\lambda}_B - \widetilde{\mu}_B \beta_k - \frac{1}{2}\sigma_B^2 \beta_k \left(\beta_k - 1\right)}, \quad k = 1, 2 \tag{65}$$

$$\omega_3 = \omega_4 = 0. \tag{66}$$

The coefficients $\{w_{1,k}, w_{2,k}\}$ are determined by the boundary conditions (39-44) for given default and investment boundaries $\left\{x_d^G, x_d^B, x_u^G, x_u^B\right\}$, which leads to a system of linear equations that can be solved in closed form.

B The Stochastic Discount Factor

The stochastic discount factor has its roots in macro asset pricing models. One way to generate the SDF process (15) is through the long-run risk model of Bansal and Yaron (2004). Chen (2009) (Proposition 1) shows that when the representative agent has recursive preferences and aggregate consumption process follows:

$$\frac{dC_t}{C_t} = \theta_C\,(s_t)\, dt + \sigma_C\,(s_t)\, dW_t, \quad s_t = G, B \tag{67}$$

we obtain the SDF as in (15).

However, this SDF is not tied to long run risk models. A number of other consumption-based models can also generate SDFs in this form, even if aggregate consumption is a random walk. For example, consider a model where the SDF is given by

$$m_t = e^{-\rho t} C_t^{-\gamma} S_t,$$

where C_t is aggregate consumption, and S_t is a slow-moving business cycle-related variable that affects the marginal value of wealth. Different theories provide different microeconomic foundations for S_t (see Cochrane (2008) for a survey), but for our purpose it suffices to assume that

$$\begin{aligned}\frac{dC_t}{C_t} &= \mu dt + \sigma dW_t, \\ \frac{dS_t}{S_t} &= \mu_{S,t} dt + \sigma_{S,t} dW_t + \delta_G(s_t)\left(e^{\kappa} - 1\right) dM_t^G + \delta_B(s_t)\left(e^{-\kappa} - 1\right) dM_t^B.\end{aligned}$$

Thus, S_t contains both a fast moving component (with shocks generated by the Brownian motion) and a slow-moving component modeled by the jump terms. Then, Ito's lemma implies

$$\frac{dm_t}{m_t} = -r(s_t)\,dt - \eta(s_t)\,dW_t^m + \delta_G(s_t)\left(e^{\kappa} - 1\right) dM_t^G + \delta_B(s_t)\left(e^{-\kappa} - 1\right) dM_t^B,$$

with $r(s_t)$ and $\eta(s_t)$ defined accordingly.

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